

Margaret L. Franquist

Monday

Tues.

Fri.

BUSINESS ARITHMETIC

BY

PRESTON E. CURRY, B. S. in Econ.

and

VICTOR M. RUBERT, B. C. S.

*FOR USE IN ALL SCHOOLS OFFERING THE
SUBJECT OF BUSINESS
ARITHMETIC*



SOUTH-WESTERN PUBLISHING COMPANY
CINCINNATI, OHIO

Copyright, 1922

SOUTH-WESTERN PUBLISHING CO.

Cincinnati, Ohio

PREFACE

The purpose of this text is to teach Arithmetic needed by the business man in the performance of business transactions. It assumes that the student has learned the fundamental principles of Arithmetic and is intended for use only in those schools offering a course in Business Arithmetic. However, practical drills in the fundamental operations are given for review purposes that greater skill on the part of the pupil may be developed in these fundamentals.

The authors have undertaken to make the text interesting and instructive by:

- (1) Selecting and arranging practical and interesting material.
- (2) The elimination of all extraneous matter.
- (3) A change in the approach to the various subjects.

The text has been addressed to the pupil, such approach being made as will make the subject of Arithmetic interesting. Problems have been selected that grow out of the everyday activities of school, home, society and business. Special care has been given to methods. The authors' aim has been to eliminate all unnecessary figures and thus save both time and labor. Where more than one method could be used, the figures have been shown side by side for comparison and discussion. Important features have been emphasized, those less important treated briefly, and all matter considered of very little use to the business man has been eliminated. Abundant drills in practical business problems are given so that the pupil may acquire skill and accuracy in making the detailed calculations of a business office. Machines are often used for the detail work, but since the machines are not always available, the pupil should be taught to handle figures rapidly. It is a known fact that a person who handles figures well can operate a machine with more ease and rapidity.

Emphasis is placed on the relation of numbers, as shown in COMMON and DECIMAL FRACTIONS and in percentage, special care being taken to show the interrelation of these three studies. Mental work is emphasized throughout. Many oral exercises are given preceding, and in conjunction with, written drills.

The subjects of Percentage, Profit and Loss, Interest, Stocks and Bonds, Investments, etc., are approached from the standpoint of accounting, business administration, and economics. This is an effort to properly correlate Arithmetic with these subjects. Authorities on these subjects were consulted freely.

The authors also are indebted to many business men and teachers who have not only given valuable suggestions, but in many instances have given material aid. Special acknowledgment is due M. E. Studebaker of the State Normal School, Muncie, Indiana, J. J. Starner of East High School, Cincinnati, Ohio, and W. W. Kennedy of Heald's Business College, Oakland, California, who read the manuscript and gave many helpful suggestions.

CONTENTS

CHAPTER I—PAGE 9

Introduction

- (a) Origin and Development of Numbers and Measurements.
- To the Pupil*
 - (a) Memory, Reasoning and Imagination in the Study of Arithmetic.
 - (b) Its Cultural and Utilitarian Values.

CHAPTER II—PAGE 13

Facility Drills in the Fundamental Operations

- (a) 27 Drills for Speed and Accuracy.

CHAPTER III—PAGE 36

Special Drills

- (a) Business Abbreviations.
- (b) Reading and Checking Numbers.
- (c) Making Change.
- (d) Averages, Modes and Medians.
- (e) Cash Account.
- (f) Short Cuts in Multiplication.
- (g) Aliquot Parts.
- (h) Chain Discounts.
- (i) Lumber Problems.
- (j) Pay Rolls.
- (k) Review Problems.
- (l) Principles of Bookkeeping.
- (m) Arithmetic of a Set of Books.
- (n) Bookkeeping Exercise.
- (o) Exercises in Making Statements.
- (p) General Problems.
- (q) Review Questions on Fundamental Principles.

CHAPTER IV—PAGE 77

Common Fractions

- (a) Introduced and Developed.
- (b) Reduction of Fractions.
- (c) Axioms for Fractions.
- (d) Addition of Fractions.
- (e) Common Denominator.
- (f) Least Common Multiple.
- (g) Greatest Common Divisor.
- (h) Multiplication of Fractions.
- (i) Mixed Numbers.
- (j) Subtraction of Fractions.
- (k) Multiplication of Fractions.
- (l) Division of Fractions.
- (m) Fractional Relations.

CHAPTER V—PAGE 98

Common Fractions—Continued

- (a) Principles of Fractions Applied.
- (b) Use of the Equation.
- (c) Approximating Results.
- (d) Drill in the Use of the Equation.
- (e) Aliquot Parts.

CHAPTER VI—PAGE 105

Common Fractions—Continued

- (a) General Problems.
- (b) 50 Oral, 114 Written.
- (c) Questions for Review.

CHAPTER VII—PAGE 121

Decimal Fractions

- (a) Introduced and Developed.
- (b) How to Use Decimals.
- (c) How to Extend Decimals.
- (d) Addition of Decimals.
- (e) Subtraction of Decimals.
- (f) Multiplication of Decimals.
- (g) Division of Decimals.
- (h) Axioms for Decimals.
- (i) Review Questions on Fractions.
- (j) Equivalent Fractions, Decimal and Common.

CHAPTER VIII—PAGE 130

Decimal Fractions—Continued

- (a) General Problems.
- (b) 90 Oral, 70 Written.
- (c) 50 Problems Reviewing Common and Decimal Fractions.

CHAPTER IX—PAGE 141

Percentage

- (a) Introduced and Developed.
- (b) Its Application Explained.
- (c) Aliquot Parts of 100 %.

CHAPTER X—PAGE 146

Percentage—Continued

- (a) 10 Groups Each of Oral and Written Problems.
- (b) Percentage and Its Applications Compared.
- (c) Trade Discount.
- (d) Methods of Figuring Discounts Compared.
- (e) Special Applications of Fractions and Percentage Through Billing.
- (f) Problems in Billing.
- (g) Commission.
- (h) Commission Problems.

CHAPTER XI—PAGE 171

Applications of Percentage—Continued

- (a) Some of the Problems of a Merchant.
- (b) Turnover.
- (c) Profit on Cost or Sales, which?
- (d) Problems.
- (e) Marking Goods.
- (f) Problems in Marking Goods.
- (g) General Review Questions on Percentage.

CHAPTER XII—PAGE 183

Graphs

- (a) Explained and Developed.
- (b) 10 Illustrations
- (c) How to Construct a Good Graph.
- (d) 12 Drills in Constructing Graphs.

CHAPTER XIII—PAGE 199

Interest

- (a) Its Use Explained.
- (b) Problems to Illustrate Its Use.
- (c) Table of Legal Rates.
- (d) 5 Kinds of Interest Compared.
- (e) Simple Interest Tables.
- (f) Bankers 60-day Interest Method Developed.
- (g) Cancellation Method.
- (h) Interest Problems.
- (i) Interest Operations Reversed.
- (j) Compound and Annual Interest Problems.
- (k) Compound Interest Tables.
- (l) Partial Payments.
- (m) Partial Payment Problems.

CHAPTER XIV—PAGE 229

Borrowing Capital

- (a) Present Worth Application of Interest.
- (b) Principles Applied.
- (c) Trade Discounts.
- (d) Problems Illustrating Present Worth.
- (e) Present Worth on Compound Interest Basis.
- (f) Notes, Drafts, and Trade Acceptances Illustrated.
- (g) Questions on Bank Discount.
- (h) Procedure in Figuring Bank Discount.
- (i) Problems in Bank Discount.

CHAPTER XV—PAGE 242

Bookkeepers' Problems in Settling Accounts

- (a) Ascertaining Balance Due.
- (b) Illustrations and Problems.
- (c) Average Date.
- (d) Questions and Problems.
- (e) Partnership Settlements.
- (f) Illustrations and Problems.

CHAPTER XVI—PAGE 254

Business Ownership and Investments

- (a) Advantages and Disadvantages of Forms of Business Organizations discussed.
- (b) How Each is Organized.
- (c) Problems Growing out of Proprietorship.
- (d) Stock and Bond Investments.
- (e) Stock and Bond Problems.
- (f) Insolvency and Bankruptcy.
- (g) Problems in Insolvency and Bankruptcy.
- (h) Questions for Discussion.

CHAPTER XVII—PAGE 277

Denominate Numbers

- (a) Principles and Operations Discussed.
- (b) Review Questions.
- (c) 5 Groups of Problems.
- (d) Measurement Tables.
- (e) Value of Foreign Coins.

APPENDIX—PAGE 292

- (a) Metric Measurement Tables and Equivalents.
- (b) Machines of Aid to the Business Man in Simplifying Arithmetical Calculations.
- (c) Machines Illustrated and Discussed as to Operation and Use.

BUSINESS ARITHMETIC

CHAPTER I

ORIGIN AND DEVELOPMENT OF NUMBERS AND MEASUREMENTS

As soon as primitive man began to compare one group of objects with another group he began to observe and think in numbers. Very soon, then, he sought to express his number-thoughts, and having neither words nor figures for the purpose he very naturally used his fingers and toes, or made *marks, scores, just* as the baseball umpire of today uses his fingers to indicate *strikes* and *balls* and keeps the *score* by *tallies*; thus, ~~IIII~~ III (=8)

The naming of each unit in a succession probably came next. Counting naturally developed along the *ten* or *decimal* scale. If man had had twelve fingers instead of ten it is quite likely that all counting would go in twelves instead of tens.

The greatest advance step in Arithmetic came with the selection of signs for the different groups, and the perfection of our present system of writing and reading numbers.

Many explanations have been offered concerning the origin of the ten digits*, figures, but the following appears to be the most probable, and is quite acceptable. Certain arbitrary forms were selected, and their values known by the number of angles shown.

0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9

*Digits originally meant fingers.

It is easy to understand how these awkward angular forms came to be written as they are today. Some of the old ear-marks still remain, however, for we often see the figure *one* with its flag at the top, and many of the latin peoples still flag the staff of the *seven*.

The idea of *place values* was a very remarkable discovery and has meant much for civilization. That a 2 in the first position should mean two; in the second position twenty, etc., seems simple to us today, but think how long it would take you to develop the plan for yourself.

Having so far mastered the problem of counting and numbering, man turned to the comparison of distance. He found it necessary to have a unit of measure as a basis, and having nothing so unvarying or so handy as his own body, he used *it*. Distance, he measured in foot-lengths, *feet*; or in step-lengths, *paces*; in hand-breadths, *hands*; or hand-reaches, *spans*; in elbow-to-finger-tips, *cubits*; or finger-tip-to-finger-tips, *fathoms*.

From time to time, standards of measurement have been set up as the progress of civilization required. *Area* began to be measured when land became scarce; *capacity*, when drink other than water came into use, and *weight* when precious metals and other commodities were exchanged in trade. The last came slowly, however; only in very recent years have fruits been sold by the pound.

It is interesting to notice the development of measurements. Today we measure the *air* pressure and determine altitudes, its density, and predict the weather; we measure the temperature, and the speed of the wind—in fact the speed of everything that moves. *Power*, too, is measured; the word *horse-power* stands as a monument to the passing horse; steam has its *pounds-pressure* and electricity its *volts*, *amperes*, *ohms* and *watts*.

Measuring has become so nearly a fine art that variations of more than a thousandth part of an inch in machine and tool manufacturing are practically out of the question. Mechanical devices for making these measurements have been invented, also numerous machines for the common computations of business. The automatic reel has replaced the old yardstick at the

dry goods counter; the automatic measuring pump fills the gasoline tank, and the adding-calculating machine does all the detail figuring of the office.

TO THE PUPIL

Fundamentally, Arithmetic consists of but three general processes: the *increasing*, *diminishing*, and *comparison* of numbers.

The first has to do with *addition* and *multiplication*; the second with *subtraction* and *division*. These are the four fundamental operations. Except for the small demands made upon the memory, they are purely mechanical, and can only be mastered by repetition.

The *comparison* of numbers, with which the real study of Arithmetic begins, has to do with fractions—both common and decimal—percentage, ratio and proportion.

In comparing numbers, the ability to reason is necessary. It is often noticed that pupils especially good in the four fundamentals fail utterly in the processes of comparison. Tell them what to do, and they will do it quickly and accurately, but they cannot determine for themselves the course of reasoning necessary to a solution of the problem. The mastery of mathematics has a distinct practical value, in that it develops the ability to reason accurately, and where it is possible for the pupil to obtain the problem-data by research, measurement or observation and then to reach a solution, the study becomes productive of a distinct educational value. This is especially true of work in *architectural* and *mechanical drawing*, *accounting* and *graphing*.

Memory should be developed only in connection with the meaning of arithmetical signs and terms, the multiplication and addition tables, the tables of denominate numbers and the fixed ratios existing between diameter and circumference—cubic feet, bushels, etc. To memorize a process in arithmetic is a mistake. Many have done this and have passed their examinations, only to discover later that memory has cast

it all away, and left them helpless in solving the problems of life. Learn to arrive at correct results by reasoning processes, rather than by memorizing, and though years pass and memory all but fails, the processes can be redeveloped at will without reference to text or aid of a teacher.

Memory has its place, but those who depend on memory alone cannot hope to go far in Mathematics; they may do very well, however, in Spelling, Geography, History, or Languages. Those who are of a reasoning turn of mind are generally successful in Mathematics, Logic, Philosophy and the General Sciences. Imagination is generally associated with the fine arts; it has a distinct value, however, in other studies, especially where constructive work is required. Without it there is no originality and very little initiative. Practical reasoning, coupled with an active, wholesome imagination, and both waited upon by a good memory, make for real personal power. You should endeavor to develop all three.

Constructive Arithmetic depends largely upon a vivid *imagination*. Before the pupil can satisfactorily figure on the material needed for any particular piece of work, he must be able to visualize it, to build it in his own mind, just as it is to appear in reality later. Pupils have too often overlooked this important feature in the study of Arithmetic.

If Arithmetic is dry and uninteresting, something is wrong. This lack of interest is generally due to the fact that you fail to make a practical application of the principles you have learned from day to day. Why not apply what is learned to the many problems daily presented in papers and magazines. Your aim should be not only to develop an appreciation of Arithmetic, skill in reasoning and in the use of figures, but also a habit of making careful observations, measuring accurately, discovering and stating problems for yourself. To know *when* to figure and what data will be necessary is more important than simply knowing *how*.

CHAPTER II

FACILITY DRILLS IN THE FUNDAMENTAL OPERATIONS

Brief daily drills in the fundamental operations, *Addition*, *Subtraction*, *Multiplication* and *Division*, are here provided for. These should be entered into with spirit and determination.

Knowing how to figure is not enough for a business man. *He must be able to get his results quickly and easily.* The shortest way is generally the best way to figure, but the result must be correct. The practical short cuts here given should be mastered. Only those short cuts are given that are simple, practical and used frequently. As future accountants and business men and women you should master them. It is not enough to memorize them; the principles on which the operations are based must be understood.

The work presented in these twenty-seven *facility drills* is intended as a review of the work done in the lower grades. A few minutes each day devoted to these exercises will develop power and speed. Many of the failures in advanced Arithmetic classes are due to inaccuracies and slowness in these fundamentals. Enter heartily into these drills, strive for leadership—study how to save a *second* of time and win over your competitor. Speed developed here will make for quick thinking in your other studies, too; just as a slow going bicycle wobbles and falls, so the slow thinking pupil generally meets defeat and failure. Don't say, "*I know how to add,*" but *drill, drill, drill*, until adding 150 figures a minute is but "good sport" for you. The joy of life comes through mastery. That means progress.

Use paper and pencil only when it is absolutely necessary. Use your head more and your pencil less; then both will be *Eversharps*.

(1)

DRILLS IN THE FUNDAMENTAL OPERATIONS

Two-figure combinations:

1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1
—	—	—	—	—	—	—	—	—

2	3	4	5	6	7	8	9
2	2	2	2	2	2	2	2
—	—	—	—	—	—	—	—

3	4	5	6	7	8	9
3	3	3	3	3	3	3
—	—	—	—	—	—	—

4	5	6	7	8	9
4	4	4	4	4	4
—	—	—	—	—	—

5	6	7	8	9
5	5	5	5	5
—	—	—	—	—

6	7	8	9
6	6	6	6
—	—	—	—

7	8	9
7	7	7
—	—	—

8	9
8	8
—	—

9
9
—

By combining the nine digits from 1 to 9 inclusive, we have 45 two-figure combinations as here shown.

These 45 two-figure combinations should be mastered.

Do not *spell* them out; *read* your results. When we first began to read we said b-o-y-s, *boys*, r-u-n, *run*. Now we read much faster for we have stopped spelling the words. Modern education teaches reading by a study of words and sentences. Why not learn to add in the same way?

Why say "2 and 3 are 5 and 4 are 9," when it is just as easy to see that 2, 3, 4 are 9 as it is to see that run is *run*. *Read your results*.

There are also 900 possible combinations in adding any of the numbers, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to numbers from 1 to 100. Of course, these include the two-figure, 45 combinations given above. These 900 combinations, if practiced diligently, will be of great aid in rapid, accurate addition.

The following drill is suggested and should be practiced from day to day by extending the drill to cover the numbers to 9.

1. Begin with 1 and 2 and count by 2's to 100.
2. Begin with 1, 2 and 3 and count by 3's to 100.
3. Begin with 1, 2, 3 and 4 and count by 4's to 100.
4. Begin with 1, 2, 3, 4 and 5 and count by 5's to 100.
5. Begin with 1, 2, 3, 4, 5 and 6 and count by 6's to 100.
6. Begin with 1, 2, 3, 4, 5, 6 and 7 and count by 7's to 100.
7. Begin with 1, 2, 3, 4, 5, 6, 7 and 8 and count by 8's to 100.
8. Begin with 1, 2, 3, 4, 5, 6, 7, 8 and 9 and count by 9's to 100.

(2)

READING AT SIGHT

Any number plus each of the digits.

<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>10</u>	<u>10</u>	<u>10</u>	<u>10</u>
2	8	3	6	9	5	7	4	2	8	3	6
<u>10</u>	<u>10</u>	<u>10</u>	<u>10</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>	<u>11</u>
9	5	7	4	2	8	3	6	9	5	7	4
<u>12</u>	<u>12</u>	<u>12</u>	<u>12</u>	<u>12</u>	<u>12</u>	<u>12</u>	<u>12</u>	<u>13</u>	<u>13</u>	<u>13</u>	<u>13</u>
2	8	3	6	9	5	7	4	2	8	3	6
<u>13</u>	<u>13</u>	<u>13</u>	<u>13</u>	<u>14</u>	<u>14</u>	<u>14</u>	<u>14</u>	<u>14</u>	<u>14</u>	<u>14</u>	<u>14</u>
9	5	7	4	2	8	3	6	9	5	7	4
<u>15</u>	<u>15</u>	<u>15</u>	<u>15</u>	<u>15</u>	<u>15</u>	<u>15</u>	<u>15</u>	<u>16</u>	<u>16</u>	<u>16</u>	<u>16</u>
2	8	3	6	9	5	7	4	2	8	3	6
<u>16</u>	<u>16</u>	<u>16</u>	<u>16</u>	<u>17</u>	<u>17</u>	<u>17</u>	<u>17</u>	<u>17</u>	<u>17</u>	<u>17</u>	<u>17</u>
9	5	7	4	2	8	3	6	9	5	7	4
<u>18</u>	<u>18</u>	<u>18</u>	<u>18</u>	<u>18</u>	<u>18</u>	<u>18</u>	<u>18</u>	<u>19</u>	<u>19</u>	<u>19</u>	<u>19</u>
2	8	3	6	9	5	7	4	2	8	3	6
<u>19</u>	<u>19</u>	<u>19</u>	<u>19</u>	<u>20</u>	<u>20</u>	<u>20</u>	<u>20</u>	<u>20</u>	<u>20</u>	<u>20</u>	<u>20</u>
9	5	7	4	2	8	3	6	9	5	7	4
<u>21</u>	<u>21</u>	<u>21</u>	<u>21</u>	<u>21</u>	<u>21</u>	<u>21</u>	<u>21</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>22</u>
6	2	8	3	9	5	7	4	2	7	3	9
<u>22</u>	<u>22</u>	<u>22</u>	<u>22</u>	<u>23</u>	<u>23</u>	<u>23</u>	<u>23</u>	<u>23</u>	<u>23</u>	<u>23</u>	<u>23</u>
4	5	6	8	8	9	2	5	7	4	6	3

Have pupils make up this table to 100 and practice adding.
Too much time cannot be devoted to this drill.

(3)

EXERCISES IN THE 45 TWO-FIGURE COMBINATIONS

2	9	3	5	6	5	9	2	2	3	4	9
2	9	2	5	7	1	1	8	7	7	7	1
8	4	6	3	8	2	2	7	6	2	6	8
1	8	7	3	7	6	9	1	1	6	3	5
7	4	4	2	9	7	8	9	5	5	8	6
2	6	2	9	8	6	1	4	9	1	2	4
4	3	8	8	4	5	3	3	4	9	7	7
8	5	2	1	1	4	9	8	8	6	5	2
—	—	—	—	—	—	—	—	—	—	—	—
5	4	6	8	4	9	2	5	7	9	7	6
8	1	4	9	5	9	2	6	3	4	3	9
4	7	3	7	3	8	5	9	4	7	4	7
7	9	5	8	4	8	5	8	5	2	5	3
3	6	2	6	2	7	4	4	6	5	2	8
6	8	4	7	3	7	4	5	4	5	7	6
2	5	1	5	1	6	3	8	8	3	5	3
5	7	3	6	2	6	1	3	2	7	5	7
—	—	—	—	—	—	—	—	—	—	—	—

Continue drilling on above until speed in group adding is obtained.

(4)

RAPID MENTAL PROGRESSION DRILLS

Count by 6's from 0 to 100; 6, 12, 18, etc.

Count by 9's from 0 to 100;

Count by 11's from 0 to 100;

Count by 15's from 0 to 100;

Count by 7's from 2 to 100;

Count by 9's from 3 to 100;

Count by 6's from 5 to 100;

Count by 8's from 7 to 100;

Count by 7's from 5 to 100;

Count by 8's from 3 to 100;

Count by 4's from 2 to 100;

Count by 12's from 5 to 100;

(5)

SUBTRACTION UPSIDE DOWN

Subtract without rewriting the problems:

49	46	13	29	36	17	33	89	76	37
<u>74</u>	<u>98</u>	<u>24</u>	<u>87</u>	<u>41</u>	<u>25</u>	<u>67</u>	<u>27</u>	<u>25</u>	<u>85</u>
49	41	77	25	75	21	34	29	72	32
<u>94</u>	<u>73</u>	<u>83</u>	<u>78</u>	<u>98</u>	<u>93</u>	<u>91</u>	<u>68</u>	<u>27</u>	<u>49</u>
56	45	29	87	78	48	45	28	37	25
<u>37</u>	<u>24</u>	<u>85</u>	<u>43</u>	<u>56</u>	<u>97</u>	<u>94</u>	<u>51</u>	<u>73</u>	<u>49</u>
84	41	37	72	98	37	72	18	73	45
<u>99</u>	<u>23</u>	<u>46</u>	<u>37</u>	<u>45</u>	<u>69</u>	<u>91</u>	<u>85</u>	<u>98</u>	<u>67</u>

(6)

TAKING FIGURES AS YOU FIND THEM

The following exercises will aid in rapid reading of figures.
Read results rapidly. First add and then subtract:

19	73	26	12	32	17	21	68	48	24
<u>7</u>	<u>9</u>	<u>13</u>	<u>4</u>	<u>25</u>	<u>11</u>	<u>4</u>	<u>53</u>	<u>11</u>	<u>12</u>
16	21	23	17	39	28	29	52	43	25
<u>9</u>	<u>47</u>	<u>14</u>	<u>8</u>	<u>51</u>	<u>11</u>	<u>9</u>	<u>9</u>	<u>6</u>	<u>6</u>
11	79	89	19	28	53	28	14	37	29
<u>5</u>	<u>5</u>	<u>11</u>	<u>6</u>	<u>23</u>	<u>11</u>	<u>7</u>	<u>8</u>	<u>22</u>	<u>12</u>
13	47	64	22	67	74	27	15	86	46
<u>7</u>	<u>96</u>	<u>11</u>	<u>5</u>	<u>4</u>	<u>5</u>	<u>8</u>	<u>6</u>	<u>9</u>	<u>11</u>
18	84	76	23	73	19	72	76	41	77
<u>9</u>	<u>7</u>	<u>11</u>	<u>11</u>	<u>14</u>	<u>46</u>	<u>95</u>	<u>13</u>	<u>28</u>	<u>39</u>
29	74	87	17	82	63	22	56	75	81
<u>13</u>	<u>25</u>	<u>36</u>	<u>15</u>	<u>57</u>	<u>19</u>	<u>13</u>	<u>39</u>	<u>35</u>	<u>57</u>

(7)

RAPID SUM READING

Practice adding the left column first, then the right, and as you add write the result:

67	75	29	35	72	37	26	71	88	36
<u>9</u>	<u>8</u>	<u>15</u>	<u>65</u>	<u>18</u>	<u>24</u>	<u>37</u>	<u>19</u>	<u>15</u>	<u>25</u>
47	87	94	25	72	39	35	66	43	27
<u>14</u>	<u>12</u>	<u>12</u>	<u>13</u>	<u>26</u>	<u>36</u>	<u>92</u>	<u>41</u>	<u>44</u>	<u>14</u>
44	57	37	59	52	41	21	31	62	91
<u>42</u>	<u>52</u>	<u>96</u>	<u>51</u>	<u>82</u>	<u>67</u>	<u>35</u>	<u>84</u>	<u>74</u>	<u>98</u>
57	67	91	66	72	44	75	73	95	25
<u>21</u>	<u>51</u>	<u>32</u>	<u>64</u>	<u>75</u>	<u>35</u>	<u>91</u>	<u>48</u>	<u>24</u>	<u>36</u>
16	35	14	22	17	17	12	9	4	3
<u>14</u>	<u>12</u>	<u>13</u>	<u>26</u>	<u>19</u>	<u>6</u>	<u>15</u>	<u>10</u>	<u>8</u>	<u>15</u>
59	59	21	99	70	14	5	12	16	12
<u>19</u>	<u>38</u>	<u>75</u>	<u>66</u>	<u>36</u>	<u>9</u>	<u>46</u>	<u>10</u>	<u>5</u>	<u>2</u>
4	2	15	14	12	12	8	25	45	26
<u>6</u>	<u>12</u>	<u>7</u>	<u>10</u>	<u>4</u>	<u>14</u>	<u>11</u>	<u>9</u>	<u>12</u>	<u>14</u>
15	14	15	82	109	73	62	11	13	25
<u>9</u>	<u>6</u>	<u>5</u>	<u>8</u>	<u>10</u>	<u>27</u>	<u>38</u>	<u>89</u>	<u>15</u>	<u>37</u>
78	41	58	38	9	37	12	15	98	47
<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>	<u> </u>

(8)

Add and subtract as you find them:

(Do not rewrite the problems.)

6491	9187	5793	3465	1567	7756	9756
<u>3129</u>	<u>7819</u>	<u>9375</u>	<u>4478</u>	<u>9193</u>	<u>7577</u>	<u>6579</u>
4578	2378	5178	1357	3692	5050	7418
<u>5739</u>	<u>4531</u>	<u>3546</u>	<u>7531</u>	<u>2963</u>	<u>7505</u>	<u>8147</u>
9876	8434	4123	4578	9823	7189	8656
<u>6789</u>	<u>7431</u>	<u>3167</u>	<u>3955</u>	<u>9831</u>	<u>5790</u>	<u>6468</u>

(10)

CONTINUED DRILL IN GROUP ADDITION

Continue the grouping into longer problems. You will be surprised at your increased speed with earnest practice.

Example:

$$5 \left\{ \begin{array}{l} 47 \\ 13 \end{array} \right\} 10$$

$$5 \left\{ \begin{array}{l} 25 \\ 34 \end{array} \right\} 9$$

$$\begin{array}{r} 19 \\ 10 \\ \hline 119 \end{array}$$

$$10 \left\{ \begin{array}{l} 84 \\ 26 \end{array} \right\} 10$$

$$10 \left\{ \begin{array}{l} 39 \\ 71 \end{array} \right\} 10$$

$$10 \left\{ \begin{array}{l} 44 \\ 63 \end{array} \right\} 7$$

In adding this problem you should say $10 + 10 + 7 = 27$. Write down your 7 and carry two and begin reading with 12 + 10 + 10.

85	17	19	22	36	77	48	71	65	75	92	82
13	28	18	88	98	18	27	37	27	84	34	25
26	34	46	36	27	46	34	53	84	21	56	31
85	13	83	14	83	12	92	21	38	72	27	48
67	92	24	97	49	24	75	84	18	38	72	62
31	85	17	14	82	63	67	96	22	56	49	37
16	31	15	35	78	87	43	41	36	24	31	49

6472	2894	2837	1927	8159	7827	8926
3895	3612	4017	3416	3926	1945	4652

7819	1347	9967	4956	1923	1627	9706
6026	8663	1738	5427	4175	9345	4908

1763	4917	8172	6206	3674	1964	8815
3249	3294	1825	6293	7014	3745	1967

4724	8291	1836	7295	1316	8157	7935
1965	7569	9214	5275	9419	1989	7627

(11)

WRITTEN DRILL

After a little practice you should add 100 figures per minute. Therefore, you should add the following in about 15 seconds each.

1	2	3	4	5	6	7	8
326	726	317	174	428	942	715	257
432	398	243	316	747	163	168	533
515	407	815	175	215	206	206	956
679	259	912	815	164	721	721	128
248	117	748	994	928	832	832	314
143	243	463	375	314	415	415	725
724	836	109	862	727	637	637	193
129	375	955	181	136	318	318	295

9	10	11	12	13	14	15	16
517	249	752	321	213	613	234	158
861	424	944	234	431	132	311	206
912	375	407	231	524	235	532	891
383	242	367	423	413	443	131	185
559	573	134	435	213	214	321	215
602	703	357	342	321	312	212	112
638	117	209	543	432	424	135	235
837	317	592	312	361	422	513	930

(12)

SMALL NUMBER GROUPINGS

After a little practice all small numbers can be just as easily grouped in three's, thereby increasing the speed in addition. Read, and time for speed and accuracy.

3	2	2	4	4	3	3	5	1	4	2	1
2	3	3	2	3	4	1	4	3	3	1	3
1	4	1	3	5	2	2	3	2	2	3	1
2	3	4	3	5	4	2	2	2	6	3	4
1	2	1	1	1	2	4	1	3	1	6	3
2	4	2	5	2	2	5	5	5	2	1	1

(13)

GROUPING DRILL

Group in three figures and time. The larger the groups you make, the faster and easier you add.

		<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>
1	6	3	3	4	2	5	5	2	6	1	2
3		4	2	1	3	1	3	1	2	2	3
2		2	1	3	4	2	3	3	1	3	4
4	10	6	2	3	2	4	4	4	1	4	2
5		2	3	1	6	2	1	3	3	1	3
1		1	4	2	1	2	5	1	4	6	1
2	9	3	2	4	5	2	6	5	1	2	2
3		1	3	2	3	1	4	2	3	1	1
4		2	1	3	1	5	2	4	1	3	3
1	8	4	4	1	4	2	6	4	3	5	2
2		2	2	3	3	3	1	1	1	1	4
5		3	3	2	2	5	3	3	6	3	3
6	9	5	3	4	1	6	1	4	1	1	3
1		1	4	4	3	1	2	3	6	2	2
2		2	2	2	4	2	6	2	3	6	4
3	10	2	3	3	3	3	3	3	3	2	1
6		1	1	2	1	6	2	6	1	2	3
1		2	2	1	1	1	1	1	2	4	2
3	10	3	1	2	2	4	4	3	5	3	4
6		1	3	1	5	3	3	2	1	1	3
1		1	2	5	1	1	1	1	3	1	2
52		3	4	1	1	5	2	4	2	3	1
		2	3	2	2	2	1	2	1	2	2
		1	2	4	6	3	6	2	6	1	3
72	34	15	22	47	82	19	95	71	95		
38	76	26	16	42	17	15	16	28	32		
96	12	78	74	39	43	96	76	84	67		
53	35	52	15	51	26	23	34	16	14		
48	18	19	29	35	18	81	18	77	66		
13	11	12	61	14	71	27	72	32	15		

(14)

DRILL IN GROUP ADDITION

1	2	3	4	5	6	7
124	431	231	412	1244	4213	3124
216	513	431	134	3152	2531	5113
341	142	424	244	3541	4315	1234
234	353	321	123	1124	1241	6725
261	425	212	125	3225	3632	1043
514	112	324	412	6321	4016	2120
315	126	124	351	1232	6521	3216
442	451	242	113	1525	1234	5451
312	332	134	142	5342	2245	2320
135	321	413	311	2143	1412	1215
126	423	353	132	1513	1235	4101
<u>321</u>	<u>422</u>	<u>311</u>	<u>431</u>	<u>2134</u>	<u>5142</u>	<u>5363</u>

(15)

SUBTRACTION BY ADDITION

Pupils should give missing numbers rapidly:

$2 + ? = 13$	$14 + ? = 40$	$? + 12 = 28$
$4 + ? = 30$	$16 + ? = 67$	$? + 9 = 27$
$5 + ? = 30$	$4 + ? = 19$	$? + 4 = 12$
$12 + ? = 36$	$6 + ? = 63$	$? + 8 = 41$
$3 + ? = 15$	$9 + ? = 32$	$? + 14 = 22$
$6 + ? = 28$	$18 + ? = 98$	$? + 35 = 78$
$17 + ? = 47$	$8 + ? = 35$	$? + 50 = 77$
$7 + ? = 19$	$13 + ? = 30$	$? + 17 = 31$
$8 + ? = 22$	$19 + ? = 25$	$? + 16 = 28$
$9 + ? = 24$	$10 + ? = 42$	$? + 7 = 19$
$10 + ? = 36$	$3 + ? = 14$	$? + 31 = 45$
$14 + ? = 28$	$9 + ? = 71$	$? + 15 = 60$
$? + 41 = 62$	$? + 33 = 78$	$? + 25 = 50$
$? + 13 = 48$	$? + 17 = 31$	$? + 16 = 34$
$? + 37 = 44$	$? + 21 = 63$	$? + 27 = 45$
$? + 19 = 55$	$? + 18 = 37$	$? + 12 = 36$

(16)

SPEED DRILL IN EASY GROUPINGS

Teachers should step to the side of any pupil and read rapidly
 while entire class adds three sums of three figures each, thus

Just as soon as the leader-pupil has finished, read another problem,
 until five or six problems have been given. Then check for
 accuracy. Step to another pupil quickly, and repeat the drill.
 This is a wonderful enlivener and makes for speed and alertness.

Horizontal and vertical addition:

1

$$\begin{array}{r}
 3+7+2+6+10+4=? \\
 9+6+4+8+9+12=? \\
 5+14+10+3+6+8=? \\
 8+9+9+6+5+3=? \\
 3+8+3+4+7+10=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

2

$$\begin{array}{r}
 18+4+9+8+6+7=? \\
 5+14+8+3+12+5=? \\
 16+8+10+5+7+11=? \\
 9+12+6+3+5+10=? \\
 10+10+7+12+3+4=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

3

$$\begin{array}{r}
 2+9+7+8+15+4=? \\
 8+6+3+7+4+14=? \\
 5+7+12+8+3+11=? \\
 11+9+4+8+6+3=? \\
 6+3+8+2+7+10=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

4

$$\begin{array}{r}
 1+3+5+7+9+11=? \\
 12+10+8+6+4+2=? \\
 3+6+9+12+18+20=? \\
 11+9+3+7+1+2=? \\
 4+6+3+2+4+5=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

5

$$\begin{array}{r}
 13+8+5+7+1+2=? \\
 4+6+7+3+2+9=? \\
 5+9+3+10+9+1=? \\
 12+11+2+5+6+10=? \\
 7+6+4+1+2+7=? \\
 3+2+10+11+9+8=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

6

$$\begin{array}{r}
 14+3+5+7+9+8=? \\
 2+9+5+4+3+1=? \\
 10+4+7+8+11+12=? \\
 1+5+12+4+5+2=? \\
 9+13+10+9+4+3=? \\
 2+4+8+11+3+5=? \\
 \hline
 ?+?+?+?+?+?=?
 \end{array}$$

(17)

Adding money columns:

1	2	3	4	5	6	7	8
\$0.79	\$18.42	\$ 1.23	\$49.15	\$13.26	\$46.50	\$23.10	\$ 1.98
2.18	.75	.94	.37	1.50	.98	.05	10.14
1.62	1.60	6.17	2.45	9.75	1.00	4.50	1.75
19.10	.57	9.18	121.16	4.80	6.50	9.75	8.45
.32	9.63	10.10	3.40	28.75	15.50	.50	1.76
1.51	8.70	9.16	.84	1.12	7.56	8.00	1.50
.08	.15	.13	17.20	8.75	.45	.25	25.50
63.07	96.24	3.21	146.35	.05	1.75	.45	7.50
14.26	9.63	9.46	1.25	87.50	71.25	19.75	4.32
12.39	17.27	98.45	22.75	1.39	1.72	3.86	1.57
1.06	37.26	17.20	39.47	14.50	15.76	27.55	19.50
<u>2.15</u>	<u>2.37</u>	<u>3.15</u>	<u>82.34</u>	<u>23.50</u>	<u>17.06</u>	<u>18.97</u>	<u>87.55</u>

(18)

SIDE ADDITION AND PROOF OF MONEY COLUMNS

Illustration:

$$\text{\$ } 9.65 + \text{\$ } 12.40 = \text{\$ } 22.05$$

$$12.90 + 14.70 = 27.60$$

$$\underline{28.00} + \underline{62.50} = \underline{90.50}$$

$$50.55 + 89.60 = 140.15 \text{ Result proves.}$$

$$1. \quad \text{\$ } 1.20 + \text{\$ } 0.40 + \text{\$ } 2.40 + \text{\$ } 0.80 = ?$$

$$.80 + .20 + .75 + .25 = ?$$

$$.75 + .35 + .70 + 1.80 = ?$$

$$\underline{1.75} + \underline{.25} + \underline{2.50} + \underline{1.75} = ?$$

$$? + ? + ? + ? = ?$$

$$2. \quad \text{\$ } 0.70 + \text{\$ } 1.00 + \text{\$ } 2.35 + \text{\$ } 0.95 = ?$$

$$2.15 + .25 + .55 + .35 = ?$$

$$.15 + 2.10 + .65 + 1.15 = ?$$

$$\underline{.85} + \underline{.75} + \underline{.35} + \underline{.45} = ?$$

$$? + ? + ? + ? = ?$$

MULTIPLICATION

A published result of an investigation made to determine the relative portion of time spent on the four fundamental operations in bookkeeping and accounting shows approximately 94% for addition and multiplication; 53% for addition and 41% for multiplication. All of the operations should be practiced, but it is very evident that if addition and multiplication can be handled rapidly little difficulty will be experienced with subtraction and division.

The following drill combines addition and multiplication and should be practiced daily.

1. Multiply the following by 2:

$$9-4-6-3-5-8-7-2$$

2. Same as No. 1, but add 1 to product, as:

$$2 \times 9 + 1 = 19, \text{ etc.}$$

3. Multiply by 3:

$$9-4-6-3-5-8-7-2$$

4. Same as No. 3, but add 2 to the product of each.

5. Multiply by 4:

$$9-4-6-3-5-8-7-2$$

6. Same as No. 5, but add 2 and 3 to product of each.

7. Multiply by 5

$$9-4-6-3-5-8-7-2$$

8. Same as No. 7, but add 3 and 4 to product of each.

9. Multiply by 6:

$$9-4-6-3-5-8-7-2$$

10. Same as No. 9, but add 4, 5 and 6 to product of each.

11. Multiply by 7:

$$9-4-6-3-5-8-7-2$$

12. Same as No. 11, but add 3, 4, 5, 6 and 7 to product of each.

(19)

RAPID TIME ORAL DRILLS

The following problems are to be completed in class as directed by the teacher.

$9+6-5+8+3-7 \times 5 = ?$	$4 \times 5 - 10 + 14 \div 12 \times 8 + 4 = ?$
$3+7+4-8+6+2 \times 3 = ?$	$7+3 \times 5 - 14 + 4 \div 4 - 5 = ?$
$7 \times 3 - 11 + 6 - 6 + 9 \times 2 = ?$	$9+6 \div 3 \times 6 - 10 + 4 \div 12 = ?$
$12+12-4-8-4+2 \times 5 = ?$	$7+9 \div 2 \times 3 + 6 + 6 - 8 = ?$
$45-15-25+5+12-18 \times 15 = ?$	$24-4 \div 5 + 12 \div 2 \times 3 + 6 = ?$
$4+5 \div 3 \times 8 + 12 \div 9 + 8 = ?$	$17+7-4 \div 5 \times 4 \div 2 + 5 = ?$
$9 \times 5 - 15 \div 6 + 13 + 7 \div 5 = ?$	$16+14 \div 10 \times 8 + 12 + 5 = ?$
$8+10+7 \div 5 \times 9 - 15 = ?$	$14 \div 2 + 3 + 14 \div 3 + 7 - 5 = ?$

The teacher may provide similar problems for additional drill in oral calculations.

MULTIPLICATION AND DIVISION DRILL

48	2)17418240
2	3)8709120
96	4)2903040
3	5)725760
288	6)145152
4	7)24192
1152	8)3456
5	9)432
5760	48
6	
34560	
7	
241920	
8	
1935360	
9	
17418240	

Multiply 48 by each of the digits 2 to 9 inclusive, beginning with 2; then divide the product obtained by the same digits in the same order.

NOTE: Frequent drills in this exercise may be given, using any number as a starting point, and returning to it. Use two figure numbers at first, three and four figures later. This exercise can be used for figuring contests.

(20)

ADDING BY CIVIL SERVICE METHOD

To be used in adding long columns or where you may be often interrupted in your work.

Illustration: Write the total of each column, dropping one column to left each time. By adding results of each column you should arrive at the correct answer. Notice that you are not troubled with carrying figure to next column by this method.

Illustration:	1	2	3	4	5
4827	28427	18572	38725	37928	89991
9638	78492	61424	49281	41672	57287
4924	27463	37296	30427	37097	84332
3725	85927	75735	35729	25562	33333
5125	42379	82046	86428	37981	78987
6849	28456	27593	72823	27438	30051
7524	48937	41275	67129	41924	54549
3579	29452	39256	43552	27096	99948
2845	38247	52938	27849	35243	56432
2454	40524	43725	37472	17241	23458
60	27846	37296	27305	22719	76983
33	52317	28579	41275	32556	70801
61	74283	46294	28517	73892	65749
45	37208	38432	36729	25727	45290
51490	<u>52172</u>	<u>17245</u>	<u>48196</u>	<u>41273</u>	<u>98541</u>

(21)

ADDITION DRILL

Dictate to the class, problems 5 figures each way; give them just 15 seconds to add, then dictate another, etc. Keep up the addition drills until the class can add ten problems at this speed with ease and accuracy.

14789	72482	88172
76294	62372	32946
38215	81924	17139
19735	21456	97146
14649	<u>78546</u>	<u>75397</u>

(22)

DRILL IN ADDITION

Take any two numbers, add them, and set the result beneath without ruling; then add the last two of the three, and repeat the operation until you have ten numbers; then add the whole column and divide by 11. The result should be the 7th number in the column. This is excellent practice for rapid board work. Interesting contests may be developed from this exercise.

Illustration:

72	(72+184=256)
184	
256	(184+256=440)
440	etc.
696	
1136	
—1832—	
2968	
4800	
7768	
11)20152	
1832	

EXERCISES COMBINING ADDITION AND SUBTRACTION

1. You collected and disbursed the following cash items during the month. Received, \$75.20, \$126.30, \$15.27, \$196.54, \$88.19, \$275.66, \$13.23, \$78.27, \$514.16. Paid \$18.47, \$3.76, \$125.00, \$93.27, \$66.28, \$14.26, \$173.21, \$256.15. What is your cash *balance at the end of the month?

2. Your bank statement shows that you deposited the following amounts during the month: \$75.80, \$163.25, \$120.16, \$38.90, \$265.30, \$189.24 and \$16.04. You drew the following checks: \$15.25, \$4.67, \$29.80, \$68.90, \$3.64, \$9.95 and \$127.50. What is the amount of your bank balance?

*Receipt less payments will give the cash balance.

(23)

DRILL IN SUBTRACTION

Take any number 5 or 6 figures in length. From it subtract $\frac{1}{10}$ of it, using the necessary decimal. From the result subtract $\frac{1}{10}$ of it. Repeat five times until you have eleven numbers. Proof: Add the digits in your result. Use the check figure 9 proof and if the same key figure shows for the result as for the first answer it is correct.

$$\begin{array}{r}
 842635 \\
 \underline{84263.5} \\
 758371.5 = 36 - 9 - 0 \\
 \underline{75837.15} \\
 682534.35 \\
 \underline{68253.435} \\
 614280.915 \\
 \underline{61428.0915} \\
 552852.8235 \\
 \underline{55285.28235} \\
 497567.54115 = 54 = 9 = 0
 \end{array}$$

See Check Figure Proof discussion, Drill 27.

(24)

COMBINING ADDITION AND SUBTRACTION

These and similar problems can be dictated for practice. Since the results prove, the pupils know when they get the correct answer.

1

$$\begin{array}{l}
 8 + 4 + 6 + 7 + 3 - 14 = ? \\
 9 + 5 + 7 + 3 + 4 - 15 = ? \\
 10 + 7 + 8 + 7 + 9 - 24 = ? \\
 12 + 8 + 10 + 9 + 4 - 27 = ? \\
 4 + 11 + 7 + 5 + 8 - 19 = ? \\
 8 + 7 + 6 + 8 + 9 - 35 = ? \\
 \hline
 ? + ? + ? + ? + ? - ? = ?
 \end{array}$$

2

$$\begin{array}{l}
 8 + 2 + 7 + 6 + 5 - 9 = ? \\
 7 + 6 + 5 + 9 + 6 - 29 = ? \\
 3 + 12 + 6 + 7 + 7 - 21 = ? \\
 9 + 9 + 7 + 5 + 8 - 35 = ? \\
 7 + 9 + 8 + 12 + 9 - 27 = ? \\
 10 + 12 + 3 + 6 + 10 - 19 = ? \\
 \hline
 ? + ? + ? + ? + ? - ? = ?
 \end{array}$$

(25)

MULTIPLICATION DRILL

Multiply any number by one of the digits. Then multiply the result by the same digit, etc., until you have ten numbers. Then add the column. Results may be proved by check figure. Excellent contest drill.

$$\begin{array}{r}
 721 \\
 (7) \\
 \hline
 5047 \\
 35329 \\
 247303 \\
 1731121 \\
 12117847 \\
 84824929 \\
 593774503 \\
 4156421521 \\
 29094950647 \\
 \hline
 33944108247
 \end{array}$$

1

$$\begin{array}{l}
 45 \times 9 = ? \\
 16 \times 17 = ? \\
 38 \times 12 = ? \\
 92 \times 13 = ? \\
 \hline
 \end{array}$$

2

$$\begin{array}{l}
 63 \times 28 = ? \\
 16 \times 14 = ? \\
 32 \times 18 = ? \\
 19 \times 17 = ? \\
 \hline
 \end{array}$$

3

$$\begin{array}{l}
 126 \times 19 = ? \\
 153 \times 17 = ? \\
 144 \times 11 = ? \\
 135 \times 35 = ? \\
 \hline
 \end{array}$$

4

$$\begin{array}{l}
 72 \times 16 = ? \\
 85 \times 45 = ? \\
 49 \times 38 = ? \\
 76 \times 24 = ? \\
 \hline
 \end{array}$$

5

$$\begin{array}{l}
 46 \times 26 = ? \\
 94 \times 73 = ? \\
 18 \times 15 = ? \\
 27 \times 55 = ? \\
 \hline
 \end{array}$$

6

$$\begin{array}{l}
 175 \times 9 = ? \\
 115 \times 44 = ? \\
 117 \times 23 = ? \\
 199 \times 49 = ? \\
 \hline
 \end{array}$$

TIME SAVING DRILLS IN DIVISION

1. Never use long division where short division will apply. Learn to divide by 11, 12 and even larger numbers by the short division method.

$\begin{array}{r} 12)84276 \\ \hline 7023 \end{array}$ (11 figures 2 lines) Half your time saved.	not $12)84276(7023$ $\begin{array}{r} 84 \\ \hline 27 \\ \hline 24 \quad (21 \text{ figures} \\ 36 \quad 5 \text{ lines}) \\ \hline 36 \end{array}$
--	--

2. To divide by multiples of 10 point off one place or more to the left and divide by the significant figure.

To illustrate:

$$\begin{array}{r} 7246 \div 20 = 2)724 \overline{)6} \\ 362 \overline{)3} \text{ Ans.} \end{array}$$

$$\begin{array}{r} 47285 \div 50 = 5)4728 \overline{)5} \\ 945 \overline{)7} \text{ Ans.} \end{array}$$

$$\begin{array}{r} 7204626 \div 90 = 9)720462 \overline{)6} \\ 80051 \overline{)4} \text{ Ans.} \end{array}$$

$$\begin{array}{r} 49273 \div 80 = 8)4927 \overline{)3} \\ 615 \overline{)9\frac{1}{8}} \left(.9\frac{1}{8} = \frac{73}{80} \right) \end{array}$$

$$\begin{array}{r} 927256 \div 700 = 7)9272 \overline{)56} \\ 1324 \overline{)65\frac{1}{7}} \left(.65\frac{1}{7} = \frac{456}{700} = \frac{114}{175} \right) \end{array}$$

3. To change pounds to cwt., point off two decimal places to the left.

To change board feet to thousands (M) point off three places to the left.

To change pounds to tons, point off three places to the left and divide by two. (See following illustrations.)

Illustration: 484 lbs. = 4.84 cwt. or $4\frac{21}{25}$ cwt.

1725 board feet = 1.725 M. ft. or $1\frac{29}{40}$ M.

4276 lbs. = $4\overline{)276}$
 $2\overline{)138}$ tons or $2\frac{69}{100}$ tons.

4. To divide by aliquot parts of 100 or 1000:

$8400 \div 75 = ?$ $8400 \div 100 = 84\overline{)00}$
 add $\frac{1}{3} = 28\overline{) }$ ($75 + \frac{1}{3}$ of itself) = 100
 $112\overline{) }$ Ans.

$7500 \div 125 = ?$ $7500 \div 100 = 75\overline{)00}$
 Deduct $\frac{1}{5} = 15\overline{) }$ ($125 - \frac{1}{5}$ of itself) = 100
 $60\overline{) }$ Ans.

It is true of a division problem, as it is of an equation or of a fraction, that dividing or multiplying either term (or side) by the same numbers does not change the relations or value, and adding or subtracting corresponding proportionate parts does not change the relationship. In dividing by 75, we see that $\frac{1}{3}$ of it added to it makes a hundred—therefore, adding $\frac{1}{3}$ to the dividend and dividing by 100 brings the proper result.

$9600 \div 220 = ?$ $9600 \div 200 = 96\overline{)00}$ ($\frac{1}{2}$ of 96) or $9600 \div 1000 = 9\overline{)600}$
 $48\overline{) }$ $\times 5 = 48\overline{) }$
 Deduct $\frac{1}{11} \quad 4\overline{) \frac{4}{11}}$ Deduct $\frac{1}{11} \quad 4\overline{) \frac{4}{11}}$
 $43\overline{) \frac{7}{11}}$ Ans. $43\overline{) \frac{7}{11}}$

NOTE: 20 is $\frac{1}{11}$ of 220; 200 is $\frac{1}{5}$ of 1000.

By a careful comparison of numbers and the factors or parts of numbers, unusual mental skill may be easily developed. Avoid all unnecessary figures—figures saved is money saved, and time is too precious to be wasted in needless labor.

(26)

WRITTEN DRILL

- | | | |
|-------------------------|------------------------|------------------------|
| 1. $740 \div 90 =$ | 7. $6245 \div 125 =$ | 13. $824 \div 1250 =$ |
| 2. $2870 \div 800 =$ | 8. $3184 \div 110 =$ | 14. $1726 \div 750 =$ |
| 3. $76500 \div 12000 =$ | 9. $87420 \div 1200 =$ | 15. $2496 \div 1500 =$ |
| 4. $4320 \div 30 =$ | 10. $7920 \div 400 =$ | 16. $475 \div 25 =$ |
| 5. $1626 \div 75 =$ | 11. $7920 \div 440 =$ | 17. $1620 \div 1125 =$ |
| 6. $4720 \div 750 =$ | 12. $7920 \div 360 =$ | 18. $540 \div 90 =$ |

(27)

CHECK FIGURE PROOFS

Proof of addition and multiplication by the check figure 9.

Illustration:

How to get the key figure

$\$ 7425.83 = 29 = 11 = 2$

$524.25 = 18 = 9 = 0$

$31.55 = 14 = 5$

$1.26 = 9 = 0$

$385.27 = 25 = 7$

$240.04 = 10 = 1$

$7526.42 = 26 = 8$

$7.50 = 12 = 3$

$18.24 = 15 = 6$

$2500.00 = 7 = 7$

$7+4+2+5+8+3=29$

$2+9=11$

$1+1=2 \text{ or}$

dropping the 9 groups shown
above, we have $8+3=11$ and

$1+1=2$

(See Note below.)

$\$18660.36$

$39, 3+9=12 \text{ and } 1+2=3 \text{ Key figure}$

$(1+8+6+6+3+6=30=3) \text{ (or dropping the 9 we have 3)}$

$(\text{Or } 6+6=12 \text{ and } 1+2=3)$

Add the following by the Civil Service method and prove by casting out the nines as illustrated above:

1	2	3	4	5	6
72467	29715	90466	19368	29247	70214
89365	16314	81726	77409	67124	89217
86542	72199	15294	63701	13271	36199
13725	90467	77899	15167	12346	94215
88294	88165	54278	82195	61214	55317
76587	47315	78245	67215	39116	64815
38925	59146	13727	56327	98915	97228
67382	66284	49604	52168	15327	89571
72468	19377	96536	75219	81583	67909
19293	76209	85579	91317	26215	32721
86153	97158	67394	46314	19181	87588
12517	37256	18257	88175	91927	15765
79407	46629	27395	27405	15375	12915
27314	26375	38245	13697	14615	18194

NOTE: The best way to prove addition is to re-add—the check figure plan saves little time. It should be understood as it might prove useful.

PROOF BY THE CHECK FIGURE 11

Illustration:

$$\begin{array}{rcl}
 & 23^* & \\
 \$ 7425.83 = & \cancel{12} - 17 = & 6 \\
 524.25 = & 14 - 4 = & 10 \\
 31.55 = & 6 - 8 = & 9 \\
 1.26 = & 7 - 2 = & 5 \\
 385.27 = & 15 - 10 = & 5 \\
 240.04 = & 6 - 4 = & 2 \\
 7526.42 = & 13 - 13 = & 0 \\
 7.50 = & 7 - 5 = & 2 \\
 18.24 = & 12 - 3 = & 9 \\
 2500.00 = & 5 - 2 = & 3 \\
 \hline
 & 24^* & 51 = (7) \\
 \$18660.36 = & \cancel{12} - 17 = (7) & \text{Key figure}
 \end{array}$$

Then add sum of check figures in last column and divide by 11. The excess is the Key figure which should equal the excess of even figures over the odd figure in the presumed answer of the problem under proof.

*If the sum of odds should be smaller than the sum of evens add 11, then subtract.

Problems in previous lesson may be proved by check figure 11.

PROOF OF MULTIPLICATION

Multiplication

$$\begin{array}{rcl}
 7284567 = & 39 = 12 = 3 & \text{Key figure} \\
 243 = & 9 = & 0 \text{ Key figure} \\
 \hline
 21853701 & 3 \times 0 = 0 & \text{Key figure} \\
 29138268 & & \\
 14569134 & & \\
 \hline
 1770149781 = & 45 = 9 = 0 &
 \end{array}$$

EXPLANATION

Adding the digits of the multiplicand we have 39, or (by dropping the two 9 groups) $8+6+7=21$. $3+9=12$, and $2+1=3$ in both cases. 3×0 (the Key figure for the multiplier $=0$, the same as the Key figure for the product.

Proof of long problems in multiplication by this method is a decided advantage. Check figure 11 may also be used.

NOTE: By observation we discover that the sum of the figures used in expressing any multiple of nine, add to nine. $9 \times 46 = 414$, $4+1+4=9$, $9 \times 846 = 7614$, $7+6+1+4=18$ and $8+1=9$, etc.

CHAPTER III

SPECIAL DRILLS

BUSINESS ABBREVIATIONS

A.....	acre	Cr.....	creditor or credit
a/c or Acct..	account	d.....	pence
Adm. Exp....	administrative expense	da.....	day; days
amt.....	amount	dft.....	draft
ad.....	advertisement	dis.....	discount
av.....	average	dm.....	decimeter
bal.....	balance	D. M.....	dekameter
bbl.....	barrel	Dr.....	debtor; debit; doctor
bdl.....	bundle; bundles	doz.....	dozen
bg.....	bag; bags	ea.....	each
bkpr., bkpg..	bookkeeper (ing)	e. e.....	errors excepted
bkt.....	basket; baskets	e. & o. e.....	errors and omissions ex- cepted
bl.....	bale; bales	e. g.....	for example
bldg.....	building	eqpt.....	equipment
blvd.....	boulevard	etc.....	and so forth
B/L.....	bill of lading	exp.....	expense; express
Bros.....	brothers	f.....	folio or page
bu.....	bushel	far.....	farthing; farthings
bx.....	box; boxes	frt.....	freight
Cap.....	capital	f. o. b.....	free on board
cd.....	cord; cords	fr.....	franc; francs
ch.....	chain; chains	ft.....	foot; feet
c. i. f.....	carriage and insurance free	gal.....	gallon; gallons
cm.....	centimeter	gen'l.....	general
Co.....	company	gi.....	gill; gills
C. O. D.....	collect on delivery	hhd.....	hogshead; hogsheads
C. of D.....	certificate of deposit	hf. cht.....	half chest; half chests
Coll.....	collection	H. M.....	hektometer

hrs.....	hour; hours	s.....	shillings; shilling
ins.....	insurance	S. B.....	sales book
int.....	interest	sec.....	second; seconds
inv't.....	inventory	sq. ch.....	square chain
jnl. or j.....	journal	sq. ft.....	square feet
lb.....	pound	sq. mi.....	square mile
m, M !.....	meter—thousand	sq. rd.....	square rod
mm.....	millimeter	sq. yd.....	square yard
mi.....	mile; miles	st. draft.....	sight draft
min.....	minute; minutes	T.....	ton
mo.....	month; months	tb.....	tub; tubs
O. K.....	all correct	twp.....	township; townships
oz.....	ounce; ounces	via.....	by way of
p.....	page	wk.....	week; weeks
P. B.....	purchase book	wt.....	weight
pp.....	pages	yd.....	yard; yards
pc.....	piece	yr.....	year; years
pd.....	paid	✓.....	check mark
per.....	by the; by	°.....	degree
per cent....	by the hundred	'.....	foot; feet; minutes
pk.....	peck; pecks	C.....	hundred
pkg.....	package	".....	inches or seconds
P. & L.....	profit and loss	#.....	number, if written be- fore a figure; pounds if written after a figure.
pr.....	pair, pairs	1 ¹	one and one-fourth
pref.....	preferred	1 ²	one and two-fourths one and one-half
Pur. Dis....	purchase discount	1 ³	one and three-fourths
pt.....	pint; pints	%.....	per cent; hundredth; hundredths
pwt.....	pennyweight	£.....	pounds sterling
pymt.....	payment	2/10 n/30...	(2% discount if paid in 10 days; due in 30 days)
qr.....	quire; quires		
qt.....	quart; quarts		
rd.....	rod; rods		
Res. for Dep.	reserve for depreciation		
rm.....	ream; reams		
K. M.....	kilometer		

(28) .

READING NUMBERS

In the rush of business the reading of numbers is much abbreviated. Instead of *One Thousand Two Hundred and Forty-five Dollars and Fifty-six cents*, the business man or accountant says *twelve forty-five fifty-six* (\$1245.56) shortening the time over one-half and eliminating errors.

For \$12.80 he reads twelve eighty.

For 1280. he reads twelve eighty dollars.

For .80 he reads eighty cents.

For 200.60 he reads two hundred sixty.

For 138460.40 he reads one thirty-eight, four sixty, forty.

For 29207.08 he reads twenty-nine, two *oh* seven, *oh* eight.

Following this plan, have the class first read and then write from dictation the following problems. Then exchange papers and have pupils check while one of the class reads aloud.

\$1237.42	\$ 9.67	\$ 38.48	\$ 84.29
427.00	109.67	208.08	186.43
400.27	1000.00	2200.22	792.05
3.08	10967.00	2006.08	80.95
308.00	666.66	700.07	11.89

Suitable material for continued drill in checking and reading numbers may be found in this text or in any bookkeeping set.

CHECKING

When verifying items, the checkmark (✓) is used to signify *examined and passed as correct*.

If after checking, any item remains unchecked, it indicates an error or disagreement.

Never place checkmarks within the money column.

In a second checking use a different checkmark position throughout, or flag the first checkmarks by drawing a short straight line through the previous checkmarks.

(29)

MAKING CHANGE

In making change, begin with the amount of the purchase and count out enough, adding as you proceed, to equal the amount handed to you in payment. If \$2.69 is bought and \$5.00 received, say \$2.69, \$2.70, \$2.75, \$3.00, \$5.00, laying down a penny, a nickel, a quarter, and a two dollar bill. (Give coins and bills of largest amount when possible.)

EXERCISES IN MAKING CHANGE

(Teacher should supply the necessary coins and bills or use imitation currency if possible.)

1. A \$20 bill for \$7.83 sold	11. A \$5 and \$2 for \$5.12 sold
2. A 10 bill for 1.11 sold	12. A 10 and 1 for \$10.01 sold
3. A 5 bill for 4.09 sold	13. A 5 bill and 1c for \$2.51 sold
4. A 2 bill for .13 sold	14. A 20 bill and 4c for \$1.04 sold
5. A 10 bill for 1.17 sold	15. 3 10 bills for \$21.71 sold
6. A 1 bill for .48 sold	16. 4 5 bills for 17.51 sold
7. A 20 bill for 3.33 sold	17. 3 2 bills for 4.76 sold
8. A 5 bill for .43 sold	18. 8 1 bills for 7.07 sold
9. A 10 bill for 4.79 sold	19. 3 20 bills for 41.01 sold
10. A 20 bill for 1.04 sold	20. A 5 bill and a $\$ \frac{1}{2}$ for \$5 39 sold

MENTAL EXERCISES IN MAKING CHANGE

You are cashier, and customers pay you in the denominations mentioned, out of which you take the amount owed.

1. \$20.00 bill to pay.....	\$12.50 account
2. *10.00 check endorsed to pay	7.20 account
3. 25.00 bank draft endorsed to pay.....	18.25 account
4. 50.00 bill to pay.....	38.60 account
5. 28.50 check to pay.....	11.05 account
6. 5.00 bill to pay.....	3.15 account
7. 10.00 gold piece to pay.....	7.35 account
8. 10.00 bill to pay.....	4.75 account
9. 20.00 pay roll check to pay.....	8.10 account
10. 100.00 bill to pay.....	73.30 account

*Not good practice to cash checks for strangers.

11. You are given \$2.00 to purchase from a grocery store the following items:

- 4 lbs. sugar at $8\frac{1}{4}$ c
- 1 loaf bread at 15c
- 1 lb. prunes at 25c
- 1 lb. butter at 65c

What is the amount of change and the denominations in which the merchant would give it to you?

12. You draw \$10.00 from your bank and make the following purchases:

- 1 pr. gloves, \$2.25
- 1 shirt, \$3.50
- 2 linen handkerchiefs, .30 ea.

After paying 16c car fare what is the amount left?

13. You started to a football game with \$6.00. You paid car fare 15c, ticket to the game \$1.65, pennant 35c, dinner with friends after the game \$1.50. What is the amount left?

14. You mail a \$10.00 bill to a certain theater with instructions to put aside 4 seats in a certain section. These are \$1.38 each including war tax. The change is handed you the following week with the tickets. How much is received?

WRITTEN

1. You had the following transactions with Smith Bros. for the month of Jan., 1922: Jan. 2, sold them goods on account 30 days, \$78.25; Jan. 9, sold on 10 days' credit, \$80.15; Jan. 18, received check for \$75.00 on account; Jan. 21, sold goods and billed at 60 days, \$42.25; Jan. 24, received \$50.00 to apply on account. At the end of the month you are handed a \$100.00 bill to balance the account with this firm. What change would you return?

(30)

FINDING AVERAGES, MODES AND MEDIANS

The amount of work done by 15 different men on a certain day was 12, 14, 12, 18, $13\frac{1}{2}$, 14, 14, $10\frac{1}{2}$, 15, 16, $18\frac{1}{2}$, 10, 8, 14 and 15 pieces respectively.

(a) To find the average:

$$\text{The sum} = 204\frac{1}{2}$$

$$204\frac{1}{2} \div 15 = 13\frac{1}{3} \text{ each, average}$$

(b) To find the mode:

What amount appears most often?

Answer, 14. Therefore 14 is the mode.

(c) To find the median:

8
10
 $10\frac{1}{2}$
12
12
 $13\frac{1}{2}$
14
—14—
14
14
15
15
16
18
 $18\frac{1}{2}$

Arrange in a column, beginning with the lowest and ending with the highest. The middle number is the median. Which of the three methods do you think should be used in determining what each man should do?

PROBLEMS

1. The price of wheat for 9 months was 90c, \$1.10, \$1.20, 96c, \$1.40, \$1.25, 98c, 85c, 98c. Find the average, mode and median.

2. The intelligence quotients for 15 boys were as follows: 60, 125, 80, 96, 110, 115, 140, 110, 105, 65, 70, 110, 105, 95, and 75. Find average, mode and median.

3. John has an average of $88\frac{1}{2}$ in four studies. If he makes 96 in the fifth, what will his final average be?

4. Blue and Gray each owned 60 chickens. Blue sold his 2 for \$1; Gray sold his 3 for \$1. Blue asked Gray to deliver his chickens for him. Gray agreed, and putting Blue's chickens in with his, said, "2 for \$1 and 3 for \$1, that makes 5 for \$2; I'll sell them that way." When he arrived home he paid Blue the \$30 due him and handed only \$18 to his wife, who accused him of spending \$2 for drink. How will he explain?

5. The report of a restaurant for one day shows:

12 meals at 20c, 18 meals at 25c, 24 meals at 30c
 25 meals at 35c, 56 meals at 40c, 55 meals at 45c
 10 meals at 50c, 4 meals at 55c, 3 meals at 65c
 2 meals at 80c, 1 meal at 95c.

Find the average price spent for a meal. The mode indicates between 40 and 45 cents. How does this compare with the average?

(31)

EXERCISE CASH ACCOUNT

What is the Cash Balance on March 15?

March

1. A. L. Graham invested \$3,500.00 in cash. (Cash Received.)
2. Paid by check for team and wagon, \$300.00.
3. Paid by check for office and store fixtures, \$650.00.
5. Received for cash sales, \$304.87; paid cash for merchandise \$1,500.00.
6. Received from Caleb Barber, \$50.00; paid sundry expenses, \$127.00.
7. Paid \$150.00 for safe; received from O. H. Arnold, \$64.00.
8. Cash purchase, \$330.75; received for cash sales, \$204.18.
9. Received from H. F. Ritter, \$15.00; cash purchase, \$28.65.
10. Cash sales, \$264.85; received from O. B. Donaldson, \$50.00.
12. Cash sales, \$15.79; paid Belknap Hardware Co., \$629.87.
14. Paid sundry expenses, \$72.50; paid C. R. Crawford, \$75.00.
15. Received from H. F. Ritter, \$20.00; borrowed \$1,000.00 from the bank.

NOTE: Add receipts and payments and take difference. Ruled paper as in Illustration page 64 may be used.

(32)

SHORT CUTS IN MULTIPLICATION

Multiplying by 11.

Example:

$$\begin{array}{r} \text{Multiply } 175 \times 11 \quad 175 \\ \quad \quad \quad 175 \\ \hline \quad \quad \quad 1925 \end{array}$$

Never write 11 as multiplier as it is a mere repetition.

Drill on the following:

- | | |
|-------------------------|------------------------|
| 1. $1246 \times 11 =$ | 8. $8794 \times 11 =$ |
| 2. $965 \times 11 =$ | 9. $256 \times 11 =$ |
| 3. $2472 \times 11 =$ | 10. $1865 \times 11 =$ |
| 4. $16749 \times 11 =$ | 11. $3963 \times 11 =$ |
| 5. $27245 \times 11 =$ | 12. $9864 \times 11 =$ |
| 6. $26315 \times 11 =$ | 13. $7004 \times 11 =$ |
| 7. $129646 \times 11 =$ | 14. $394 \times 11 =$ |

To multiply by 111:

Here you have the same principle as in multiplying by 11. The operations save time in writing figures. For example: we have 4625 to multiply by 111. Why repeat all the figures when we can write down in columns and add as:

$$\begin{array}{r} 4625 \\ 4625 \\ 4625 \\ \hline 513375, \text{ Ans.} \end{array}$$

Multiply the following by 111:

- | | | |
|---------|----------|-----------|
| 1. 798 | 4. 16728 | 7. 762425 |
| 2. 1675 | 5. 49257 | 8. 175315 |
| 3. 2928 | 6. 81215 | 9. 825614 |

To multiply any number by 9, 19, 29, 39, 49, etc. up to 100:

$725 \times 9 = 7250 - 725 = 6525$. Just add a cipher and deduct the multiplicand.

$725 \times 19 = 725 \times 20 = 14500 - 725 = 13775$. Since your multiplier is one greater it is necessary to subtract once the multiplicand.

Drill:

(33)

- | | | |
|-----------------------|------------------------|-----------------------|
| 1. $825 \times 99 =$ | 7. $926 \times 59 =$ | 13. $54 \times 19 =$ |
| 2. $624 \times 49 =$ | 8. $327 \times 59 =$ | 14. $87 \times 79 =$ |
| 3. $836 \times 29 =$ | 9. $206 \times 29 =$ | 15. $126 \times 79 =$ |
| 4. $1064 \times 19 =$ | 10. $754 \times 89 =$ | 16. $853 \times 29 =$ |
| 5. $210 \times 69 =$ | 11. $1275 \times 69 =$ | 17. $837 \times 59 =$ |
| 6. $870 \times 79 =$ | 12. $247 \times 39 =$ | 18. $159 \times 39 =$ |

Multiplying any number by 21, 31, 41, etc., up to 91:

$$735 \times 31 = 735$$

$$\begin{array}{r} 2205 \\ \hline 22785 \end{array}$$

Never repeat writing the number for the one, as it is merely a duplicate.

Drill:

(34)

- | | | | |
|-----------------------|------------------------|------------------------|-------------------------|
| 1. $845 \times 21 =$ | 7. $74 \times 61 =$ | 13. $3864 \times 91 =$ | 19. $1794 \times 21 =$ |
| 2. $607 \times 91 =$ | 8. $127 \times 71 =$ | 14. $246 \times 31 =$ | 20. $864 \times 81 =$ |
| 3. $184 \times 31 =$ | 9. $158 \times 51 =$ | 15. $1059 \times 51 =$ | 21. $1479 \times 61 =$ |
| 4. $726 \times 51 =$ | 10. $180 \times 31 =$ | 16. $785 \times 71 =$ | 22. $3359 \times 51 =$ |
| 5. $815 \times 81 =$ | 11. $214 \times 61 =$ | 17. $69 \times 91 =$ | 23. $758 \times 101 =$ |
| 6. $1026 \times 41 =$ | 12. $2249 \times 41 =$ | 18. $2046 \times 31 =$ | 24. $2438 \times 111 =$ |

(35)

VALUABLE SHORT-CUTS IN MULTIPLICATION

Often in billing, items are found where this plan can be employed to good advantage. This is only possible where the units add 10 and the figures in the tens and hundreds columns are alike.

For example: 32

$$\begin{array}{r} 38 \\ \hline 1216 \end{array}$$

Observe that $8+2=10$ and the figures in the tens column are alike.

Operation: $8 \times 2 = 16$. Bring down, add one to either of figures in tens column, multiply and bring down $4 \times 3 = 12$.

Drill on the following:

- | | |
|-----------------------|-----------------------|
| 1. $25 \times 25 =$ | 8. $99 \times 91^* =$ |
| 2. $27 \times 23 =$ | 9. $36 \times 34 =$ |
| 3. $82 \times 88 =$ | 10. $45 \times 45 =$ |
| 4. $75 \times 75 =$ | 11. $33 \times 37 =$ |
| 5. $125 \times 125 =$ | 12. $52 \times 58 =$ |
| 6. $65 \times 65 =$ | 13. $59 \times 51 =$ |
| 7. $77 \times 73 =$ | 14. $13 \times 17 =$ |

*91 Where the first column is less than 10 bring down 99
9009 cipher for second place.

Sometimes the two right-hand figures are alike and the two left-hand figures add to ten, thus:

$$\begin{array}{r} 76 \\ 36 \\ \hline 2736 \end{array}$$

Here we multiply the right-hand figures for the 36, and $3 \times 7 + 6 =$ the 27.

(36)

Work the following problems by the above method:

28	45	*73	29	61	24	77	32	34	36
<u>88</u>	<u>65</u>	<u>33</u>	<u>89</u>	<u>41</u>	<u>84</u>	<u>37</u>	<u>72</u>	<u>74</u>	<u>76</u>
62	16	54	48	23	71	92	96	87	93
<u>42</u>	<u>96</u>	<u>54</u>	<u>68</u>	<u>83</u>	<u>31</u>	<u>12</u>	<u>16</u>	<u>27</u>	<u>13</u>

*When the units multiplied together do not give an amount into tens, a cipher is written in tens place. In this problem we have $3 \times 3 = 9$. $3 \times 7 + 3 = 24$. We write 2409 for answer.

(37)

When neither of these combinations occur, cross multiplication may be employed, thus:

$$\begin{array}{r} 37 \\ 42 \\ \hline 1554 \end{array}$$

$7 \times 2 = 14$, place the 4 and carry the 1.
 $4 \times 7 = 28$ and $3 \times 2 = 6$.
 $28 + 6 + 1 = 35$, place the 5 and carry the 3.
 $4 \times 3 = 12 + 3 = 15$.

Work the following problems by the above method.

24	55	73	59	81	56	59	29	32	72
<u>37</u>	<u>28</u>	<u>24</u>	<u>73</u>	<u>24</u>	<u>45</u>	<u>28</u>	<u>16</u>	<u>17</u>	<u>19</u>
35	24	63	82	21	37	81	35	18	45
<u>58</u>	<u>31</u>	<u>24</u>	<u>45</u>	<u>36</u>	<u>42</u>	<u>25</u>	<u>19</u>	<u>29</u>	<u>33</u>

TO MULTIPLY NUMBERS ENDING WITH 5

Study and practice until you can use these readily and thereby save much time and labor.

If sum of tens figures is even, the last two numbers are 25 as in first problem. If tens figures are odd, then the last two figures will be 75 as in the second problem.

$$\begin{array}{r}
 85 \\
 45 \\
 \hline
 3825
 \end{array}
 \begin{array}{l}
 \text{In the first } 5 \times 5 = 25 \text{ and write down. } 4 \times 8 = 32 \quad \frac{1}{2} \\
 \text{of } (8+4) \text{ or } 6+32=38. \\
 \text{In the second we have the same operation, except that} \\
 \frac{1}{2} \text{ of 9 gives } 4\frac{1}{2}. \text{ The } \frac{1}{2} \text{ is carried forward as 5. Or to} \\
 \text{show figures we have:} \\
 \begin{array}{r}
 65 \\
 35 \\
 \hline
 1825
 \end{array}
 \text{ Units and tens multiplied and brought down.} \\
 \begin{array}{r}
 65 \\
 35 \\
 \hline
 2275
 \end{array}
 \begin{array}{r}
 45 \\
 \hline
 2275
 \end{array}
 \frac{1}{2} \text{ of } (6+3) \text{ or } 4\frac{1}{2}
 \end{array}$$

(38)

DRILL

95	65	125	55	75	85	105	145	25	115
35	45	45	35	65	25	65	35	15	95

1. What will be the cost of the following:

(Use short cuts in all of these)

- 89 pounds of bacon at 29c per pound
- 62 pounds of butter at 42c per pound
- 37 pounds of prunes at 16c per pound
- 75 pounds of lard at 15c per pound

2. Find the cost of:

- 35 dozen eggs at 35c per dozen
- 37 pounds of beef at 33c per pound
- 25 cans of tomatoes at 15c per can
- 25 cans of peaches at 25c per can

3. Find the cost of:

- 65 bushels of oats at 45c
- 125 bushels of wheat at 95c
- 48 pounds of ham at 37½c

(39)

ALIQUTS OF \$1.

What part of \$1 is 50c? $33\frac{1}{3}$ c? $66\frac{2}{3}$ c? 25c? 75c? 20c?
 40c? 60c? 70c? 80c? 90c? $12\frac{1}{2}$ c? $37\frac{1}{2}$ c? $62\frac{1}{2}$ c? $87\frac{1}{2}$ c? $16\frac{2}{3}$ c?
 $\frac{5}{6}$ $83\frac{1}{3}$ c? $8\frac{1}{3}$ c? $\frac{1}{12}$

How many yards at $16\frac{2}{3}$ c can be bought for \$4?

(NOTE: Multiply 4×6 yds. = 24 yds., ans.)

How many yards at $12\frac{1}{2}$ c can be bought for \$8?

How many yards at 25c can be bought for \$4 $\frac{1}{2}$?

How many yards at $33\frac{1}{3}$ c can be bought for \$3 $\frac{1}{3}$?

How many yards at $8\frac{1}{3}$ c can be bought for \$8?

What will 540 yds. cost at $33\frac{1}{3}$ c?

(NOTE: 540 yds. $\div 3$ = \$180.00, ans.)

ORAL DRILL

- | | |
|--------------------------------------|-------------------------------------|
| 1. 96 yds. at \$1. $33\frac{1}{3}$? | 6. 840 yds. at \$1.50? |
| 2. 44 yds. at $12\frac{1}{2}$ c? | 7. 75 yds. at $16\frac{2}{3}$ c? |
| 3. 84 yds. at $8\frac{1}{3}$ c? | 8. 720 yds. at 25c? |
| 4. 720 yds. at 1.25? | 9. 95 yds. at 1.10? |
| 5. 72 yds. at $1.12\frac{1}{2}$? | 10. 114 yds. at $1.16\frac{2}{3}$? |

ALIQUT PARTS OF 18, 24, 36, 48, 50, 60, 72, 80, 90 AND 96.

If you had any one of the following numbers given, what fractional part of it would you add to it or subtract from it to produce each of the numbers listed after it?

- 18 to produce 24, 6, 12, 9, 28, 15, 16, 20, 21, 27
 24 to produce 12, 28, 30, 36, 18, 32, 16, 21, 27, 20
 36 to produce 30, 33, 24, 45, 42, 39, 48, 32, 54, 40
 48 to produce 42, 40, 44, 32, 72, 54, 45, 64, 52, 56
 48 to produce 24, 36, 80, 60, 40, 32, 42, 56, 72, 44
 50 to produce 45, 60, 55, 15, 25, 75, $62\frac{1}{2}$, $37\frac{1}{2}$
 60 to produce 90, 45, 75, 54, 66, 48, 80, 70, 72, 40
 60 to produce 66, 55, 57, 30, 65, 56, 62, 64, 58, 63
 72 to produce 90, 36, 48, 54, 60, 96, 84, 81, 63, 78
 80 to produce 72, 50, 60, 90, 76, 88, 96, 100, 120, 64
 90 to produce 72, 80, 99, 60, 81, 100, 108, 120, 135
 96 to produce 90, 80, 72, 48, 84, 108, 144, 48, 64

(40)

FOUR ALIQUOT PART DRILLS

(a) Let a series of numbers like the following represent articles or units sold, as cwt., bu., bbl., yds., etc., 480, 960, 1200, 1800, 360, 1720, 540, 240. Multiply each of the above by the aliquots of \$100.00, \$10.00, \$1.00 and 10c shown in the table below.

To illustrate:

480 lbs. at 50c = \$240.

960 bu. at $33\frac{1}{3}$ c = \$320.

1800 gals. at $12\frac{1}{2}$ c = \$225.

360 yds. at $3\frac{1}{3}$ c = \$12.00

Base \$100	{	\$50.	\$ 5.	Base \$1	{	\$.50	\$.05
		$33\frac{1}{3}$	45.			$.33\frac{1}{3}$	$.02\frac{1}{2}$
		25.	4.			.25	$.01\frac{1}{4}$
		20.	2.			.20	.75
		12.50	$3\frac{1}{3}$			$.12\frac{1}{2}$	.80
		87.50	90.			.10	.90
		95.00	2.50			$.07\frac{1}{2}$	$.87\frac{1}{2}$
Base \$10	{	\$5.	\$ 4.50	Base 10c	{	\$.05	$$.06\frac{2}{3}$
		$3\frac{1}{3}$	7.50			$.03\frac{1}{3}$	$.08\frac{3}{4}$
		2.50	15.			$.02\frac{1}{2}$	.09
		2.	12.50			.02	.08
		1.25	12.00			$.01\frac{1}{4}$	$.07\frac{1}{2}$
		$6.66\frac{2}{3}$	11.00			.01	
		8.75	9.00				

(b) Let the amounts under (a) at the top represent dollars invested and divide by the prices to get units purchased.

To illustrate:

How many bushels of oats at 50c per bu. can be bought for \$480.00, \$960.00, \$1200., etc.

$480 \times 2 = 960$ bu., Ans.

$960 \times 2 = 1920$ bu., Ans.

How many yards of ribbon at $33\frac{1}{3}$ c can be bought for \$480, etc.

(41)

DRILL IN CHAIN DISCOUNTS

From each of the amounts below, deduct one of the following discounts five times: 50%, 12½%, 11⅓%, 5%, 4%, 2%, 1%, 20%, 8⅓%, 10%, 16⅔%, 33⅓%.

Do not use the same rate in more than two problems.

From each of these amounts deduct any five of the above discounts, taking series in any order desired.

\$480.	\$540.60	\$212.45	\$340.80
960.	32.80	87.25	1124.18
1240.	5000.	196.33	79.96
450.	140.20	427.90	200.80
1860.	84.40	3524.85	1013.45
840.	175.50	872.18	888.89

1. What is the net amount due on an invoice of \$400.00 with the following discounts: 25%, 16⅔% and 2½% for cash?

2. Find the net amount you will have to pay to Jno. W. Brown on the following bill. Total \$640.00 less 12½%, 5%, with an additional 1% for cash.

3. You sold a customer goods amounting to \$80.00 subject to 20%, 12½% and 5%. What should you collect?

4. You sold another bill of \$120.00 subject to 16⅔%, 7.5% with 2.5% cash. What should you collect within 10 days? (10 days usually considered cash.)

*2½% is also ¼ of 10. Note that you could point off one place \$25.0 and take ¼ of \$25.00 or \$6.25.

Example:

$$\begin{array}{r}
 6) \$720.40 \\
 \underline{120.066} + = 16\frac{2}{3}\% \\
 6) 600.333 \\
 \underline{100.055} + \\
 6) 500.277 \\
 \underline{83.379} + \\
 6) 416.897 \\
 \underline{69.482} + \\
 6) 347.414 \\
 \underline{57.902} + \\
 289.511
 \end{array}$$

Illustration:

$$\begin{array}{r}
 25\% = \frac{1}{4}) 400 \\
 \underline{100} \\
 16\frac{2}{3}\% = \frac{1}{6}) 300 \\
 \underline{50} \\
 2\frac{1}{2}\% = \frac{1}{4} (250 \\
 \underline{6.25} \\
 243.75, \text{Ans.}
 \end{array}$$

FIGURING LUMBER

Lumber is sold either by the piece or by the thousand board feet. A board foot is 12 inches square and one inch (or less) in thickness, or the equivalent thereof in cubic inches; 144 cu. in. = 1 board foot. Widths are always in an even number of inches and lengths in even numbers of feet.

1. A 1-in. board 12 in. wide and 16 ft. long would contain 16 bd. ft.

2. A 2-in. plank 12 in. wide and 16 ft. long would contain 32 bd. ft.

3. A 3-in. plank 12 in. wide and 16 ft. long would contain 48 bd. ft.

4. A 1-in. board 6 in. wide and 16 ft. long would contain 8 bd. ft.

5. A 2-in. plank 6 in. wide and 16 ft. long would contain 16 bd. ft.

6. A 3-in. plank 6 in. wide and 16 ft. long would contain 24 bd. ft.

From these statements you may observe that the number of board feet in a piece of lumber is to its length as its width multiplied by its thickness in inches is to 12. If width multiplied by thickness = 12 then the length shows the board feet. If width multiplied by thickness is more or less than 12, the length must be increased or decreased in like proportion to give board feet.

(42)

PROBLEMS

1. How many feet of lumber in 46 timbers 3"x8"x14'?
SOLUTION: $3 \times 8 = 24$ or twice 12, therefore twice the length or 28, is the number of board feet in each timber, and 46×28 or 1288 in all.

This method enables you to work a large part of your lumber problems mentally, and therefore easily; or cancellation may be employed, as:

$$\frac{\overset{2}{3} \times \overset{2}{8} \times 14' \times 46}{\underset{4}{12}} = 1288, \text{ Ans.}$$

2. How many feet of lumber in 25, 2"x4"s. each 18 ft. long?

Find the cost of the following:

No.	Pieces	DESCRIPTION	Price per M Feet
3	400	2" x 6", soft pine, 8 ft. long.....	\$ 48
4	125	10-in. boards, 12 ft. long.....	50
5	12	4" x 8", hemlock, 16 ft. long.....	60
6	4	3" x 6", oak, 8 ft. long.....	80
7	5	18-in. boards, oak, 8 ft. long.....	125
8	2	3" x 10" planks, pine, 16 ft. long....	85
* 9	72	2" x 4", pine, 14 ft. long.....	90
10	18	4" x 10" pine timbers, 18 ft. long....	75

*In this problem you cannot easily take $\frac{2}{3}$ of 14, so instead, take $\frac{2}{3}$ of 72, (48) and multiply by 14 and 90; or take $\frac{2}{3}$ of 90 (60) and multiply by 14 and 72. The change can be made in either the number of timbers, the length, or the price.

When the plan given above does not apply easily, the cancellation method may be used.

To illustrate:

11. Find cost of seventeen 2x8's 14 ft. long at \$85.20 per M.

Solution:

$$\begin{array}{r}
 17 \times 2 \times 8 \times 14 \times \$85.20 \\
 \hline
 12 \times 1000 \\
 6 \quad 250 \\
 3 \quad 125 \\
 \hline
 25
 \end{array}
 \quad
 \begin{array}{r}
 4 \quad 7 \quad 17.04 \quad 5.68 \\
 17 \times 2 \times 8 \times 14 \times \$85.20 \\
 \hline
 17 \times 7 \times 5.68 = \frac{67592}{25} = \$27.04
 \end{array}$$

Even here the other method is just as easy and much safer.

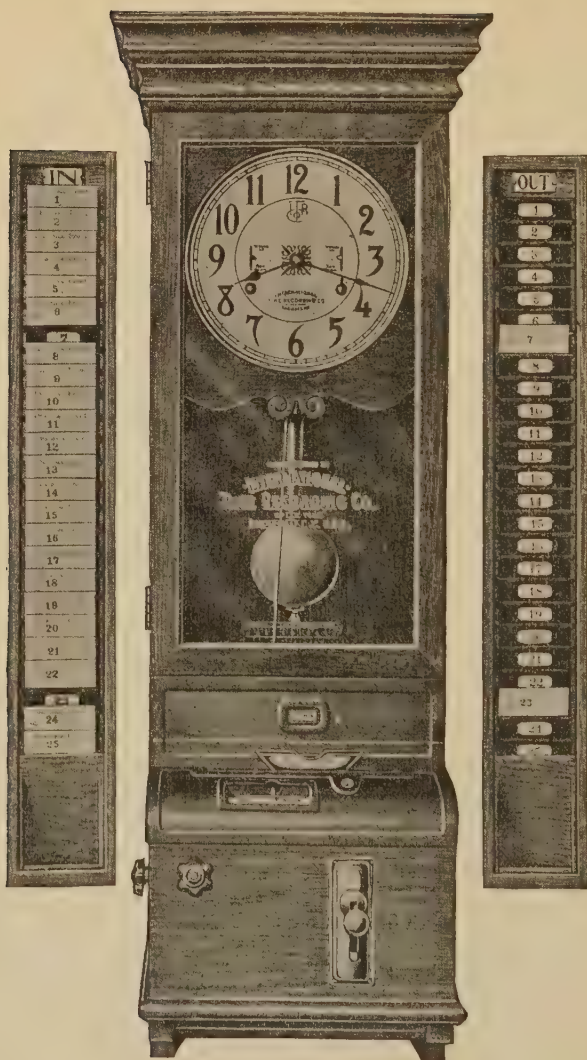
$$2 \times 8 = 16 = (1\frac{1}{3} \times 12). \quad \$85.20 + \frac{1}{3} \text{ of itself} = \$113.60, \quad \frac{\$113.60 \times 17 \times 14}{1000} = \$27.04$$

12. Find the cost of nineteen 4" boards 16 ft. long at \$40 per M.

13. Sold twenty-nine 10" boards, 8 ft. long at \$43.25 per M.

14. Bought eleven 3x6's 14 ft. long at \$51.75 per M.

15. Bought five $\frac{1}{2}$ "x10" boards, 8 ft. long at \$49.50 per M.



TIME CLOCK

The illustration above shows a time clock with cards used by employees for registering at the beginning of the day's work and also registering upon leaving work. This is known as "punching in" and "punching out." Upon arriving, the card corresponding with the employee's number is removed

from the holder beside the clock and placed in the slot at the bottom of the clock, and upon pressing down the knob the time is registered. This is then placed in the holder at the side where it remains until the end of the day, when it is again punched, registering the quitting time. The card is returned to the holder where it remains until the end of the week, when the pay roll clerk makes up the pay roll from the information given on the cards.

WEEK ENDING		April 10		1920		
I. T. R. Co. Form No. 1212						
No.		24				
NAME						
B. BARNES						
	MORNING IN	NOON OUT	NOON IN	NIGHT OUT	EXTRA IN	EXTRA OUT
	Σ 7 01	Σ 12 02	Σ 12 50	Σ 5 03		8 3/4
	Π 6 59	Π 12 05	Π 12 55	Π 4 59		8 3/4
	Σ 6 45	Σ 11 30	Σ 12 54	Σ 5 01	Σ 6 00	Σ 9 05 1 1/2
	Π 6 55	Π 12 01	Π 12 50	Π 5 05		9
	Π 6 53	Π 12 00	Π 12 59	Π 5 04		9
	Σ 6 58	Σ 12 02	Σ 1 02	Σ 5 00		8 3/4
TOTAL TIME 55 3/4 HRS.						
RATE 60						
TOTAL WAGES FOR WEEK \$ 33.45						

TIME CARD

The illustration above shows the completed time card for the week. You will note this shows the hours worked per day and the total hours for the week for employee No. 24. From this card the pay roll clerk enters on the pay roll the number of hours, the rate paid per hour and the amount due for the week's work. This card may be filed or destroyed, depending upon the details given on the pay roll. A new card is placed in the holder where the process described above is repeated.

(44)

PROBLEMS IN FIGURING PAY ROLL

From the following data construct properly ruled pay roll sheets and coin sheets: (See coin sheet on page 56.)

1

No.	Name	M.	T.	W.	Th.	F.	S.	Rate
1	Alice Stone.....	8	8	8	$7\frac{1}{2}$	8	8	\$0.42 $\frac{1}{2}$
2	Alice Bateman.....	$6\frac{3}{4}$	8	8	...	...	...	.37 $\frac{1}{2}$
3	Martha Wright.....	8	$8\frac{1}{2}$	8	8	$8\frac{1}{2}$	$8\frac{1}{2}$	.40
4	Nora Kirk.....	8	$6\frac{2}{3}$	7	8	7	$7\frac{1}{2}$	.28
5	Etta Brown.....	8	8	8	...	8	8	.33 $\frac{1}{3}$
6	Lucy Maurer.....	7	7	7	7	7	7	.40
7	Mabel Davis.....	8	8	8	8	$8\frac{1}{4}$	$8\frac{1}{2}$	.50
8	Alice Pringle.....	8	8	8	8	8	$8\frac{3}{4}$	.50
9	Hazel Howard.....	$7\frac{1}{4}$	8	8	$7\frac{1}{4}$	8	...	.46 $\frac{2}{3}$
10	Viva Baker.....	8	...	$3\frac{1}{2}$	8	8	8	.50
11	Mary Wheaton.....	8	8	8	8	8	8	.45
12	Rose Lee.....	8	$8\frac{1}{2}$	$8\frac{1}{4}$	8	$7\frac{1}{2}$	8	.48

2

No.	Name	M.	T.	W.	Th.	F.	S.	Rate
1	Roland Ward.....	8	8	8	$8\frac{1}{2}$	$8\frac{1}{4}$	8	\$0.54
2	Dick Rowan.....	$7\frac{3}{4}$	$7\frac{1}{2}$	8	8	$8\frac{1}{2}$	8	.52
3	Ben Putnam.....	8	8	...	$7\frac{1}{2}$	8	8	.45
4	Thos. Morgan.....	4	8	8	8	8	4	.47 $\frac{1}{2}$
5	T. E. Castle.....	8	8	8	8	8	8	.50
6	W. R. Allen.....	8	8	8	8	...	...	.55
7	Roy De Sort.....	$8\frac{1}{4}$	$8\frac{1}{4}$	8	8	$7\frac{3}{4}$	8	.48
8	Pat Hagan.....	...	...	...	$7\frac{1}{2}$	8	8	.33 $\frac{1}{3}$
9	Chas. Titch.....	8	8	8	$\frac{1}{2}$	...	...	.33 $\frac{1}{3}$
10	G. E. Davis.....	8	8	8	8	8	$8\frac{1}{2}$	.56
11	Tom Lee.....	8	$7\frac{1}{4}$	8	8	$7\frac{1}{4}$	8	.50

(45)

THE MELLON NATIONAL BANK

Pittsburgh, Pa.

Pay Roll Memorandum

wanted by

192...

Currency		
Twenties.....	\$	
Tens.....		
Fives.....		
Twos.....		
Ones.....		
Silver		
Dollars.....		
Halves.....		
Quarters.....		
Dimes.....		
Nickels.....		
Pennies.....		
Total.....		

Prepared by Pay Roll clerk, showing change he will need to make up Pay Roll.

Supply names for each of the following employe numbers,
construct complete pay roll and coin sheet.

3

No.	Name	M	T	W	T	F	S	Rate	No.	Name	M	T	W	T	F	S	Rate
1		8	8	8	4 $\frac{1}{2}$	8	7 $\frac{3}{4}$	.54 $\frac{1}{2}$	8		7 $\frac{3}{4}$	7 $\frac{1}{2}$	8	8	8	7 $\frac{1}{2}$	.57 $\frac{1}{2}$
2		8	7 $\frac{3}{4}$	8 $\frac{1}{2}$	7 $\frac{1}{4}$	7 $\frac{3}{4}$	8	.57 $\frac{2}{3}$	9		8	8	...	7 $\frac{1}{2}$	7 $\frac{1}{4}$		.53 $\frac{1}{4}$
3		5 $\frac{3}{4}$	8	8	8	8	8	.57 $\frac{1}{2}$	10		8 $\frac{1}{4}$	8	8	...	4 $\frac{1}{4}$	8	.47 $\frac{1}{2}$
4		8	8	7 $\frac{1}{4}$	...	...	...	.46 $\frac{3}{4}$	11		8	8	8	8	8	5	.53 $\frac{1}{3}$
5		..	8	8	8 $\frac{1}{4}$	8 $\frac{1}{2}$	8	.48	12		8	8	8	8	8	8	.48 $\frac{1}{4}$
6		8	8 $\frac{1}{3}$	7 $\frac{1}{2}$	8 $\frac{1}{2}$	8	8 $\frac{1}{4}$	.52 $\frac{1}{2}$	13		8	7 $\frac{1}{2}$	8	8	8	6 $\frac{3}{4}$	.57 $\frac{1}{2}$
7		8 $\frac{1}{4}$	8	8 $\frac{1}{4}$	8 $\frac{1}{2}$	8	..	.56	14		7 $\frac{1}{4}$	7 $\frac{1}{4}$	8 $\frac{1}{4}$	8	7 $\frac{3}{4}$	8	.57 $\frac{1}{2}$

REVIEW PROBLEMS

1. Copy the following columns, correcting any errors:

1	2	3	Total
3426892	54123	81726894	85207909
18925	348857	4675	371457
8952148	4165915	15749	133330813
9071786	22431947	323487	33814745
7417	93826	8348168	83638540
142634	235694	92547629	92925957
4276509	9126945	6705694	201091488
524657	71142673	32292612	10395984
8272693	3046794	51690217	63009704
817654	82014769	34852734	11768515
<u>35311315</u>	<u>194492432</u>	<u>308507858</u>	<u>538311605</u>

2. Find the net amount of an invoice billed at \$1860, if the discounts allowed are 25%, 20%, and 10%.

3. After paying the following bills and accounting for cash sales, the Smith Manufacturing Co. finds its cash balance to be \$12,267.50. If the amount of cash invested was \$10,500 what was the amount of the sales?

Union Gas and Electric Co., for light, \$35; Jones Realty Co., for rent, \$400; sundry clerk hire, \$150; for goods bought during the month, \$4872.50.

4. A carpenter contractor ordered the following lumber for a job. Find the cost of the entire bill:

100 pieces 4" x 6", soft pine, 10' long, @ \$48 per M
 50 pieces 2" x 4", white oak 12' long, @ \$69 per M
 40 pieces $\frac{1}{2}$ " x 6", hemlock, 10' long, @ \$30 per M
 20 bundles red cedar shingles @ \$4 per bundle

5. John Jones' grades for the month of January were as follows: Reading, 80%; Writing, 75%; Arithmetic, 95%; Geography, 90%; History, 95%; Drawing, 70%, and Music, 75%. Find his general average for the month.

6. In making a purchase of \$1.83 from your dealer you hand him a twenty dollar bill. Name the pieces he would give you in making the change.

ARITHMETIC OF BOOKKEEPING

Purpose of a Business. Those engaged in business undertakings provide the public with services or goods from the sale of which the owner or owners expect to make a profit. If the services or goods are sold at less than cost, the business is said to have been operated at a loss.

Business Transactions refer to the exchange of equivalent values. When the business man gives services or goods, he expects to receive in return a value for such goods or services. If a shoe merchant sells a pair of shoes for cash, he gives a value in form of shoes and receives in return a value in form of money.

Bookkeeping. In order that the business man may keep a record of his transactions and know the results of his operations from year to year, he employs a bookkeeper who records on properly ruled blank books the transactions from day to day.

Debit and Credit are terms to designate the left and right side of an account. Debit refers to the left side (abbreviated Dr.) and credit to the right side (abbreviated Cr.). In recording transactions, we debit the value received and credit the value parted with.

An Account is a record of the debits and credits relating to property, persons, profits or losses, collected under an appropriate heading or title. For example, the cash account is a record of cash received and paid. The expense account is a record of the cost of operating the business, etc.

The Balance of an Account is the difference between the debits and credits. If the debit side is larger, the difference is called a debit balance. If the credit side is larger, it is known as a credit balance.

Books of Original Entry are the blank books in which the transactions are recorded as they occur; such as the Cash Book, Journal, Sales Book, etc.

The Ledger, or Book of Accounts, as it is sometimes called, contains the names of the accounts to which are transferred or posted, all the debits and credits from the books of original entry. If the business is small, the accounts in the ledger will not be numerous. However, with a large business concern, the accounts may number hundreds and even thousands.

Trial Balance is a list of the accounts in the ledger usually made monthly to test the accuracy of the posting and at the end of the fiscal period furnishes a basis for the preparation of the necessary statements.

Mdse. Inventory is the value of goods in stock at the close of the fiscal period.

Resources or Assets refer to values owned by an individual or business. These values may represent property, such as cash, merchandise, land, buildings, furniture, stock, machinery, or they may be in the form of debts due from others, such as accounts receivable, notes, drafts, mortgages, etc.

Liabilities refer to debts owed to others. These debts are usually in the form of accounts payable and notes payable.

Present Worth or Present Capital also referred to as *Proprietorship* is the difference between the total Resources and total Liabilities.

Business Period or Fiscal Period is that time between statements, and may be from one month in length to one year. One year or half-year, perhaps, is the time most commonly used for the fiscal period. A Balance Sheet and Profit and Loss Statements are made for the period in order to compare the progress of the business with past and future fiscal periods.

Balance Sheet is a statement of the assets, liabilities and Proprietorship made at the end of each fiscal period of the business. See model on page 69.

Profit and Loss Statement is a report of profits and losses for the fiscal period which shows the results of the operations for the period. See model, page 70.

(46) ,

THE ARITHMETIC OF A SET OF BOOKS

The following are the transactions for B. A. Mann for the month of January, 1921. These are correctly recorded in the forms that follow:

- Jan. 1 B. A. Mann invested cash \$5600.00.
- " 2 Paid by check 1 month's rent, \$150.00.
- " 2 Paid by check bill for desk and files, \$225.50.
- " 2 Paid cash for stationery, books, etc., \$17.20.
- " 2 Received bill for mdse. bought of W. T. Briggs, Detroit, Mich., 2/10, n/60, amounting to \$5246.25.
- " 4 Sold J. B. Betts, 1242 Main St., cash 10 days, the following items: 84 lbs. sugar at $8\frac{1}{2}$ c, 120 yds. calico at $33\frac{1}{2}$ c, 78 yds. gingham at 45c, 420 yds. linen at 75c.
- " 4 Bought of W. R. Rowan, 124 Walnut St., N. Y., on a/c 30 days, mdse. amounting to \$3472.18.
- " 5 Sold S. O. Stark, 540 Hilldale Ave., 2/10, n/30, the following items: 4845 lbs. codfish at \$7.50 per cwt., 4625 lbs. lard at \$25 per cwt., 51024 lbs. ice at \$12.50 per ton.
- " 5 Bought of J. H. Davis, Peoria, Ill., 2/10, n/30, goods amounting to \$1807.29.
- " 5 Bought mdse. amounting to \$479.68 from J. H. Davis, Peoria, Ill. Terms on a/c.
- " 5 Bought for cash upon delivery mdse. amounting to \$826.75. Paid by writing check No. 4.
- " 6 Borrowed from local bank on 90-day note \$3,000. Gave check in advance for interest, \$45.00.
- " 6 Sold M. A. Drain, R. R. 17, Box 8, on a/c 60 days the following items: 2725 lbs. egg coal at $\$6.66\frac{2}{3}$ per ton. 734 lbs. nut coal at \$9.00 per ton, 1725 lbs. mine-run at 8c a bushel.
- " 6 Bought on 10 days of O. M. Gault, Chicago, Ill., goods amounting to \$1524.19.

- Jan. 7 Sold P. D. Quortz, 726 Walnut St., on a/c 60 days: 724 bars soap at 55 cts. a dozen, 1924 cans of Gee Whizz at \$11 a gross, 923 fly swatters at \$7.50 a C.
- " 8 Sold W. J. Kent, 15 Barclay St., 2/10, n/60: $17\frac{2}{3}$ yds. duck at $\$1.16\frac{2}{3}$, $4\frac{1}{4}$ yds. flannel at $\$1.33\frac{1}{3}$, 7214 bone buttons at \$5.20 a M.
- " 8 Paid by check invoice No. 2, due W. T. Briggs, less 2% discount. Check \$5141.32; discount \$104.93.
- " 12 Bought of W. R. Rowan, 124 Walnut St., on a/c goods amounting to \$57.24
- " 12 Paid J. H. Davis \$1,000.00 on a/c, less 2%. (Divide 1000 by 98%; will give the amount Davis will credit you, or \$1020.41.)
- " 12 Received of J. B. Betts a check for \$397.10 in full of a/c.
- " 14 Bought of J. P. Sharp, on 30 day note, goods amounting to \$3661.46. The note has been signed and delivered.
- " 15 Sold B. F. Cole, 407 Jeff Ave., note 90 days: Two 9' x 12' Wilton Rugs at \$33.75, one E. Z. Swing Washer at \$16.43, three porch chairs at \$7.60. Less 10%.
- " 15 Received of S. O. Stark in full of a/c, less 2%.
- " 16 Received of W. J. Kent in full of a/c, less 2%.
- " 16 Paid O. M. Gault on a/c by check, \$1024.19.
- " 17 Received a 90-day note from B. F. Cole for mdse. sold him on Jan. 15.
- " 20 Bought Ford Truck for cash, \$675.50.
- " 22 Bought goods for cash (check), \$1424.80.

Each transaction here given should be studied carefully and traced to the record on the following pages. A transaction involving cash is entered in the Cash Book. If it be a sale it is recorded in the Sales Book and a purchase of goods the record is made in the Purchase Book. In the journal you record special transactions that cannot be recorded in the other books. As a rule few transactions are found in the journal. The book where all items are posted from the above books is known as the Ledger or book of Accounts.

Journal, B. A. Mann, Jan. 15, 1921.

8	J. P. Sharp	3661 46		
6	Notes Payable		3661 46	
	Give note #1, 30 days, on account			
	17			
6	Notes Receivable	10673		
2	B. F. Cole		10673	
	Rec'd his 90-day note on account Reg. #1			

JOURNAL

The above entries are the correct form for the general journal entries. The date of recording the transaction is placed in the center between the entries. The figures at the left indicate the ledger page on which the accounts may be found in the ledger.

January, 1921

Date	L F	Purchaser	Address	Terms	Due	Items	Total
2	2	W. T. Briggs	Detroit, Mich.	2/5, m. p.	23	124625	
11	3	W. K. Rowan	Wichita, Kan.	2/5, 30 d.	13	347215	
5	4	J. H. Davis	Peoria, Ill.	2/10, 2/15	14	180729	
5	5	J. H. Davis	Peoria, Ill.	2/5	15	47968	
6	6	B. F. Cole	Chicago, Ill.	10 days	17	152419	
12	7	W. K. Rowan	Wichita, Kan.	2/5	11	5724	
14	8	J. P. Sharp	Evie, Pa.	2/5, 30 d.		366146	
		Total purchases					122415

PURCHASE BOOK

In this book is recorded the merchandise that is bought on account. Add the "item" column which should agree with the amount on the debit (left) side of the Purchase account in the ledger (page 66). Check the items from this record to the individual accounts in the ledger, pages 66-67.

January, 1921.

2	J. B. Betts, 1242 Main St. Cash 10 day			397 10
	84 lbs sugar @ $8\frac{1}{3}$ ¢	7		
	120 yds. calico @ $33\frac{1}{3}$ ¢	40		
	78 yds. gingham @ 45¢	35 10		
	420 yds. linen @ 75¢	315		
	5			
2	S. S. Stark, 540 Hillside Ave., 1/10 30 d			
	4845 lbs. codfish @ \$7.50 per cwt.			
	4625 lbs. lard @ \$2.25 per cwt.			
	51024 lbs. ice @ \$12.50 per ton			
	6			
2	M. O. Drains, R.R. 17, Box 8 1/10 60 d			
	2725 lbs. egg coal @ \$6.66 2/3 per ton			
	734 lbs. nut coal @ \$9.00 per ton			
	1725 lbs. mine-run @ 8¢ a bu.			
	7			
2	P. D. Quortz, 726 Walnut St., 1/10 60 d			
	724 bars soap @ 55 cts a dozen			
	1924 cans of Blue Whiz @ \$1.00 a gross			
	923 fly swatters @ \$7.50 a C.			
	8			
2	H. J. Kent, 15 Barclay St., 2/10 2/60			
	17 1/2 yds. duck @ \$1.16 2/3 cents			
	4 1/4 yds. flannel @ \$1.33 1/3 cents			
	7214 bone buttons @ \$5.20 M			
	15			
2	B. B. Cole, 407 Jeff. Ave., note 90 d			
	2 9 x 12 Wilton Rugs @ \$33.75			
	1 E. J. Swing Washer @ 16.43			
	3 Torch Chairs @ 7.60			
	Total Sales, Credit			2667 64

SALES BOOK

The first sale to J. B. Betts is properly extended. Place your sheet alongside this record and extend the other sales in a similar manner. The six sales should add to the amount shown as "Total Sales." Trace these items to their respective accounts in the ledger, pages 66-67.

Cash Receipts

Date	Account Name	Explanation	L F	Items	Total
1921 Jan 1	B. A. Mann	Investment	1	5600	
6	Notes Payable	Borrowed at Bank	1	3000	
8	Purchases Disc	On Briggs a/c	2	10493	
12	" "	On Davis paymt	2	2041	
12	J. B. Betts	In full of a/c	3	39710	
15	S. O. Stark	" " "	3	183853	
16	W. J. Kent	" " " "	3	6379	
	Total Receipts, Cash Dr				xxxxxx
					xxxxxx
Jan 23	Balance				33111

The above is the record of cash received; on the opposite page is the Cash Payments record. The Cash Receipts record combined with the Cash Payments record on the following page is often referred to as the Cash Book. They are also spoken of separately as the Cash Receipts Journal and the Cash Payments Journal. The total receipts less payments will give what is called Cash Balance. Add both sides and subtract. You should have the same balance as entered opposite Cash Balance on the payment side and brought down as net Cash Receipts.

1. What do the figures in the L. F. column indicate?
2. Why is it necessary to clearly explain the transaction in the explanation column?
3. Explain what should always be written in each of the seven columns.

Cash Payments

Date	Account Name	Explanation	L F	Items	Total
Jan 2	General Expense	Rent 1 mo	2	1 50	
2	Office Furniture	Desk & Chairs	2	2 25	50
2	General Expense	Stationery, Etc	2	1 72	0
5	Purchases	Inv #1, Ch #4	1	8 26	75
6	Int & Dis	Int on note	2	45	
8	W T Briggs	Inv #2, 2% Dis	2	5 24	62 5
12	J H Davis	\$1000 Dis 2% Inv #4	2	10 20	41
15	Sales Dis	On Starks Bill	2	3 67	77
16	" "	On Kent's Bill	2	1 28	
16	O M Gault	On a/c	3	10 24	19
20	Delivery, Eggt	Ford Truck	2	6 75	50
22	Purchases	Inv. #9	1	1 42	480
	Payments, Cash Cr				xxxxxx
	Cash Balance				33111
					xxxxxx

Check each individual item from both Cash Receipts and Payments to the respective ledger account. You will observe that the receipt items are posted to the credit (right) side of each account. Also note that the total received is posted to the left (debit) side of the cash account. Therefore the several credit items equal the one debit item to the cash account. The payments above are posted to the debit side of the separate accounts and the total to the credit of cash or just the reverse of Cash Receipts.

1. Explain what is meant by Cash Balance and how you arrive at such a balance.

2. Can you show that the accounts debited from the cash payments equal the amount credited to the cash account?

B. A. Mann, Capital

					1921 Jan. 1			C3	5600		
--	--	--	--	--	----------------	--	--	----	------	--	--

Cash

1921 Jan. 22			C2	110247.6	1921 Jan. 22			C3	10693.65		
-----------------	--	--	----	----------	-----------------	--	--	----	----------	--	--

Purchases

1921 Jan. 5			C3	826.75							
22			C3	1424.80							
22	Total Pur. Book P.			16248.29							

Sales

					1921 Jan. 22			S1	2669.64		
--	--	--	--	--	-----------------	--	--	----	---------	--	--

Notes Receivable

1921 Jan. 15			f1	106.73							
-----------------	--	--	----	--------	--	--	--	--	--	--	--

Notes Payable

					1921 Jan. 6			C2	3000		
					15			f1	3661.46		

General Expense

1921 Jan. 2			C3	150							
2			C3	17.20							

Office Furniture

1921 Jan. 2			C3	225.50							
----------------	--	--	----	--------	--	--	--	--	--	--	--

Delivery Equipment

1921 Jan.	20		G3	67550						
--------------	----	--	----	-------	--	--	--	--	--	--

Interest & Discount

[illegible]

Sales Discount

<i>Jan.</i>	15	C.3	3677						
	16	C.3	128						

Purchases Discount

[illegible]

M. O. Drain

[illegible]

P.D. Quartz

[illegible]

J. K. Davis

¹⁹²¹				¹⁹²¹					
Jan	12	C3	10.2041	Jan	5		P1	180729	
					5		P1	47968	

W. R. Rowan

[illegible]

O. M. Gault

1921 Jan 16		C3	102419	1921 Jan 6		P1	152419
----------------	--	----	--------	---------------	--	----	--------

Trial Balance of B. A. Mann Jan. 23, 1921

<i>B. A. Mann, Capital</i>				
<i>Cash</i>				
<i>Purchases</i>				
<i>Sales</i>				
<i>Notes Receivable</i>				
<i>Notes Payable</i>				
<i>General Expense</i>				
<i>Office Furniture</i>				
<i>Delivery Eqp't.</i>				
<i>Interest & Discount</i>				
<i>Sales Discount</i>				
<i>Purchases Discount</i>				
<i>M. O. Drain</i>				
<i>P. D. Quortz</i>				
<i>J. H. Davis</i>				
<i>W. R. Rowan</i>				
<i>O. M. Gault</i>				
		x x x x x	x x x x x	x x x x x

TRIAL BALANCE

If ruled paper cannot be had, rule a sheet like the above and complete the Trial Balance. The purpose of the Trial Balance is to prove that the ledger is in balance; that is, if both sides are equal. By adding the accounts in the ledger on the two preceding pages and entering the amounts in the respective columns you should arrive at a balance. Place the amounts on the left side of the account in the left column of the Trial Balance and the amounts on the right side of the account in the right column. Now add both sides of the Trial Balance. Your footings should show the same amount in both columns.

1. Can you explain why the debit (left) column in the above Trial Balance equals the credit (right) column?

Balance Sheet, B. A. Mann, Jan. 23, 1921

<u>Assets</u>			
Cash		xxxxxx	
Accts Receivable		xxxxxx	
M. O. Drain	xx.xx		
P. D. Quortz	<u>xxx.xx</u>		
Notes Receivable		xxxxxx	
Merch. Inventory (List of stock in store)		1754050	
Office Furniture (Cost) \$225.50			
Deduct 5% for Dep.	<u>11.28</u>	xxxxxx	
Delivery Eght. (Cost) \$675.50			
Deduct 2% for Dep.	<u>13.51</u>	xxxxxx	
Int. & Disc. paid in advance		2120	
<u>Total Resources</u>			1714924
<u>Liabilities</u>			
Notes Payable		xxxxxx	
Accounts Payable		xxxxxx	
J. W. Davis	xxxx.xx		
W. K. Rowan	xxxx.xx		
O. M. Gault	<u>xxx.xx</u>		
<u>Total Liabilities</u>			195744
B. A. Mann's Pres. Cap. (Assets less Liabilities)			719180
" " " Investment		5600	
" " " <u>Net Profit</u>		<u>159180</u>	
		<u>719180</u>	<u>719180</u>

The purpose of the Balance Sheet is to show the Worth or Capital of the owner of the business. His property or assets less his debts or liabilities gives his Present Worth or Capital as shown above. Rule a sheet and enter the missing items indicated by the crosses. These items can be had from the Trial Balance. The asset items should add to the total as given. The liability items should also add to the amount given.

Merchandise Inventory is found by making a list of goods in the store.

Profit and Loss Statement

<u>Profits</u>			
Sales	xxxxxx		
Deduct Sales Disc	xxxx		
Net Sales		263159	
Purchases	xxxxxx		
Deduct Inventory	1754050		
Cost of Sales		95934	
Profit on trading		167225	
Profit on Purchases Discounts		12534	
Total Profits		179759	
<u>Losses</u>			
General Expense	xxxxxx		
Depreciation 5% on Office Fur	xxxx		
Depreciation 2% on Deliv Eqp.	xxxx		
Interest & Discount paid \$45.			
Deduct Int. & Dis. in advance 31.20	1380		
Total Losses		20579	
Net Profit		159180	

The purpose of this statement is to show how the net profit is made. In other words, it shows a list of accounts recording the profits and losses. These accounts are those remaining on the Trial Balance after the Balance Sheet is made. From the Trial Balance enter the missing items as indicated by the crosses and see if you arrive at the result given. This statement, as the previous one, should be studied closely. Note that the profit here shown is the same as increased capital on the Balance Sheet.

Also note that the merchandise unsold (Inventory) deducted from purchases is the same amount as found in the Balance Sheet. The merchandise is listed and Inventory value found as indicated on the following page.

(47)

Copy and find the total value of stock.

MERCHANDISE INVENTORY*, JANUARY 31, 1921

73 lbs. Granulated Sugar... (per 100 lbs.)	\$ 5.25
196 lbs. Brown Sugar..... (per 100 lbs.)	5.10
1 doz. Ivory Soap..... (per doz.)	.35
3 doz. Blacking..... (per doz.)	.30
46 lbs. Arbuckle Coffee..... @	.15
13 lbs. Tea..... @	.25
38 gal. Coal Oil..... @	.11
646 lbs. Bacon..... @	.25
1388 lbs. Shoulder..... @	.33 $\frac{1}{3}$
119 lbs. Lard..... @	.18
24 doz. Cans Tomatoes..... (per can)	.16
35 gal. Syrup..... @	.35
8 bu. Irish Potatoes..... @	1.25
10 bu. Sweet Potatoes..... @	1.40
127 lbs. Trent Tobacco..... @	.35
42 lbs. Smoking Tobacco..... @	.30
3863 Daniel Boone Cigars..... (per 1000)	33.00
1896 Key West Cigars..... (per 1000)	26.00
23 bbls. Roller King Flour..... @	6.25
24 bbls. White Lily Flour..... @	5.85
36 bu. Meal..... @	.75
80 lbs. Creamery Butter..... @	.48
98 doz. Eggs..... @	.25
3 bu. Beans..... @	2.50
9261 lbs. Hay..... (per ton of 2000 lbs.)	17.65
73 bu. Corn..... @	.90
88 bu. Oats..... @	.37 $\frac{1}{2}$
5963 lbs. Bran..... (per 1000 lbs.)	7.50

Value of merchandise on hand.....

*Means the value of the merchandise unsold in the store.

(48)

BOOKKEEPING EXERCISE

Enter the following transactions for the month of January, 1922. The Cash Book, Sales Book, and Purchases Book may be used as in preceding models.

1922

- January 1. H. E. Dennison invested \$4,000.00 in the retail grocery business.
2. Bought of Norris, Inc., on account, merchandise per invoice of this date, \$154.60.
 3. Bought of M. C. Kaiser & Co., on account, merchandise per invoice received this date, \$269.90.
 4. Paid \$40.00 cash, city and state license for one year. Sold merchandise to John R. Byington, on account, \$28.50.
 5. Bought of McCord-Steward & Co., for cash per invoice of this date, \$384.00.
 6. Received \$60.00 for sundry cash sales to date. Paid Norris, Inc., \$154.60 in full of account.
 8. Sold merchandise to W. S. Kell, on account, \$114.70.
 9. Received \$20.00 from J. R. Byington to apply as part payment of the amount he owes us.
 10. Bought of the American Sugar Co., on account, merchandise per invoice of this date, \$457.20.
 11. Sold merchandise to J. R. Byington, on account, \$108.50.
 12. Received from W. S. Kell, \$60.00, in part payment of the bill sold them on the 8th.
 13. Received \$80.00 for sundry cash sales to date. Paid M. C. Kaiser & Co. \$100.00 on account. Sold merchandise to Dr. Wm. Ownes, on account, \$104.80.
 15. Bought of Norris, Inc., on account, merchandise per invoice of this date, \$492.00.
 16. Sold merchandise to T. B. Loggins, on account, \$98.82.

- Jan. 17. Bought of J. P. Allen & Co., on twenty days' time, merchandise per invoice of January 14, \$56.00.
18. Received of Dr. Wm. Ownes \$75.00 to apply on account.
19. Sold merchandise to T. N. Colley, on account, \$84.60.
20. Paid M. C. Kaiser & Co. the balance due them.
Received \$100.00 for sundry cash sales to date.
22. Sold merchandise to J. F. Cannon, on account, \$48.10.
23. Bought of McCord-Stewart & Co., at 10 days, merchandise per invoice of this date, \$294.20.
24. Received \$30.00 from W. S. Kell on account.
Paid American Sugar Co. \$250.00 on account.
25. Sold merchandise to T. B. Loggins, on account, \$160.00.
26. Received \$8.50 from J. R. Byington in full of balance due on bill sold him January 4.
Sold merchandise to P. F. Nichols, on account, \$156.50.
27. Sold merchandise to G. A. Denfield, on account, \$84.50.
Received \$85.00 for sundry cash sales to date.
Bought of S. P. Richards & Co., on account, merchandise per invoice of January 24, \$345.50.
29. Received \$100.00 from P. F. Nichols in part payment of the amount he owes us.
30. Paid Norris, Inc., \$300.00 to apply on amount we owe them.
31. Paid bookkeeper's salary, \$50.00, and rent, \$70.00.

A. Open following accounts in the ledger, leaving space as indicated: H. E. Dennison, Proprietor, top of first page, eight lines; Cash, Purchases, Sales, Inventory, Expense, Profit and Loss, 15 lines each. The following accounts, four lines each: Norris, Inc.; M. C. Kaiser & Co.; American Sugar Co.; J. P. Allen & Co.; McCord-Stewart & Co.; S. P. Richards & Co.; J. R. Byington; W. S. Kell; Dr. William Ownes; T. B. Loggins; T. N. Colley; J. F. Cannon; G. A. Denfield; P. F. Nichols. Post and take a Trial Balance. Prepare a Profit and Loss Statement and Balance Sheet as of January 31, 1922.

By going through the stock of goods and valuing the goods on hand, the inventory was found to be \$1,764.92.

(49)

EXERCISES IN MAKING STATEMENTS

1. Find the Present Worth of J. B. Aaron, who gives the following Assets and Liabilities: (Total Assets—Liabilities = Present Worth) Cash, \$325.00; Mdse. Inventory, \$1250.00; Furniture at cash \$250.00 (Allow 10% depreciation on Furniture); Notes Receivable \$180.00; Office Supplies (at cost), \$18.50; Due from A. B. Davis, \$35.50, C. E. Caram, \$15.40 and L. B. Coons, \$52.20. Liabilities: Notes Payable, \$350.00; C. D. Daniels, \$195.50; A. Smith, \$260.00, and W. W. Brown, \$29.30. See form of Balance Sheet, page 69.

2. W. W. Kennedy reports the following for the month of May: Purchases, \$1654.10; Sales, \$996.58; Mdse. in the Store, \$927.36; and Expense, \$107.50. What was his profit for the month?

3. J. W. O'Dell's ledger contains the following accounts after posting on December 31, 1921: J. W. O'Dell's Capital, Cr., \$1500.00; Cash, Dr., \$1215.50; Purchases, Dr., \$1116.75; Sales, Cr., \$718.10; Expense, Dr., \$50.50; Johnston Bros., Cr., \$218.75; Rice and May, Cr., \$197.62; E. K. Isaacs, Dr., \$125.18; J. C. Sisk, Dr., \$110.15; W. W. Moore, Dr., \$62.50; A. L. Peer, Cr., \$46.11. From the above accounts prepare a Trial Balance as illustrated on page 68.

4. O'Dell reports merchandise in the store amounting to \$565.40. With this Inventory and the Trial Balance given in No. 3, prepare the Balance Sheet and Profit and Loss Statements.

5. A retail grocer at the end of the fiscal period gives you the following list of his assets and liabilities:

Cash on hand.....	\$320.00	Int. due on Notes Rec.	\$15.00
Mdse. Inventory.....	2850.00	Accounts Payable....	640.00
Accounts Receivable..	675.20	Notes Payable.....	800.00
Notes Receivable.....	360.00	Int. on Notes Payable	10.00

Prepare a statement showing his total resources, total liabilities and his Present Worth.

His investment at the beginning of the period was \$2500.00. Did he gain or lose during the year, and how much?

(50)

GENERAL DRILL PROBLEMS

1. 25% of \$840, is what per cent. of \$210?
2. Sold 84 dozen eggs at $33\frac{1}{3}$ c and bought cloth at \$1.50. How many yards were bought?
3. Interest 6% on \$920 for 3 days.
4. Cost to fence a quarter section at \$3.00 a rod?
5. Cost to plow a half section at \$2.50 an acre?
6. $7 \times 8 \times 0 + 8 = ?$
7. Find the sum of the prime numbers between 30 and 50.
8. Interest at 3% on \$1000 for 90 days.
9. Sold a hat for \$2 and lost 40c. Find loss per cent. on cost.
10. $2\frac{1}{3}$ is what per cent. of $7\frac{1}{2}$?
11. Spent $\frac{2}{5}$ of my money and had \$36 left. How much did I spend?
12. Find the difference between 2 sq. yards and 2 yds. square.
13. Cost of 4120 pounds of ore at \$22 a ton.
14. Cost of 780 pounds coal at 9c a bushel.
15. $12\frac{1}{2}\%$ of \$420 is $33\frac{1}{3}\%$ of?
16. From .87275 take $\frac{1}{33}$.
17. $23\frac{4}{5}$ is $\frac{2}{3}$ of?
18. Interest on \$1820 for 2 months, 10 days at 6% = ?
19. Cost of 40 stoves at \$90 less 20%, 12% and 10%?
20. Multiply $186\frac{2}{3}$ by $66\frac{2}{3}$.

Your attention has been called to the fact that you can use as an aliquot part either the price or quantity. For example: 16 pounds of prunes at 25c. We know that 25c is $\frac{1}{4}$ of a dollar and know that $\frac{1}{4}$ of 16 equals \$4, the answer. Now notice that 25 lbs. of prunes at 16c can be handled in the same way. Be careful with the decimals. Look at the problem carefully and approximate your answer. You know that 150 lbs. of sugar at $6\frac{1}{2}$ c per lb. would be near \$10 and not near \$1,000 or \$1.00.

(51)

Give the results to the following without the aid of a pencil:

- | | |
|--|--|
| 1. 90 bu. of oats at $33\frac{1}{3}c$ | 6. $16\frac{2}{3}$ lbs. of butter at $42c$ |
| 2. $33\frac{1}{3}$ yds. of silk at $90c$ | 7. 24 doz. eggs at $37\frac{1}{2}c$ |
| 3. 120 lbs. of candy at $25c$ | 8. $37\frac{1}{2}$ yds. ribbon at $32c$ |
| 4. 25 yds. of cloth at \$1.20 | 9. $8\frac{1}{3}$ lbs. bacon at $48c$ |
| 5. 36 lbs. of lard at $16\frac{2}{3}c$ | 10. 24 lbs. apples at $8\frac{1}{3}c$ |

11. Change 9427 lbs. to cwt., to tons.

12. Change 4384 lbs. to cwt., to tons.

13. Change 134 lbs. to cwt., to tons.

14. Change 72 lbs. to cwt., to tons.

15. Change 8 lbs. to cwt., to tons.

QUESTIONS ON THE FUNDAMENTAL OPERATIONS

1. What is meant by *reading results* in adding a column?
2. What are the fundamental operations?
3. What is meant by a combination in adding? In what way are they an advantage?
4. In adding do you look at one figure only, or at the column?
5. What is the Civil Service Method of addition? When should it be used?
6. When should *long division* be employed?
7. How do you divide by multiples of 10?
8. How are lbs. changed to cwt. and tons?
9. Explain how you would divide 480 by $133\frac{1}{3}$ and by $66\frac{2}{3}$.
10. How do you prove the accuracy of a problem in addition by the check figure 9? By the check figure 11?
11. How do you prove accuracy in multiplication by the check figure 9? By the check figure 11?
12. Can you add 100 figures a minute accurately?

CHAPTER IV

COMMON FRACTIONS

Fractions of all kinds are constantly used in business computations. Practice the drills and exercises given until they can be handled with all ease, for no one can expect to handle ordinary business transactions without a working knowledge of fractions.

A unit is any single quantity by which another quantity of the same kind is measured. 1 inch is the unit of 4 inches, 1 yard the unit of 10 yards, 1 the unit of 10, etc.

A fraction is one or more of the equal parts of a unit. Therefore, fractional numbers are the result of the division of units into parts.

1											
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$			
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$	
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

What is the difference between $\frac{1}{1}$, $\frac{3}{3}$, $\frac{6}{6}$, $\frac{12}{12}$?

Numerator (Number) expresses the number of fractional units considered.

Denominator (Denominates) or names the equal parts into which the unit or whole is divided.

In $\frac{3}{4}$, and 5 gallons, the denominators 4 and gallons, express the parts of the unit (base), while 3 and 5, the numerators, express the number of these parts belonging to the fraction.

To analyze further: $\frac{3}{4}$ indicates that the fraction contains 3 units, 4 being the denominator shows that the value of each unit is $\frac{1}{4}$ of an integral unit.

A fraction may also arise through the *comparison* of numbers. When we compare 3 with 8 we have $\frac{3}{8}$. It may be thought of as being an indicated division as $3 \div 4 = \frac{3}{4}$, which if divided gives us .75. This aids materially in showing the close relation existing between *division* (out of which fractions grow), *common* and *decimal fractions*, *percentage* and *ratio*.

A Common Fraction: $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$. A fraction the terms of which are expressed in figures and whose denominator is not a power of 10.

A Complex Fraction: $\frac{5\frac{1}{2}}{4}$, $\frac{2}{7\frac{3}{8}}$, $\frac{\frac{3}{8}}{2\frac{1}{2}}$. One with a fraction in one or both of its terms.

A Decimal Fraction: $\frac{2}{10}$, $\frac{20}{100}$, $\frac{500}{1000}$. A fraction the denominator of which is a power of 10, generally expressed by the use of the decimal points; .4, .20, .050, etc.

A Proper Fraction: $\frac{2}{5}$, $\frac{1}{7}$, $\frac{7}{11}$. A fraction the numerator of which is less than its denominator.

An Improper Fraction: $\frac{3}{3}$, $\frac{8}{7}$, $\frac{6}{5}$. A fraction the numerator of which is equal to or greater than its denominator.

Mixed Number: $4\frac{2}{3}$, $7\frac{1}{8}$, $16\frac{2}{3}$. One containing a whole number and a fraction.

NOTE: The use of improper fractions should be avoided whenever possible. Reduction of a mixed number to an improper fraction is not a good practice, especially where the whole number is larger than 5, as it increases the work unnecessarily and leads to serious blunders.

REDUCTIONS OF FRACTIONS

Reduction of Fractions is the process of changing their forms to higher or lower terms without altering their values.

To *Lower Terms*: $\frac{5}{20} = \frac{1}{4}$, $\frac{2}{10} = \frac{1}{5}$, $\frac{4}{6} = \frac{2}{3}$, etc.

To *Higher Terms*: $\frac{1}{3} = \frac{2}{6}$, $\frac{2}{3} = \frac{4}{6}$, $\frac{3}{8} = \frac{1}{2}$, etc.

Notice that the value is not changed by multiplying or dividing both the numerator and denominator by the same number.

It should be noted that the value of any fraction depends upon the ratio or relation existing between the numerator and denominator. The approximate value of any fraction can thus be quickly given.

$\frac{71}{279}$ is non-reducible, but by observation it is discovered that the numerator is about one-fourth of the denominator and the fraction nearly equivalent to one-fourth. In many instances an approximate result is sufficient. You will therefore do well to keep in mind the value of deciding approximately the answer of your problem before you begin to solve it.

(52)

ORAL

Reduce to lowest terms:

1. $\frac{6}{12} =$	7. $\frac{9}{24} =$	13. $\frac{5}{10} =$	19. $\frac{4}{10} =$
2. $\frac{24}{32} =$	8. $\frac{25}{30} =$	14. $\frac{15}{20} =$	20. $\frac{28}{48} =$
3. $\frac{16}{24} =$	9. $\frac{14}{35} =$	15. $\frac{18}{63} =$	21. $\frac{21}{49} =$
4. $\frac{4}{10} =$	10. $\frac{2}{6} =$	16. $\frac{3}{9} =$	22. $\frac{7}{8} =$
5. $\frac{18}{48} =$	11. $\frac{9}{27} =$	17. $\frac{14}{28} =$	23. $\frac{2}{10} =$
6. $\frac{21}{49} =$	12. $\frac{3}{36} =$	18. $\frac{6}{24} =$	24. $\frac{3}{14} =$

Change to higher terms:

1. $\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{2}{3}$ to 12ths
2. $\frac{5}{6}$	$\frac{6}{7}$	$\frac{2}{3}$	$\frac{1}{6}$ to 42ds
3. $\frac{1}{9}$	$\frac{2}{6}$	$\frac{3}{4}$	$\frac{5}{6}$ to 36ths
4. $\frac{2}{3}$	$\frac{11}{12}$	$\frac{5}{6}$	$\frac{5}{9}$ to 72ds

(53)

Reduce to lower terms by approximation:*

To illustrate: $\frac{17}{72}$ = about $\frac{1}{4}$.

- | | | | | |
|-----------------------|-------------------------|--------------------------|------------------------|---------------------------|
| 1. $\frac{17}{72} =$ | 5. $\frac{19}{150} =$ | 9. $\frac{630}{941} =$ | 13. $\frac{98}{787} =$ | 17. $\frac{784}{2380} =$ |
| 2. $\frac{25}{96} =$ | 6. $\frac{49}{198} =$ | 10. $\frac{196}{485} =$ | 14. $\frac{39}{160} =$ | 18. $\frac{1524}{2280} =$ |
| 3. $\frac{23}{116} =$ | 7. $\frac{702}{2098} =$ | 11. $\frac{890}{1001} =$ | 15. $\frac{5}{19} =$ | 19. $\frac{49}{65} =$ |
| 4. $\frac{78}{233} =$ | 8. $\frac{411}{3685} =$ | 12. $\frac{14}{79} =$ | 16. $\frac{39}{385} =$ | 20. $\frac{51}{331} =$ |

Change: $\frac{11}{12}$ to 108thsSolution: $108 \div 12 = 9$ (indicating that both terms of the fraction $\frac{11}{12}$ must be multiplied by 9), $\frac{11}{12} \times \frac{9}{9} = \frac{99}{108}$, Ans.

Change:

- $\frac{7}{15}$ to 150ths
 $\frac{6}{12}$ to 144ths
 $\frac{2}{3}$ to 126ths
 $\frac{11}{23}$ to 299ths
 $\frac{26}{35}$ to 350ths

Change:

- $\frac{11}{15}$ to 450ths
 $\frac{9}{11}$ to 660ths
 $\frac{7}{8}$ to 736ths
 $\frac{8}{9}$ to 828ths
 $\frac{15}{21}$ to 441sts

AXIOMS FOR FRACTIONS

1. Multiplying both numerator and denominator of a fraction by the same number does not change their relation or the value of the fraction.

How this applies:

 $\frac{7}{25}$ is how many hundredths?

We can divide 7 by 25 or, seeing that to multiply 25 by four will change 25 to 100, we also multiply the 7 by 4 and get $\frac{28}{100}$.

Drill:

- | | |
|--|---|
| 1. $\frac{3}{10}$ = how many hundredths? | 4. $\frac{5}{33\frac{1}{3}}$ = how many hundredths? |
| 2. $\frac{5}{4}$ = how many hundredths? | 5. $\frac{5}{12\frac{1}{2}}$ = how many hundredths? |
| 3. $\frac{4}{8}$ = how many forty-eighths? | 6. $\frac{5}{9}$ = how many sixty-thirds? |

*In all your work you should first approximate your answer.

If a fraction appears in either numerator or denominator, eliminate by multiplying both numerator and denominator by the denominator of the fraction within the fraction.

For example: $\frac{8\frac{1}{3}}{50} = \frac{25}{150}$ and $\frac{4}{4\frac{1}{6}} = \frac{24}{25}$.

2. Increasing or decreasing the numerator and denominator in like part or proportion does not change their relation or the value of the fraction.

How this applies:

$\frac{3}{75}$ = how many hundredths?

75 increased by $\frac{1}{3}$ of itself equals 100, and 3 increased by $\frac{1}{3}$ of itself equals 4, therefore, $\frac{3}{75}$ is equivalent to $\frac{4}{100}$.

Drill:

(54)

1. $\frac{18}{120}$ = how many hundredths? 5. $\frac{240}{1200}$ = how many hundredths?

2. $\frac{25}{125}$ = how many hundredths? 6. $\frac{141}{150}$ = how many hundredths?

3. $\frac{16}{80}$ = how many hundredths? 7. $\frac{1\frac{1}{2}}{4}$ = how many hundredths?

4. $\frac{45}{72}$ = how many sixty-fourths? 8. $\frac{15}{24}$ = how many thirty-seconds?

3. Dividing both numerator and denominator by the same number does not change their relation or the value of the fraction. This principle is commonly known and much used in the reduction of fractions.

4. If the numerator of a fraction is less than $\frac{1}{10}$ of the denominator, the equivalent decimal cannot be written without a cipher to the right of the decimal point. $\frac{7}{84} = .08\frac{1}{3}$. $\frac{9}{91} = .09\frac{81}{91}$.

5. Multiplying the numerator or dividing the denominator, multiplies the fraction. Multiplying $\frac{7}{45}$ by 5, we have:

$$\frac{7 \times 5}{45} = \frac{35}{45} = \frac{7}{9} \text{ or } \frac{7}{45 \div 5} = \frac{7}{9}$$

6. Dividing the numerator or multiplying the denominator divides the fraction. Dividing $\frac{8}{38}$ by 4 we have:

$$\frac{8}{38 \times 4} = \frac{8}{152} = \frac{1}{19} \text{ or } \frac{8 \div 4}{38} = \frac{2}{38} = \frac{1}{19}.$$

7. In general, therefore, any change in the numerator produces a like change in the value of the fraction, and any change in the denominator produces an exactly opposite change in the value of the fraction.

(55)

WRITTEN

Change to lowest terms:

$$\begin{array}{llll} 1. \frac{72}{99} = & 6. \frac{121}{132} = & 11. \frac{316}{428} = & 16. \frac{264}{936} = \\ 2. \frac{147}{252} = & 7. \frac{528}{660} = & 12. \frac{924}{1386} = & 17. \frac{97}{582} = \\ 3. \frac{144}{216} = & 8. \frac{153}{204} = & 13. \frac{153}{675} = & 18. \frac{414}{999} = \\ 4. \frac{144}{396} = & 9. \frac{684}{1026} = & 14. \frac{437}{551} = & 19. \frac{355}{568} = \\ 5. \frac{783}{2889} = & 10. \frac{603}{1005} = & 15. \frac{729}{972} = & 20. \frac{1016}{1248} = \end{array}$$

ORAL DRILL

Read rapidly:

1. $\frac{1}{2}$ = how many 4ths, 10ths, 6ths, 12ths, 8ths, 24ths, 16ths, 14ths, 20ths, 32ds, 26ths, 40ths, 72ds, 18ths, 22ds, 34ths, 30ths, 88ths, 56ths, 38ths, 76ths, 64ths?
2. $\frac{1}{3}$ = how many 9ths, 30ths, 6ths, 15ths, 21sts, 18ths, 45ths, 12ths, 27ths, 72ds, 51sts, 33ds, 42ds, 39ths, 54ths, 60ths, 36ths, 57ths, 48ths, 69ths, 75ths, 93ds?
3. $\frac{1}{4}$ = how many 8ths, 16ths, 24ths, 44ths, 12ths, 56ths, 64ths, 32ds, 40ths, 52ds, 84ths, 96ths, 76ths, 36ths, 48ths, 28ths, 60ths, 68ths, 80ths, 88ths, 92ds?
4. $\frac{1}{5}$ = how many 5ths, 30ths, 45ths, 20ths, 80ths, 65ths, 15ths, 35ths, 75ths, 90ths, 55ths, 70ths, 25ths, 50ths, 10ths, 60ths, 95ths, 125ths, 100ths, 110ths?
5. $\frac{1}{6}$ = how many 18ths, 36ths, 42ds, 12ths, 90ths, 72ds, 84ths, 24ths, 96ths, 120ths, 30ths, 48ths, 54ths, 126ths, 78ths, 108ths, 66ths, 114ths, 132ds, 144ths?
6. $\frac{1}{7}$ = how many 21sts, 140ths, 49ths, 42ds, 84ths, 105ths, 147ths, 112ths, 28ths, 91sts, 35ths, 126ths, 63ds, 77ths, 70ths, 98ths, 119ths, 133ds, 154ths, 210ths?

7. $\frac{1}{8}$ = how many 88ths, 160ths, 72ds, 24ths, 168ths, 240ths, 112ths
16ths, 120ths, 56ths, 48ths, 40ths, 248ths, 320ths
32ds, 64ths, 80ths, 96ths, 104ths, 128ths, 136ths?
8. $\frac{1}{9}$ = how many 72ds, 108ths, 270ths, 450ths, 27ths, 117ths
180ths, 207ths, 45ths, 18ths, 234ths, 36ths
90ths, 54ths, 81sts, 126ths, 63ds, 153ds, 99ths?
9. $\frac{1}{12}$ = how many 24ths, 144ths, 240ths, 48ths, 156ths, 72ds
60ths, 36ths, 84ths, 192ds, 108ths, 168ths
96ths, 120ths, 180ths, 132ds, 276ths, 360ths?

(56)

EASY PROBLEMS IN FRACTIONS

Like Denominators:

$\frac{1}{8} + \frac{2}{8} + \frac{5}{8} + \frac{9}{8} = \frac{1+2+5+9}{8} = \frac{17}{8}$ or $2\frac{1}{8}$, Ans. Why are these easy to add?

$\frac{5}{8} - \frac{3}{8} = \frac{5-3}{8} = \frac{2}{8}$, Ans. Why is this operation simple?

$\frac{9}{8} \div \frac{1}{8} = (9 \div 1)$ or 9, Ans. Why is this operation simple?

When adding fractions with the same denominators add the numerators and divide by the denominator of either fraction.

In subtracting when the denominators are alike subtract the numerators as shown above.

In dividing fractions when the denominators are the same, disregard the denominator and write for the result the quotient of the numerators, as:

$\frac{10}{8} \div \frac{2}{8} = (10 \div 2) = \frac{10}{2} = 5$, Ans. Notice here that the denominator (8) is dropped; in adding and subtracting it is not dropped.

ORAL

Add:

1. $\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} =$

2. $\frac{1}{10} + \frac{2}{10} + \frac{4}{10} + \frac{6}{10} =$

3. $\frac{7}{6} + \frac{2}{6} + \frac{4}{6} + \frac{1}{6} =$

Subtract:

4. $\frac{9}{16} - \frac{7}{16} =$

5. $\frac{4}{53} - \frac{1}{53} =$

6. $\frac{13}{64} - \frac{5}{64} =$

Divide:

7. $\frac{18}{12} \div \frac{3}{12} =$

8. $\frac{21}{43} \div \frac{7}{43} =$

9. $\frac{105}{124} \div \frac{35}{124} =$

ADDITION WHEN DENOMINATORS ARE NOT ALIKE

1. Why can you not add 4 ft. and 3 yds.?
2. Why can you not add $\frac{1}{2}$ day and 4 hrs.?
3. Why can you not add $\frac{3}{4}$ and $\frac{1}{2}$?

Before adding, the fractions must be expressed in similar units.

1. As we cannot add 4 ft. and 3 yds., we must change feet to yards or yards to feet, thus:

4 feet = $1\frac{1}{3}$ yards + 3 yds. = $4\frac{1}{3}$ yds., Ans.; or 3 yards = 9 feet + 4 feet = 13 feet, Ans.

2. $\frac{1}{2}$ day = 12 hours + 4 hours = 16 hours, Ans.; or 4 hours = $\frac{1}{6}$ day + $\frac{1}{3}$ day or $\frac{1}{6} + \frac{2}{6} = \frac{3}{6}$ or $\frac{1}{2}$ day, Ans.

3. $\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4} = \frac{5}{4}$ or $1\frac{1}{4}$, Ans.

(57)

PROBLEMS FOR RAPID ORAL DRILL

Add the following by reducing to a Common Denominator:

Example: $(\frac{3}{4} + \frac{1}{2} + \frac{2}{3}) = (\frac{9}{12} + \frac{6}{12} + \frac{8}{12}) = \frac{23}{12}$ or $1\frac{11}{12}$, Ans.

1. $\frac{4}{5} + \frac{1}{2} = 10.$ $\frac{1}{2} + \frac{1}{4} = 19.$ $\frac{1}{8} + \frac{3}{4} = 28.$ $\frac{4}{5} + \frac{2}{3} = 37.$ $\frac{1}{3} + \frac{1}{2} =$
2. $\frac{3}{4} + \frac{2}{3} = 11.$ $\frac{1}{4} + \frac{2}{3} = 20.$ $\frac{1}{2} + \frac{3}{6} = 29.$ $\frac{5}{8} + \frac{1}{2} = 38.$ $\frac{3}{4} + \frac{5}{7} =$
3. $\frac{1}{6} + \frac{2}{3} = 12.$ $\frac{3}{4} + \frac{5}{8} = 21.$ $\frac{9}{12} + \frac{1}{6} = 30.$ $\frac{2}{3} + \frac{7}{8} = 39.$ $\frac{1}{6} + \frac{2}{3} =$
4. $\frac{3}{8} + \frac{1}{4} = 13.$ $\frac{2}{3} + \frac{1}{6} = 22.$ $\frac{5}{8} + \frac{3}{4} = 31.$ $\frac{5}{6} + \frac{2}{3} = 40.$ $\frac{4}{5} + \frac{1}{2} =$
5. $\frac{5}{8} + \frac{7}{16} = 14.$ $\frac{5}{4} + \frac{3}{8} = 23.$ $\frac{3}{7} + \frac{1}{14} = 32.$ $\frac{3}{14} + \frac{6}{7} = 41.$ $\frac{7}{11} + \frac{1}{5} =$
6. $\frac{1}{12} + \frac{2}{3} = 15.$ $\frac{5}{6} + \frac{1}{3} = 24.$ $\frac{6}{11} + \frac{1}{2} = 33.$ $\frac{1}{3} + \frac{3}{8} = 42.$ $\frac{3}{21} + \frac{6}{7} =$
7. $\frac{7}{8} + \frac{1}{4} = 16.$ $\frac{1}{6} + \frac{5}{12} = 25.$ $\frac{2}{14} + \frac{5}{7} = 34.$ $\frac{5}{7} + \frac{7}{8} = 43.$ $\frac{3}{15} + \frac{4}{5} =$
8. $\frac{8}{9} + \frac{2}{3} = 17.$ $\frac{1}{12} + \frac{1}{12} = 26.$ $\frac{2}{9} + \frac{1}{6} = 35.$ $\frac{5}{6} + \frac{9}{10} = 44.$ $\frac{1}{12} + \frac{3}{8} =$
9. $\frac{4}{7} + \frac{4}{5} = 18.$ $\frac{6}{10} + \frac{6}{7} = 27.$ $\frac{2}{7} + \frac{2}{5} = 36.$ $\frac{8}{9} + \frac{1}{3} = 45.$ $\frac{3}{32} + \frac{5}{16} =$

FINDING THE LEAST COMMON DENOMINATOR

The common denominator of a series of fractions is not always easily determined as in the case of $\frac{4}{21}$, $\frac{1}{35}$ and $\frac{8}{15}$. It is always true that the product of the several denominators is a common denominator, but to eliminate overlarge numbers and facilitate the work, the Least Common Denominator, or Multiple, should be used.

How the L. C. M. is found:

$$3) 24 - 35 - 15$$

$$5) 8 - 35 - 5$$

$$2) 8 - 7 - 1$$

$$2) 4 - 7 - 1$$

$$2 - 7 - 1$$

NOTE: Where the divisor will not evenly divide one of the numbers, bring that number down.

The product of the divisors and remainders (all of which are prime numbers) is the L. C. M., $3 \times 5 \times 2 \times 2 \times 2 \times 7 = 840$

(Had we multiplied $24 \times 35 \times 15$ we would have used 12600 instead.)

(58)

Problems in Finding L. C. M.

$$1. \frac{19}{72}, \frac{14}{45}, \frac{5}{9}$$

$$2. \frac{24}{51}, \frac{17}{79}, \frac{15}{81}$$

$$3. \frac{48}{95}, \frac{9}{57} \text{ (See Note)}$$

$$4. \frac{4}{793}, \frac{11}{1037}$$

$$5. \frac{1}{144}, \frac{3}{360}, \frac{5}{1800}$$

$$6. \frac{8}{105}, \frac{9}{70}, \frac{6}{315}$$

$$7. \frac{5}{341}, \frac{14}{527}$$

$$8. \frac{5}{34}, \frac{7}{51}, \frac{2}{85}$$

$$9. \frac{7}{58}, \frac{14}{84}, \frac{5}{140}, \frac{16}{70}$$

$$10. \frac{7}{11}, \frac{5}{13}, \frac{4}{29}$$

NOTE: If any one of the denominators is a factor of one of the others, it may be omitted in the division, as 9 in problem number one.

Where factoring is difficult as in number three, test the divisibility of the numbers as follows:

$$95 \div 57$$

$$57 \overline{) 138}$$

$$38 \overline{) 19} = \text{last division} = \text{G. C. D.}$$

$$38 \overline{) 2} \quad (19 = \text{prime factor.})$$

is the G. C. D. If the last divisor is 1, the numbers are prime and their product is the L. C. M.

EXPLANATION: The remainder in each division is used as the divisor in the next, dividing the previous divisor, $95 \div 57$, then $57 \div 38$, etc. The last remaining divisor

NOTE: The process is based on this axiom; the factors of any two numbers are also factors of their sum, difference, or product; i. e., 7 is a factor of 21 and 98, and also of their difference 77; their sum, 119; and their product, 2058.

(59)

WRITTEN

Try some of these mentally:

$$1. \frac{1}{6} + \frac{2}{3} + \frac{3}{4} + \frac{3}{2} =$$

$$2. \frac{3}{5} + \frac{1}{10} + \frac{1}{2} + \frac{5}{6} =$$

$$3. \frac{4}{3} + \frac{5}{8} + \frac{7}{12} + \frac{1}{6} =$$

$$4. \frac{5}{7} + \frac{2}{3} + \frac{7}{9} + \frac{5}{8} =$$

$$5. \frac{1}{9} + \frac{7}{12} + \frac{2}{3} + \frac{7}{6} =$$

$$6. \frac{5}{24} + \frac{3}{8} + \frac{5}{16} + \frac{8}{9} =$$

$$7. \frac{3}{16} + \frac{3}{5} + \frac{5}{10} + \frac{2}{15} =$$

$$8. \frac{3}{72} + \frac{9}{24} + \frac{7}{8} + \frac{5}{12} =$$

$$9. \frac{24}{80} + \frac{2}{5} + \frac{7}{16} + \frac{1}{6} =$$

$$10. \frac{32}{65} + \frac{3}{13} + \frac{4}{5} + \frac{9}{10} =$$

(60)

IMPROPER FRACTIONS

Reduce mentally to improper fractions:

1. $1\frac{2}{3}, 3\frac{1}{8}, 6\frac{1}{3}, 4\frac{1}{5}, 2\frac{1}{3}, 3\frac{1}{5}, 5\frac{1}{5}, 4\frac{1}{6}, 3\frac{1}{8}, 29\frac{3}{8}$.

2. $2\frac{1}{9}, 3\frac{7}{8}, 6\frac{1}{4}, 7\frac{2}{3}, 7\frac{1}{5}, 6\frac{2}{9}, 3\frac{1}{7}, 8\frac{1}{3}, 9\frac{1}{2}, 17\frac{3}{5}$.

3. $10\frac{1}{4}, 5\frac{4}{5}, 6\frac{7}{8}, 9\frac{5}{7}, 10\frac{3}{8}, 12\frac{1}{2}, 11\frac{1}{9}, 12\frac{3}{4}, 16\frac{3}{4}, 85\frac{1}{4}$.

4. $11\frac{3}{9}, 12\frac{5}{6}, 9\frac{2}{3}, 9\frac{6}{7}, 10\frac{4}{5}, 10\frac{7}{8}, 11\frac{5}{5}, 12\frac{7}{9}, 23\frac{7}{8}, 77\frac{2}{3}$.

5. $12\frac{1}{12}, 12\frac{6}{7}, 13\frac{1}{4}, 12\frac{1}{9}, 13\frac{2}{3}, 14\frac{1}{7}, 14\frac{1}{6}, 12\frac{3}{5}, 19\frac{2}{3}, 39\frac{1}{7}$.

NOTE: A decided advantage is generally gained in avoiding the use of improper fractions, especially where the numbers are large. Examine your work frequently, and where improper fractions appear change the operation, if possible, until time-saving habits are formed. Where the improper fraction method is the shortest, use it, but avoid it elsewhere.

**TO SHOW THE TIME AND LABOR SAVED IN
AVOIDING IMPROPER FRACTIONS**

Find the Sum, Difference, Product and Quotient of $27\frac{2}{3}$ and $81\frac{3}{4}$ (an average problem).

THE LONG METHOD

1. $27\frac{2}{3} = \frac{8}{12}$
 $81\frac{3}{4} = \frac{9}{12}$

$109\frac{5}{12} = \text{Sum}$

or, $\frac{83}{3} + \frac{327}{4} = \frac{332}{12} + \frac{981}{12} = \frac{1313}{12} =$
 $109\frac{5}{12}, \text{Sum.}$

A saving of $\frac{1}{3}$ of your time.

2. $81\frac{3}{4} = \frac{9}{12}$
 $27\frac{2}{3} = \frac{8}{12}$

$54\frac{1}{12} = \text{Difference}$

or, $81\frac{3}{4} - 27\frac{2}{3} = \frac{327}{4} - \frac{83}{3} = \frac{981}{12} - \frac{332}{12} = \frac{649}{12} =$
 $54\frac{1}{12}, \text{Difference.}$

A saving of about $\frac{1}{3}$ of your time and the work at the left is much more businesslike in appearance.

3. $81\frac{3}{4}$
 $27\frac{2}{3}$

567

162

$54 = (\frac{2}{3} \text{ of } 81)$

$20\frac{1}{4} = (\frac{3}{4} \text{ of } 27)$

$\frac{6}{12} = (\frac{3}{4} \text{ of } \frac{2}{3})$

$2261\frac{3}{4} = \text{Product}$

or, $81\frac{3}{4} \times 27\frac{2}{3} = \frac{327}{4} \times \frac{83}{3} = \frac{27141}{12} =$
 $2261\frac{3}{4}, \text{Product.}$

Here (No. 3) the operations are of equal length, but the reduction of the improper fraction generally requires long division, and then the advantage is decidedly with the method shown at the left. Cancellation sometimes aids materially in reducing the large numerators.

4. $81\frac{3}{4} \div 27\frac{2}{3} = \frac{327}{4} \times \frac{3}{83} = \frac{981}{332} = 332)981(2\frac{317}{332}, \text{Quotient.}$

$27\frac{2}{3}$	$81\frac{3}{4}$ $\times 12$
324	972
8	9
332	981
	664
	317

$\overline{23\frac{17}{32}}$, Quotient

Here there is a saving of about $\frac{1}{4}$, and while there are problems more easily worked by the improper fraction method, most problems are more quickly worked by avoiding improper fractions. The business man dislikes to see the signs $+$, $-$, $\times$ and $\div$ used. To him they represent a loss of time. They are easily eliminated. Learn how to do it.

Work the following problem by different methods and compare the amount of work required.

Find the Sum, Difference, Product and Quotient of $182\frac{5}{6}$ and $71\frac{3}{5}$.

(61)

CHANGING IMPROPER FRACTIONS TO MIXED NUMBERS

ORAL PROBLEMS

(a)	(b)	(c)	(a)	(b)	(c)	(a)	(b)	(c)
1. $\frac{20}{3} = \frac{100}{8} = \frac{15}{4} =$			5. $\frac{16}{3} = \frac{601}{7} = \frac{50}{4} =$			9. $\frac{20}{9} = \frac{65}{12} = \frac{88}{10} =$		
2. $\frac{36}{5} = \frac{21}{6} = \frac{19}{4} =$			6. $\frac{81}{7} = \frac{125}{5} = \frac{31}{4} =$			10. $\frac{95}{8} = \frac{61}{5} = \frac{45}{7} =$		
3. $\frac{77}{6} = \frac{145}{11} = \frac{78}{15} =$			7. $\frac{62}{9} = \frac{80}{14} = \frac{39}{4} =$			11. $\frac{250}{12} = \frac{196}{5} = \frac{845}{12} =$		
4. $\frac{49}{13} = \frac{176}{7} = \frac{163}{2} =$			8. $\frac{945}{11} = \frac{328}{6} = \frac{293}{4} =$			12. $\frac{875}{12} = \frac{139}{7} = \frac{637}{9} =$		

WRITTEN

Change to whole or mixed numbers:

$$\begin{array}{r}
 \text{MODEL. } \frac{1267}{16} = 16)1267 \\
 \underline{112} \\
 147 \\
 \underline{144} \\
 3
 \end{array}$$

(a)	(b)	(c)	(d)	(e)	(f)
1. $\frac{1965}{18} =$	$\frac{3964}{23} =$	$\frac{1875}{13} =$	$\frac{2764}{25} =$	$\frac{13075}{27} =$	$\frac{18092}{14} =$
2. $\frac{49826}{22} =$	$\frac{1401}{16} =$	$\frac{27351}{17} =$	$\frac{30603}{24} =$	$\frac{1906}{34} =$	$\frac{7907}{23} =$
3. $\frac{9463}{45} =$	$\frac{1653}{19} =$	$\frac{87645}{56} =$	$\frac{9321}{59} =$	$\frac{17062}{26} =$	$\frac{37964}{97} =$
4. $\frac{38915}{27} =$	$\frac{18146}{33} =$	$\frac{19067}{45} =$	$\frac{16123}{36} =$	$\frac{27241}{18} =$	$\frac{6407}{27} =$
5. $\frac{2946}{28} =$	$\frac{3694}{34} =$	$\frac{1876}{29} =$	$\frac{7645}{37} =$	$\frac{84906}{139} =$	$\frac{12539}{113} =$

SUBTRACTION OF FRACTIONS

What kind of whole numbers can be added ?

What kind of fractions can be added ?

Remember that in adding fractions you find the sum of the numerator. So in subtracting fractions you must find the difference of the numerators.

(62)

ORAL

Find difference:

$$\begin{array}{l}
 1. \frac{8}{9} - \frac{2}{3} = 7. \frac{4}{7} - \frac{2}{14} = 13. \frac{7}{6} - \frac{2}{3} = 19. \frac{12}{16} - \frac{1}{4} = 25. \frac{9}{11} - \frac{5}{22} = \\
 2. \frac{7}{8} - \frac{3}{4} = 8. \frac{1}{3} - \frac{1}{6} = 14. \frac{11}{12} - \frac{5}{6} = 20. \frac{18}{20} - \frac{8}{10} = 26. \frac{3}{8} - \frac{1}{4} = \\
 3. \frac{9}{12} - \frac{1}{6} = 9. \frac{1}{2} - \frac{1}{5} = 15. \frac{3}{5} - \frac{1}{2} = 21. \frac{5}{10} - \frac{2}{5} = 27. \frac{1}{8} - \frac{2}{25} = \\
 4. \frac{3}{5} - \frac{1}{10} = 10. \frac{3}{8} - \frac{1}{4} = 16. \frac{3}{10} - \frac{1}{5} = 22. \frac{3}{6} - \frac{1}{2} = 28. \frac{4}{14} - \frac{1}{7} = \\
 5. \frac{11}{15} - \frac{3}{5} = 11. \frac{11}{16} - \frac{3}{8} = 17. \frac{7}{9} - \frac{2}{3} = 23. \frac{2}{3} - \frac{3}{8} = 29. \frac{4}{6} - \frac{1}{3} = \\
 6. \frac{5}{4} - \frac{3}{8} = 12. \frac{4}{9} - \frac{1}{3} = 18. \frac{2}{8} - \frac{1}{16} = 24. \frac{7}{9} - \frac{1}{18} = 30. \frac{1}{4} - \frac{1}{8} =
 \end{array}$$

(63)

WRITTEN

Explain why the following fractions are more difficult.

Find difference:

$$\frac{13}{15} - \frac{12}{21} =$$

105 is the L. C. M.

$$\frac{13}{15} = \frac{91}{105}; \frac{12}{21} = \frac{60}{105}; 91 - 60 = \frac{31}{105}, \text{Ans.}$$

Observe that the fractions must be changed to like fractions.

Take difference of their numerators, and reduce result to simplest form.

$$\begin{array}{l}
 1. \frac{8}{9} - \frac{3}{4} = 6. \frac{11}{15} - \frac{3}{8} = 11. \frac{1}{2} - \frac{5}{11} = 16. \frac{11}{16} - \frac{1}{18} = 21. \frac{4}{3} - \frac{5}{7} = \\
 2. \frac{7}{8} - \frac{2}{3} = 7. \frac{10}{11} - \frac{3}{4} = 12. \frac{7}{16} - \frac{1}{6} = 17. \frac{9}{10} - \frac{11}{15} = 22. \frac{13}{15} - \frac{12}{21} = \\
 3. \frac{9}{12} - \frac{3}{5} = 8. \frac{11}{12} - \frac{9}{13} = 13. \frac{14}{25} - \frac{13}{45} = 18. \frac{11}{24} - \frac{1}{6} = 23. \frac{7}{45} - \frac{1}{8} = \\
 4. \frac{4}{7} - \frac{1}{6} = 9. \frac{24}{56} - \frac{2}{7} = 14. \frac{89}{108} - \frac{18}{72} = 19. \frac{16}{45} - \frac{2}{12} = 24. \frac{19}{20} - \frac{2}{3} = \\
 5. \frac{7}{9} - \frac{3}{5} = 10. \frac{21}{27} - \frac{2}{9} = 15. \frac{6}{32} - \frac{2}{11} = 20. \frac{74}{75} - \frac{7}{8} = 25. \frac{5}{6} - \frac{3}{10} =
 \end{array}$$

(64)

ADDITION OF MIXED NUMBERS

Add mentally:

(Drill on the following until the results can be read without difficulty.)

Example:

$$49\frac{2}{3} + 16\frac{2}{3} = 66\frac{1}{3}$$

$$53\frac{14}{15} + 35\frac{1}{5} = 89\frac{2}{15}$$

Operation:

Add fraction first $\frac{2}{3} + \frac{2}{3} = 1\frac{1}{3}$

$$1\frac{1}{3} + 49 = 50\frac{1}{3} + 16 = 66\frac{1}{3}, \text{ Ans.}$$

$$\frac{14}{15} + \frac{3}{5} = \frac{17}{15} \text{ or } 1\frac{2}{15}$$

$$1\frac{2}{15} + 53 = 54\frac{2}{15} + 35 = 89\frac{2}{15}, \text{ Ans.}$$

- | | | |
|--|---|---|
| 1. $3\frac{1}{2} + 18\frac{1}{2} =$ | 13. $35\frac{5}{6} + 33\frac{1}{3} =$ | 25. $45\frac{5}{6} + 13\frac{5}{12} =$ |
| 2. $16\frac{2}{3} + 24\frac{1}{3} =$ | 14. $17\frac{1}{5} + 18\frac{3}{10} =$ | 26. $43\frac{14}{15} + 17\frac{4}{5} =$ |
| 3. $37\frac{1}{8} + 12\frac{1}{4} =$ | 15. $37\frac{5}{8} + 19\frac{1}{4} =$ | 27. $58\frac{1}{2} + 6\frac{5}{6} =$ |
| 4. $775\frac{1}{5} + 18\frac{2}{5} =$ | 16. $17\frac{1}{2} + 45\frac{3}{4} =$ | 28. $39\frac{3}{8} + 15\frac{1}{6} =$ |
| 5. $19\frac{3}{4} + 17\frac{1}{8} =$ | 17. $18\frac{1}{6} + 14\frac{2}{3} =$ | 29. $27\frac{5}{6} + 17\frac{2}{3} =$ |
| 6. $64\frac{1}{7} + 46\frac{5}{7} =$ | 18. $22\frac{3}{8} + 19\frac{3}{4} =$ | 30. $15\frac{1}{2} + 15\frac{2}{3} =$ |
| 7. $49\frac{2}{3} + 16\frac{2}{3} =$ | 19. $19\frac{3}{10} + 75\frac{4}{5} =$ | 31. $71\frac{1}{9} + 12\frac{5}{6} =$ |
| 8. $19\frac{3}{8} + 27\frac{7}{8} =$ | 20. $77\frac{3}{7} + 15\frac{9}{14} =$ | 32. $45\frac{2}{3} + 11\frac{1}{4} =$ |
| 9. $14\frac{6}{7} + 21\frac{5}{7} =$ | 21. $78\frac{1}{10} + 12\frac{1}{5} =$ | 33. $39\frac{4}{7} + 93\frac{5}{7} =$ |
| 10. $78\frac{5}{9} + 12\frac{2}{3} =$ | 22. $68\frac{7}{10} + 11\frac{9}{10} =$ | 34. $75\frac{1}{8} + 45\frac{3}{4} =$ |
| 11. $81\frac{5}{6} + 1\frac{1}{4} =$ | 23. $28\frac{5}{6} + 21\frac{2}{3} =$ | 35. $39\frac{5}{8} + 16\frac{1}{2} =$ |
| 12. $19\frac{6}{7} + 17\frac{5}{14} =$ | 24. $20\frac{6}{10} + 35\frac{4}{5} =$ | 36. $47\frac{6}{7} + 40\frac{1}{14} =$ |

(65)

WRITTEN

- | | | |
|---|--|---|
| 1. $192\frac{3}{4} + 17\frac{5}{6} =$ | 14. $214\frac{9}{32} + 69\frac{3}{8} =$ | 27. $137\frac{5}{6} + 78\frac{6}{7} =$ |
| 2. $187\frac{1}{6} + 96\frac{2}{3} =$ | 15. $163\frac{18}{31} + 724\frac{2}{3} =$ | 28. $194\frac{15}{21} + 183\frac{6}{7} =$ |
| 3. $98\frac{3}{5} + 85\frac{3}{8} =$ | 16. $147\frac{3}{10} + 127\frac{2}{7} =$ | 29. $186\frac{15}{17} + 75\frac{9}{108} =$ |
| 4. $126\frac{6}{7} + 74\frac{3}{5} =$ | 17. $193\frac{14}{15} + 67\frac{16}{25} =$ | 30. $87\frac{3}{4} + 67\frac{9}{25} =$ |
| 5. $79\frac{6}{11} + 81\frac{15}{22} =$ | 18. $98\frac{19}{28} + 77\frac{31}{35} =$ | 31. $128\frac{11}{77} + 76\frac{11}{83} =$ |
| 6. $96\frac{1}{4} + 77\frac{9}{20} =$ | 19. $714\frac{4}{19} + 69\frac{1}{2} =$ | 32. $126\frac{9}{12} + 47\frac{7}{8} =$ |
| 7. $73\frac{1}{45} + 106\frac{8}{9} =$ | 20. $215\frac{9}{18} + 75\frac{3}{4} =$ | 33. $195\frac{8}{10} + 726\frac{5}{11} =$ |
| 8. $915\frac{6}{7} + 94\frac{1}{15} =$ | 21. $89\frac{7}{8} + 45\frac{2}{15} =$ | 34. $624\frac{9}{24} + 115\frac{5}{72} =$ |
| 9. $726\frac{11}{12} + 86\frac{4}{5} =$ | 22. $215\frac{11}{12} + 230\frac{15}{16} =$ | 35. $312\frac{3}{32} + 65\frac{13}{28} =$ |
| 10. $123\frac{7}{8} + 49\frac{6}{7} =$ | 23. $318\frac{9}{10} + 64\frac{7}{8} =$ | 36. $163\frac{3}{11} + 83\frac{3}{10} =$ |
| 11. $215\frac{9}{54} + 67\frac{33}{40} =$ | 24. $64\frac{29}{30} + 86\frac{122}{48} =$ | 37. $134\frac{24}{43} + 526\frac{45}{63} =$ |
| 12. $945\frac{35}{48} + 212\frac{3}{16} =$ | 25. $86\frac{14}{25} + 76\frac{35}{45} =$ | 38. $915\frac{16}{21} + 49\frac{75}{175} =$ |
| 13. $712\frac{17}{25} + 127\frac{27}{36} =$ | 26. $126\frac{27}{62} + 815\frac{62}{124} =$ | 39. $715\frac{65}{72} + 146\frac{7}{12} =$ |

(66)

SUBTRACTION OF MIXED NUMBERS

ORAL

Find difference:

- | | | |
|---------------------------------------|--|--|
| 1. $12\frac{1}{2} - 6\frac{1}{4} =$ | 13. $28\frac{9}{10} - 19\frac{2}{5} =$ | 25. $41\frac{1}{10} - 39\frac{1}{5} =$ |
| 2. $28\frac{2}{3} - 9\frac{1}{3} =$ | 14. $76\frac{5}{8} - 67\frac{3}{8} =$ | 26. $79\frac{1}{5} - 18\frac{2}{3} =$ |
| 3. $75\frac{6}{7} - 55\frac{2}{7} =$ | 15. $31\frac{4}{5} - 17\frac{3}{10} =$ | 27. $66\frac{1}{6} - 47\frac{1}{12} =$ |
| 4. $18\frac{5}{6} - 7\frac{2}{3} =$ | 16. $92\frac{1}{9} - 87\frac{1}{3} =$ | 28. $32\frac{5}{6} - 17\frac{1}{2} =$ |
| 5. $77\frac{7}{8} - 42\frac{3}{4} =$ | 17. $84\frac{1}{12} - 28\frac{5}{6} =$ | 29. $17\frac{9}{10} - 8\frac{4}{5} =$ |
| 6. $89\frac{7}{10} - 46\frac{1}{5} =$ | 18. $79\frac{3}{8} - 62\frac{1}{2} =$ | 30. $92\frac{3}{16} - 17\frac{3}{4} =$ |
| 7. $42\frac{7}{8} - 36\frac{1}{4} =$ | 19. $55\frac{2}{3} - 46\frac{4}{9} =$ | 31. $23\frac{1}{8} - 5\frac{1}{4} =$ |
| 8. $29\frac{5}{6} - 15\frac{2}{3} =$ | 20. $16\frac{2}{9} - 7\frac{2}{3} =$ | 32. $31\frac{1}{2} - 16\frac{1}{9} =$ |
| 9. $42\frac{7}{9} - 19\frac{5}{18} =$ | 21. $21\frac{7}{12} - 4\frac{1}{6} =$ | 33. $44\frac{1}{5} - 17\frac{1}{4} =$ |
| 10. $86\frac{2}{3} - 19\frac{1}{9} =$ | 22. $33\frac{1}{3} - 16\frac{2}{3} =$ | 34. $21\frac{3}{5} - 16\frac{1}{15} =$ |
| 11. $10\frac{3}{5} - 7\frac{4}{5} =$ | 23. $7\frac{5}{6} - 6\frac{1}{12} =$ | 35. $69\frac{1}{3} - 28\frac{1}{6} =$ |
| 12. $91\frac{2}{3} - 78\frac{1}{3} =$ | 24. $56\frac{5}{6} - 26\frac{3}{12} =$ | 36. $91\frac{5}{7} - 74\frac{6}{7} =$ |

(67)

WRITTEN

- | | | |
|--|--|---|
| 1. $18\frac{5}{12} - 9\frac{3}{5} =$ | 19. $214\frac{5}{9} - 85\frac{3}{10} =$ | 37. $201\frac{6}{7} - 81\frac{3}{4} =$ |
| 2. $92\frac{1}{18} - 36\frac{4}{5} =$ | 20. $76\frac{7}{22} - 29\frac{3}{11} =$ | 38. $219\frac{7}{18} - 120\frac{1}{36} =$ |
| 3. $11\frac{8}{9} - 6\frac{2}{3} =$ | 21. $725\frac{1}{2} - 637\frac{1}{36} =$ | 39. $312\frac{1}{2} - 93\frac{5}{6} =$ |
| 4. $71\frac{1}{15} - 47\frac{3}{7} =$ | 22. $369\frac{2}{17} - 177\frac{1}{5} =$ | 40. $106\frac{7}{8} - 97\frac{9}{10} =$ |
| 5. $18\frac{5}{8} - 12\frac{7}{9} =$ | 23. $211\frac{1}{15} - 99\frac{1}{12} =$ | 41. $171\frac{3}{20} - 86\frac{7}{12} =$ |
| 6. $41\frac{7}{10} - 24\frac{1}{6} =$ | 24. $86\frac{3}{8} - 47\frac{6}{31} =$ | 42. $92\frac{1}{15} - 62\frac{9}{10} =$ |
| 7. $197\frac{1}{4} - 88\frac{1}{12} =$ | 25. $102\frac{6}{11} - 38\frac{5}{12} =$ | 43. $175\frac{3}{14} - 68\frac{3}{4} =$ |
| 8. $279\frac{5}{7} - 93\frac{5}{8} =$ | 26. $87\frac{1}{16} - 14\frac{1}{9} =$ | 44. $169\frac{7}{11} - 86\frac{3}{8} =$ |
| 9. $456\frac{3}{11} - 277\frac{1}{6} =$ | 27. $76\frac{3}{8} - 49\frac{7}{16} =$ | 45. $121\frac{1}{2} - 66\frac{9}{11} =$ |
| 10. $121\frac{1}{5} - 87\frac{5}{9} =$ | 28. $109\frac{5}{36} - 81\frac{3}{8} =$ | 46. $85\frac{1}{6} - 16\frac{4}{9} =$ |
| 11. $120\frac{3}{13} - 84\frac{9}{16} =$ | 29. $160\frac{4}{21} - 75\frac{9}{28} =$ | 47. $423\frac{2}{26} - 177\frac{5}{52} =$ |
| 12. $261\frac{1}{8} - 39\frac{1}{9} =$ | 30. $715\frac{5}{16} - 526\frac{1}{20} =$ | 48. $176\frac{7}{32} - 97\frac{7}{96} =$ |
| 13. $87\frac{4}{75} - 29\frac{3}{100} =$ | 31. $320\frac{8}{9} - 231\frac{1}{30} =$ | 49. $923\frac{4}{41} - 647\frac{1}{82} =$ |
| 14. $189\frac{6}{7} - 160\frac{1}{12} =$ | 32. $219\frac{1}{60} - 145\frac{1}{11} =$ | 50. $621\frac{1}{17} - 377\frac{2}{51} =$ |
| 15. $145\frac{2}{7} - 96\frac{3}{8} =$ | 33. $515\frac{2}{21} - 387\frac{4}{9} =$ | 51. $212\frac{1}{15} - 68\frac{5}{6} =$ |
| 16. $137\frac{9}{14} - 95\frac{1}{11} =$ | 34. $413\frac{1}{16} - 371\frac{3}{5} =$ | 52. $195\frac{7}{8} - 88\frac{5}{12} =$ |
| 17. $628\frac{7}{9} - 438\frac{1}{15} =$ | 35. $219\frac{1}{2} - 172\frac{3}{8} =$ | 53. $379\frac{2}{5} - 284\frac{5}{11} =$ |
| 18. $162\frac{3}{8} - 94\frac{5}{6} =$ | 36. $712\frac{11}{16} - 639\frac{3}{40} =$ | 54. $314\frac{6}{13} - 173\frac{8}{11} =$ |

MULTIPLICATION OF FRACTIONS

To find the fractional part of a fraction:

What is $\frac{1}{4}$ of 8 dollars? Ans. \$2.00 (See Note).

What is $\frac{1}{4}$ of 8 tenths of a dollar. Ans. 20c.

NOTE: Do not be deceived here into thinking that you divided 8 by $\frac{1}{4}$. You divided 8 by 4, which is equivalent to multiplying 8 by $\frac{1}{4}$. Of means to multiply. Likewise $8 \div \frac{1}{4}$ is equivalent to 8×4 .

This means that you must take $\frac{1}{4}$ of the 8 parts of a unit which has been divided into ten equal parts. Hence:

$$\frac{1}{4} \text{ of } \frac{8}{10} = \frac{8}{40} \text{ or } \frac{1}{5}. \quad \frac{1}{5} \text{ of } \$1.00 = 20c.$$

Arrive at the answer 20c in another way.

(68)

ORAL

In working these problems you should not have to follow the steps in detail, but by inspection only, quickly arrive at the correct results.

To illustrate:

$$\frac{4}{5} \times \frac{10}{12} = \frac{2}{3}, \text{ Ans.}$$

Think to yourself; do not use your pencil.

$$10 \text{ less } \frac{1}{5} \text{ of itself} = 8, \frac{8}{12} = \frac{2}{3}, \text{ Ans.}$$

$$\text{or } 12 + \frac{1}{4} \text{ of itself} = 15, \frac{10}{15} = \frac{2}{3}, \text{ Ans.}$$

$$\text{or } (10 \div 5) = 2, \text{ over } (12 \div 4) = \frac{2}{3}, \text{ Ans.}$$

$$\text{or } 4 \text{ tens} = 40 \text{ over } 5 \text{ twelves } (60) = \frac{40}{60} = \frac{2}{3}, \text{ Ans.}$$

- | | | | |
|---------------------------------------|---|--|--|
| 1. $\frac{1}{3}$ of $\frac{3}{4}$ = ? | 9. $\frac{4}{5}$ of $\frac{2}{3}$ = ? | 17. $\frac{2}{5}$ of $\frac{1}{3}$ = ? | 25. $\frac{1}{8}$ of $\frac{1}{3}$ = ? |
| 2. $\frac{2}{3}$ of $\frac{3}{4}$ = ? | 10. $\frac{1}{5}$ of $\frac{3}{4}$ = ? | 18. $\frac{3}{5}$ of $\frac{2}{3}$ = ? | 26. $\frac{2}{3}$ of $\frac{5}{6}$ = ? |
| 3. $\frac{1}{3}$ of $\frac{1}{2}$ = ? | 11. $\frac{1}{5}$ of $\frac{5}{6}$ = ? | 19. $\frac{1}{2}$ of $\frac{1}{8}$ = ? | 27. $\frac{3}{8}$ of $\frac{4}{9}$ = ? |
| 4. $\frac{2}{3}$ of $\frac{1}{2}$ = ? | 12. $\frac{2}{5}$ of $\frac{5}{6}$ = ? | 20. $\frac{1}{3}$ of $\frac{1}{2}$ = ? | 28. $\frac{4}{5}$ of $\frac{10}{12}$ = ? |
| 5. $\frac{1}{4}$ of $\frac{2}{3}$ = ? | 13. $\frac{3}{5}$ of $\frac{5}{6}$ = ? | 21. $\frac{1}{3}$ of $\frac{1}{4}$ = ? | 29. $\frac{9}{10}$ of $\frac{2}{3}$ = ? |
| 6. $\frac{2}{4}$ of $\frac{2}{3}$ = ? | 14. $\frac{3}{5}$ of $\frac{5}{12}$ = ? | 22. $\frac{1}{3}$ of $\frac{1}{8}$ = ? | 30. $\frac{5}{8}$ of $\frac{7}{10}$ = ? |
| 7. $\frac{3}{4}$ of $\frac{2}{3}$ = ? | 15. $\frac{1}{2}$ of $\frac{1}{2}$ = ? | 23. $\frac{1}{2}$ of $\frac{1}{3}$ = ? | 31. $\frac{7}{9}$ of $\frac{18}{35}$ = ? |
| 8. $\frac{1}{5}$ of $\frac{1}{3}$ = ? | 16. $\frac{1}{2}$ of $\frac{1}{4}$ = ? | 24. $\frac{1}{4}$ of $\frac{1}{3}$ = ? | 32. $\frac{3}{4}$ of $\frac{11}{12}$ = ? |

(69)

WRITTEN

A fractional part of a fraction is found by multiplying the numerators and denominators of the fractions together.

Cancel when possible as shown in given example:

$$\text{Find } \frac{5}{6} \text{ of } \frac{12}{35} = \frac{5}{\cancel{6}} \times \frac{\cancel{12}^2}{\cancel{35}_7} = \frac{2}{7}, \text{ Ans.}$$

- | | | |
|--|--|--|
| 1. $\frac{3}{5}$ of $\frac{7}{9} =$ | 11. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4} =$ | 21. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of $\frac{3}{8} =$ |
| 2. $\frac{125}{25}$ of $\frac{65}{72} =$ | 12. $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{5}{6} =$ | 22. $\frac{1}{6}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{9}{10} =$ |
| 3. $\frac{35}{48}$ of $\frac{12}{25} =$ | 13. $\frac{9}{10}$ of $\frac{3}{5}$ of $\frac{2}{3} =$ | 23. $\frac{1}{8}$ of $\frac{2}{4}$ of $\frac{3}{10}$ of $\frac{5}{6} =$ |
| 4. $\frac{9}{11}$ of $\frac{15}{27}^* =$ | 14. $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{5}{7} =$ | 24. $\frac{7}{14}$ of $\frac{2}{7}$ of $\frac{2}{8}$ of $\frac{7}{9} =$ |
| 5. $\frac{16}{21}$ of $\frac{29}{48} =$ | 15. $\frac{3}{16}$ of $\frac{8}{9}$ of $\frac{9}{11} =$ | 25. $\frac{5}{12}$ of $\frac{4}{5}$ of $\frac{7}{11}$ of $\frac{22}{25} =$ |
| 6. $\frac{7}{18}$ of $\frac{10}{21} =$ | 16. $\frac{5}{21}$ of $\frac{3}{10}$ of $\frac{7}{9} =$ | 26. $\frac{1}{9}$ of $\frac{18}{19}$ of $\frac{5}{6}$ of $\frac{4}{5} =$ |
| 7. $\frac{12}{13}$ of $\frac{27}{36} =$ | 17. $\frac{3}{7}$ of $\frac{2}{3}$ of $\frac{4}{5} =$ | 27. $\frac{6}{7}$ of $\frac{5}{6}$ of $\frac{10}{11}$ of $\frac{17}{20} =$ |
| 8. $\frac{23}{28}$ of $\frac{27}{35} =$ | 18. $\frac{8}{9}$ of $\frac{5}{8}$ of $\frac{3}{5} =$ | 28. $\frac{5}{4}$ of $\frac{11}{12}$ of $\frac{16}{19}$ of $\frac{38}{39} =$ |
| 9. $\frac{13}{16}$ of $\frac{14}{39} =$ | 19. $\frac{4}{7}$ of $\frac{3}{5}$ of $\frac{9}{10} =$ | 29. $\frac{25}{26}$ of $\frac{13}{15}$ of $\frac{7}{8}$ of $\frac{12}{28}^* =$ |
| 10. $\frac{81}{83}$ of $\frac{45}{51}^* =$ | 20. $\frac{12}{21}$ of $\frac{7}{15}$ of $\frac{3}{8} =$ | 30. $\frac{40}{41}$ of $\frac{1}{15}$ of $\frac{7}{10}$ of $\frac{1}{8} =$ |

*Reduce to lowest terms before multiplying.

(70)

MISCELLANEOUS ORAL PROBLEMS

Read the results rapidly:

- | | | | | |
|---------------------------------------|---|--|--|--|
| 1. $\frac{1}{2} \times \frac{3}{4} =$ | 5. $\frac{1}{8} \times \frac{9}{10} =$ | 9. $\frac{5}{8} \times \frac{4}{5} =$ | 13. $\frac{1}{2} \times \frac{10}{12} =$ | 17. $\frac{3}{4} \times \frac{3}{5} =$ |
| 2. $10 \times \frac{3}{4} =$ | 6. $\frac{1}{6} \times \frac{4}{5} =$ | 10. $\frac{2}{3} \times \frac{3}{4} =$ | 14. $\frac{7}{8} \times \frac{3}{8} =$ | 18. $\frac{2}{3} \times 2 =$ |
| 3. $\frac{7}{8} \times \frac{2}{3} =$ | 7. $\frac{7}{8} \times \frac{3}{4} =$ | 11. $\frac{6}{7} \times \frac{5}{6} =$ | 15. $\frac{2}{3} \times 1 =$ | 19. $\frac{5}{9} \times \frac{2}{5} =$ |
| 4. $\frac{1}{2} \times 6 =$ | 8. $\frac{10}{11} \times \frac{1}{5} =$ | 12. $20 \times \frac{4}{5} =$ | 16. $\frac{3}{5} \times \frac{2}{7} =$ | 20. $\frac{6}{7} \times \frac{6}{7} =$ |

(71)

MIXED NUMBERS MULTIPLIED BY WHOLE NUMBERS

Illustration: $(72\frac{2}{5} \times 10 = 720 + (\frac{2}{5} \text{ of } 10) = 724.$

- | | | | |
|--------------------------------|--------------------------------|-------------------------------|---------------------------------|
| 1. $45\frac{1}{2} \times 10 =$ | 4. $35\frac{2}{3} \times 3 =$ | 7. $17\frac{3}{4} \times 5 =$ | 10. $12\frac{1}{3} \times 9 =$ |
| 2. $18\frac{2}{3} \times 6 =$ | 5. $112\frac{1}{2} \times 7 =$ | 8. $9\frac{7}{8} \times 4 =$ | 11. $21\frac{2}{3} \times 6 =$ |
| 3. $72\frac{3}{4} \times 4 =$ | 6. $42\frac{1}{3} \times 4 =$ | 9. $6\frac{4}{5} \times 6 =$ | 12. $113\frac{2}{3} \times 7 =$ |

(72)

WHOLE NUMBERS MULTIPLIED BY MIXED NUMBERS

Illustration: $36 \times 2\frac{2}{3} = 72 + (\frac{2}{3} \text{ of } 36) = 96.$

- | | | | |
|-------------------------------|--------------------------------|-------------------------------|--------------------------------|
| 1. $72 \times 3\frac{1}{3} =$ | 4. $33 \times 3\frac{3}{11} =$ | 7. $3 \times 2\frac{2}{3} =$ | 10. $14 \times 2\frac{2}{7} =$ |
| 2. $18 \times 2\frac{1}{5} =$ | 5. $21 \times 4\frac{2}{3} =$ | 8. $18 \times 2\frac{3}{4} =$ | 11. $5 \times 2\frac{1}{2} =$ |
| 3. $96 \times 1\frac{1}{2} =$ | 6. $15 \times 5\frac{4}{5} =$ | 9. $27 \times 3\frac{4}{9} =$ | 12. $15 \times 5\frac{3}{4} =$ |

(73)

MIXED NUMBERS HAVING LIKE FRACTIONS

- | | | | |
|---|--|--|--|
| 1. $6\frac{1}{4} \times 6\frac{1}{4} =$ | 6. $16\frac{1}{4} \times 8\frac{1}{4} =$ | 11. $16\frac{1}{6} \times 8\frac{1}{6} =$ | 16. $9\frac{2}{3} \times 9\frac{2}{3} =$ |
| 2. $10\frac{1}{2} \times 10\frac{1}{2} =$ | 7. $25\frac{1}{2} \times 3\frac{1}{2} =$ | 12. $8\frac{1}{5} \times 7\frac{1}{5} =$ | 17. $7\frac{1}{7} \times 2\frac{1}{7} =$ |
| 3. $12\frac{1}{8} \times 12\frac{1}{8} =$ | 8. $30\frac{1}{8} \times 2\frac{1}{8} =$ | 13. $15\frac{1}{2} \times 9\frac{1}{2} =$ | 18. $12\frac{1}{4} \times 2\frac{1}{4} =$ |
| 4. $2\frac{1}{2} \times 2\frac{1}{2} =$ | 9. $12\frac{1}{3} \times 3\frac{1}{3} =$ | 14. $7\frac{1}{5} \times 8\frac{1}{5} =$ | 19. $11\frac{1}{2} \times 9\frac{1}{2} =$ |
| 5. $5\frac{1}{4} \times 7\frac{1}{4} =$ | 10. $6\frac{1}{3} \times 9\frac{1}{3} =$ | 15. $10\frac{1}{4} \times 10\frac{1}{4} =$ | 20. $12\frac{1}{2} \times 10\frac{1}{2} =$ |

Where the fractions have the same denominators as in $6\frac{1}{4} \times 6\frac{1}{4}$, the work can be shortened:

$6\frac{1}{4}$	$12\frac{1}{8}$	$19\frac{1}{3}$
<u>$6\frac{1}{4}$</u>	<u>$12\frac{1}{8}$</u>	<u>$17\frac{1}{3}$</u>
36	144	133
$3 = (\frac{1}{4} \text{ of } 6 + 6)$	$3 = (\frac{1}{8} \text{ of } 24)$	19
$\frac{1}{16} = (\frac{1}{4} \times \frac{1}{4})$	$\frac{1}{64} = (\frac{1}{8} \times \frac{1}{8})$	$12 = (\frac{1}{3} \text{ of } 36)$
		$\frac{1}{9} = (\frac{1}{3} \text{ of } \frac{1}{3})$
<u>$39\frac{1}{16}$</u>	<u>$147\frac{1}{64}$</u>	<u>$335\frac{1}{9}$</u>

If fractions are both $\frac{1}{2}$, and the integers the same, as in $10\frac{1}{2} \times 10\frac{1}{2}$:

$$\begin{array}{r}
 10\frac{1}{2} \\
 10\frac{1}{2} \\
 \hline
 110 = (10 \times 11) \\
 \frac{1}{4} = (\frac{1}{2} \times \frac{1}{2}) \\
 \hline
 110\frac{1}{4} = \text{Ans.}
 \end{array}$$

MULTIPLICATION OF MIXED NUMBERS

WRITTEN

Example: $12\frac{3}{4} \times 25\frac{7}{8} = \frac{51}{4} \times \frac{207}{8} = \frac{10557}{32}$ or $329\frac{29}{32}$.

Or:

$ \begin{array}{r} 12\frac{3}{4} \\ 25\frac{7}{8} \\ \hline 60 \\ 24 \\ 10\frac{1}{2} \text{ } (\frac{7}{8} \text{ of } 12) \frac{84}{8} \\ 18\frac{3}{4} \text{ } (\frac{3}{4} \text{ of } 25) \frac{75}{4} \\ \hline \frac{21}{32} \text{ } (\frac{3}{4} \times \frac{7}{8}) \\ \hline 329\frac{29}{32}, \text{ Ans.} \end{array} $	$ \begin{array}{r} 329\frac{29}{32}, \text{ Ans.} \\ 32)10557 \\ \underline{96} \\ 95 \\ \underline{64} \\ 317 \\ \underline{288} \\ 29 \end{array} $
---	--

Compare the above operations carefully. Work your problems by the shortest method.

(74)

$$\begin{aligned}
 1. 14\frac{3}{8} \times 16\frac{1}{2} &= 6. 17\frac{1}{10} \times 45\frac{1}{2} = 11. 125\frac{3}{4} \times 75\frac{3}{8} = 16. 22\frac{1}{11} \times 47\frac{5}{11} = \\
 2. 25\frac{1}{6} \times 12\frac{2}{3} &= 7. 45\frac{4}{5} \times 25\frac{5}{6} = 12. 62\frac{1}{12} \times 12\frac{1}{7} = 17. 76\frac{3}{20} \times 45\frac{5}{9} = \\
 3. 26\frac{5}{6} \times 15\frac{2}{5} &= 8. 27\frac{1}{2} \times 23\frac{1}{2} = 13. 78\frac{1}{6} \times 15\frac{1}{9} = 18. 29\frac{2}{7} \times 81\frac{7}{12} = \\
 4. 15\frac{1}{4} \times 15\frac{1}{4} &= 9. 62\frac{1}{4} \times 68\frac{1}{4} = 14. 27\frac{8}{9} \times 16\frac{7}{12} = 19. 80\frac{5}{8} \times 21\frac{1}{12} = \\
 5. 20\frac{3}{10} \times 25\frac{1}{6} &= 10. 80\frac{1}{8} \times 25\frac{1}{8} = 15. 18\frac{1}{10} \times 75\frac{1}{6} = 20. 64\frac{1}{9} \times 32\frac{5}{7} =
 \end{aligned}$$

DIVISIONS OF FRACTIONS

I. To divide an integer by a fraction:

$$\begin{array}{c}
 2 \\
 10 \div \frac{5}{6} = \frac{10}{1} \times \frac{6}{5} = 12, \text{ Ans. or } \frac{5}{6}) \frac{10}{12} \text{ (Multiply terms by 6)} \\
 \hline
 12 = \text{Ans.}
 \end{array}$$

$$\begin{array}{c}
 5 \\
 \text{(Compare } 10 \times \frac{5}{6} = \frac{10}{1} \times \frac{5}{6} = \frac{25}{3} = 8\frac{1}{3}, \text{ Ans.)} \\
 \hline
 3
 \end{array}$$

II. To divide a fraction by an integer:

$$\frac{5}{6} \div 10 = \frac{5}{6} \times \frac{1}{10} = \frac{1}{12}, \text{ Ans.}$$

(Compare $\frac{5}{6} \times 10 = \frac{50}{6}$ or $8\frac{1}{3}$, Ans.)

III. To divide a fraction by a fraction:

$$\frac{5}{6} \div \frac{1}{2} = \frac{5}{6} \times \frac{2}{1} = \frac{5}{3} \text{ or } 1\frac{2}{3}, \text{ Ans.})$$

IV. To divide an integer by a mixed number:

$$84 \div 4\frac{1}{3} = 84 \div \frac{13}{3} = \frac{84}{1} \times \frac{3}{13} = \frac{252}{13} = 19\frac{5}{13}, \text{ Ans.}$$

By multiplying both numbers by 3 the fraction is eliminated and the problem simplified.

Or:

$$\frac{84}{4\frac{1}{3}} \times \frac{3}{3} = \frac{252}{13} = 19\frac{5}{13}, \text{ Ans.}$$

V. To divide a mixed number by an integer:

$$46\frac{1}{3} \div 15 = \frac{139}{3} \div 15 = \frac{139}{3} \times \frac{1}{15} = 3\frac{4}{45}, \text{ Ans.}$$

Or:

$$\begin{array}{r} 15) 46\frac{1}{3} \\ 45) 139(3\frac{4}{45}, \text{ Ans.} \\ \underline{135} \\ 4 \end{array}$$

Eliminate fractions by multiplying both sides by 3.

VI. To divide a mixed number by a mixed number:

$$42\frac{1}{2} \div 17\frac{2}{3} = \frac{85}{2} \div \frac{53}{3} = \frac{85}{2} \times \frac{3}{53} = \frac{255}{106} = 2\frac{43}{106}, \text{ Ans.}$$

Or:

$$\begin{array}{r} 17\frac{2}{3} \quad 42\frac{1}{2} \quad 2\frac{43}{106}, \text{ Ans.} \\ \underline{6} \quad \underline{6} \quad 106) 255 \\ 102 \quad 252 \quad \underline{212} \\ 4 \quad 3 \quad \underline{43} \\ 106 \quad 255 \end{array}$$

(Multiplying both sides by 6)

(75)

FRACTIONS DIVIDED BY FRACTIONS

ORAL

- | | | | | |
|-------------------------------------|--------------------------------------|--------------------------------------|---------------------------------------|--|
| 1. $\frac{1}{2} \div \frac{1}{2} =$ | 7. $\frac{1}{2} \div \frac{2}{3} =$ | 13. $\frac{1}{2} \div \frac{1}{5} =$ | 19. $\frac{1}{2} \div \frac{5}{6} =$ | 25. $10 \div \frac{5}{8} =$ |
| 2. $\frac{1}{2} \div \frac{1}{4} =$ | 8. $\frac{3}{4} \div \frac{1}{4} =$ | 14. $\frac{1}{4} \div \frac{1}{5} =$ | 20. $\frac{5}{8} \div \frac{1}{2} =$ | 26. $\frac{2}{3} \div \frac{10}{12} =$ |
| 3. $\frac{1}{4} \div \frac{1}{2} =$ | 9. $\frac{1}{4} \div \frac{3}{4} =$ | 15. $\frac{1}{8} \div \frac{1}{5} =$ | 21. $\frac{1}{2} \div \frac{5}{8} =$ | 27. $\frac{1}{8} \div \frac{3}{8} =$ |
| 4. $\frac{1}{2} \div \frac{1}{3} =$ | 10. $\frac{1}{5} \div \frac{1}{2} =$ | 16. $\frac{1}{6} \div \frac{1}{3} =$ | 22. $\frac{2}{5} \div \frac{5}{10} =$ | 28. $\frac{3}{8} \div \frac{1}{8} =$ |
| 5. $\frac{1}{3} \div \frac{1}{2} =$ | 11. $\frac{1}{5} \div \frac{1}{4} =$ | 17. $\frac{1}{3} \div \frac{1}{6} =$ | 23. $10 \div \frac{1}{2} =$ | 29. $\frac{1}{7} \div \frac{1}{7} =$ |
| 6. $\frac{2}{3} \div \frac{1}{2} =$ | 12. $\frac{1}{5} \div \frac{1}{8} =$ | 18. $\frac{5}{6} \div \frac{1}{2} =$ | 24. $\frac{1}{2} \div 10 =$ | 30. $7 \div \frac{1}{7} =$ |

Additional drill can be given if necessary. Accuracy is gained only through practice.

(76)

MISCELLANEOUS PROBLEMS IN DIVISION

WRITTEN

- | | | |
|--|--|--|
| 1. $1\frac{2}{3} \div 5\frac{1}{2} =$ | 8. $14\frac{1}{9} \div \frac{2}{3} =$ | 15. $100 \div 33\frac{2}{3} =$ |
| 2. $6\frac{7}{8} \div 1\frac{1}{2} =$ | 9. $75\frac{1}{5} \div \frac{9}{10} =$ | 16. $16\frac{2}{3} \div 8\frac{1}{3} =$ |
| 3. $3\frac{1}{7} \div 12\frac{1}{2} =$ | 10. $8\frac{1}{6} \div \frac{2}{3} =$ | 17. $8\frac{1}{3} \div 16\frac{2}{3} =$ |
| 4. $12\frac{1}{2} \div 3\frac{1}{7} =$ | 11. $\frac{21}{25} \div \frac{14}{15} =$ | 18. $20\frac{5}{8} \div \frac{5}{8} =$ |
| 5. $\frac{9}{11} \div 14 =$ | 12. $73\frac{1}{2} \div 9\frac{4}{5} =$ | 19. $13\frac{1}{3} \div 12\frac{1}{8} =$ |
| 6. $14 \div \frac{9}{11} =$ | 13. $78\frac{5}{6} \div 10\frac{1}{3} =$ | 20. $17\frac{5}{6} \div \frac{11}{12} =$ |
| 7. $12\frac{3}{8} \div 18 =$ | 14. $54\frac{3}{8} \div 25\frac{5}{6} =$ | 21. $129\frac{7}{8} \div 8\frac{1}{3} =$ |

(If the class cannot arrive at the results quickly and accurately, the teacher should give many similar problems. Accuracy and confidence can be gained only by persistent practice.)

FRACTIONAL RELATIONS

What part of 20 is 5? $5 \div 20 = \frac{5}{20}$ or $\frac{1}{4}$

What part of 30 is 10? $10 \div 30 = \frac{10}{30}$ or $\frac{1}{3}$

What part of $\frac{1}{2}$ is $\frac{1}{3}$? $\frac{1}{3} \div \frac{1}{2} = \frac{1}{3} \times \frac{2}{1} = \frac{2}{3}$

So it is evident that the smaller number must be divided by the larger. It is also evident that the result of the division shows the ratio, or relation, existing between the two numbers; i. e., 20 is 4 times 5 and 5 is $\frac{1}{4}$ of 20.

What part of (77)

- | | | | |
|--------------|-------------|---------------|---------------|
| 1. 40 is 4 | 6. 120 is 8 | 11. 205 is 5 | 16. 368 is 8 |
| 2. 27 is 3 | 7. 116 is 4 | 12. 198 is 11 | 17. 714 is 7 |
| 3. 15 is 5 | 8. 75 is 5 | 13. 732 is 12 | 18. 396 is 6 |
| 4. 150 is 50 | 9. 125 is 5 | 14. 210 is 15 | 19. 112 is 4 |
| 5. 39 is 13 | 10. 24 is 3 | 15. 198 is 9 | 20. 912 is 12 |

(78)

WRITTEN

1. If $\frac{2}{3}$ of a number is 20, what is the number?

If $\frac{2}{3}$ of the number is 20, $\frac{1}{3}$ is a half of 20 or 10. If it is ten, then $\frac{3}{3}$ is 3 times ten or 30, Ans.

It should be noticed here that we have reasoned from many to one, and then from the one back to many, the desired number.

$$\frac{2}{3} = 20 = \text{many (2)}$$

$$\frac{1}{3} = 10 = \text{unit (1)}$$

$$\frac{3}{3} = 30 = \text{many (3) Required}$$

2. If $\frac{3}{4}$ is $\frac{2}{3}$ of a number, find the number.

If $\frac{2}{3}$ of the number is $\frac{3}{4}$, then $\frac{1}{3}$ of the number is one-half of $\frac{3}{4}$ or $\frac{3}{8}$. $\frac{3}{8} = 3$ times $\frac{3}{8}$ or $\frac{9}{8}$ the number.

Proof (as in the previous exercises, divide the smaller by the larger):

$$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$$

Give similar proofs to all your results.

Find the number of which:

- | | | | |
|------------------------------------|---------------------------------------|---------------------------------------|--|
| 1. $\frac{8}{9}$ is $\frac{3}{4}$ | 4. $6\frac{7}{8}$ is $\frac{1}{8}$ | 7. $87\frac{1}{2}$ is $\frac{7}{8}$ | 10. $\frac{7}{9}$ is $\frac{1}{3}$ |
| 2. $3\frac{1}{3}$ is $\frac{5}{6}$ | 5. $12\frac{5}{6}$ is $\frac{7}{8}$ | 8. $33\frac{1}{3}$ is $\frac{1}{3}$ | 11. $195\frac{9}{10}$ is $\frac{3}{4}$ |
| 3. 204 is $\frac{4}{11}$ | 6. $120\frac{1}{2}$ is $\frac{3}{16}$ | 9. $145\frac{1}{2}$ is $\frac{9}{11}$ | 12. $\frac{25}{2}$ is $\frac{5}{16}$ |

CHAPTER V

THE PRINCIPLES OF FRACTIONS APPLIED

Having mastered the fundamental operations, you are now prepared to make a careful study of the uses of these operations in solving the practical problems of life.

You will now discover that, though you may know how to reduce, add, subtract, multiply and divide fractions you do not always know the relation of the facts given, which operations apply to the problem presented, or the order in which the operations are to be performed.

Information on which problems are based comes to us in certain related or unrelated facts, figures and numbers. The quick discovery of these relationships depends upon a general knowledge of things about us, common sense, a practical turn of mind and the ability to readily visualize the problem—to see the objects before us. Having discovered the related facts given in the problem, we must compare them carefully and express the relationship found. *Comparison is the first step in successful analysis.*

How do we compare \$50 with \$150? If oranges are 60c a dozen what is the relation of price to the number of oranges? If the relation is 5, then we have the key to the solution of the problem. What will 17 oranges cost? $17 \times 5 = 85$ cents. It is this *ratio-value* or *key-figure* that is so important in our study, and especially so when we consider how many ways it may be expressed.

In comparing 50 with 150 we have the ratio value of 3; 50 is $\frac{1}{3}$ of 150, and 150 is 3 times 50. There are five ways of expressing this relationship: (a) 50:150; (b) $\frac{50}{150}$; (c) $.3333\frac{1}{3}$; (d) $33\frac{1}{3}\%$ and (e) by use of a *graph*.



Graph

(79)

Compare the following numbers, expressing each relation in at least three ways.

- | | | |
|---------------------------------------|---|----------------------------------|
| 1. $7\frac{1}{2}$ and $22\frac{1}{2}$ | 5. $112\frac{1}{2}$ and $37\frac{1}{2}$ | 9. 45 and 900 |
| 2. 8 and 64 | 6. 75 and $2\frac{1}{2}$ | 10. $2\frac{1}{2}$ and 10 |
| 3. 175 and 25 | 7. 48 and $\frac{1}{2}$ | 11. 14 and 16 |
| 4. $\frac{3}{13}$ and $33\frac{1}{3}$ | 8. $3\frac{3}{4}$ and $7\frac{1}{2}$ | 12. 3 in. and $1\frac{1}{2}$ ft. |

For a more extensive treatment of *graphs* see Chapter XII.

APPROXIMATING RESULTS

Attention has been called to the necessity of approximating the result of your problem before beginning the solution. By a careful mental analysis you can determine approximately what the answer should be. If the problem should be long, write down your approximate answer to be compared with your result after solving.

To illustrate: Suppose you purchase 6500 lbs. of coal at \$7.50 per ton. At a glance you observe that you have three and a fraction tons. Three tons at \$7.50 will amount to \$22.50. So you know your answer should be near this amount. One pupil gave as a result for this problem \$48,750.00 or just 2000 times what it should have been. In other words he was paying \$7.50 per pound for the coal instead of \$7.50 per ton. Only a few seconds of reasoning would have avoided such an absurd result.

Again suppose you want to know the interest on \$2,500 at 7% for 123 days. At a glance you observe that \$2,500 for 60 days at 6% equals \$25.00. 123 days is about twice 60 and that 7% is $\frac{1}{6}$ greater than 6%. So you know that the answer should be about \$60.00 and not \$6.00 or \$600.00.

Give the approximate result of:

- (1) 2,500 lbs. of hay at \$15.50 per ton.
- (2) 9,600 lbs. of coal at \$8.25 per ton.
- (3) 50 bu. corn at $37\frac{1}{2}$ cents per bu.
- (4) 1,850 ft. of lumber at \$27.50 per M.
- (5) $12\frac{1}{2}$ lbs. of cheese at 32 cents per lb.

THE USE OF THE EQUATION

The Equation is a statement of equality existing between two numbers or quantities, and this statement offers the basis for all arithmetical reasoning. The equality need exist in but one particular.

To illustrate: 7 tons=\$50.00 doesn't refer to weight, distance, area, or cubical content, but to *value*. $\frac{7}{100}=84$ acres, does not express equality of measure or of value, but equivalent relationship.

AXIOMS FOR EQUATIONS

1. Multiplying or dividing both sides of an equation by the same number does not alter the relation.

$$\frac{7}{100}=84 \text{ acres.}$$

$$\frac{1}{100}=12 \text{ acres.}$$

$$\frac{100}{100}=1200 \text{ acres.}$$

2. Adding or subtracting like parts or proportions of each side, to or from its respective base, does not change the relation expressed in the equation.

If $\frac{14}{100}=\$700$ then $\frac{21}{100}=\$1050$.

$$\frac{14}{100}=\$700$$

Here we added $\frac{1}{2}$ of each respectively and saved one step. Following 1 above the operation would be

$$\frac{1}{100}=50$$

$$\frac{21}{100}=1050$$

NOTE: For a more extensive treatise on the equation and its use, consult any standard algebra. What is given here is considered sufficient for the needs of business.

After a study of the above give a similar analysis to the following problem:

If the expense of selling a piece of property is \$45.00, which is $\frac{5}{24}$ of sales, find the selling price.

EXAMPLES IN THE USE OF EQUATIONS

1. If oranges are 40c a dozen, what will 15 cost?

a. $12 = (\text{or cost}) 40c$

$1 = (\text{or cost}) 3\frac{1}{3}c$

$15 = (\text{or cost}) 50c$

NOTE: Notice that we have reasoned from many to one and from the one to the many required; from the known to the unknown.

$12 : 15 :: 40 : ?$

b. $15 \times 40 = 600$ (product of means) using ratio and proportion or the old rule of three.

$600 \div 12 = 50$ missing term

c. $15 \div 12 = \frac{15}{12} = 1\frac{1}{4}$ doz.

$1\frac{1}{4} \text{ doz.} \times 40c = 50c$

Reduction of units to price unit given; very often an awkward, long-drawn-out and inaccurate method.

THE ORDER OF PROCEDURE FOR YOUR ANALYSIS

1. Determine what term represents unity, what is to be considered as 1, $\frac{3}{8}$, $\frac{7}{7}$, $\frac{5}{5}$, etc.

2. Construct a true equation.

3. Change the equation so that the correct side is reduced to unity. The name of the unit-part must correspond with the name of the unit-whole as found in step one. To illustrate: $\frac{3}{4} = 5\frac{2}{3}$ acres, can be changed to read $\frac{1}{4} = 1\frac{8}{9}$ acres, or $\frac{9}{64} = 1$ acre.

4. Change the equation until the desired quantity appears on one side or the other. Be sure to follow the rules for changing an equation previously stated.

PROBLEM TO ILLUSTRATE

A merchant marked a hat and then sold it at $\frac{1}{5}$ off; what was it marked if it sold for \$4.80?

SOLUTION

Does the merchant, the hat, or the \$4.80 represent our unit basis for reasoning? Neither. It isn't $\frac{1}{5}$ of the merchant, $\frac{1}{5}$ of the hat, or even $\frac{1}{5}$ of the \$4.80, but $\frac{1}{5}$ of the Marked Price which

is not given. The Base not being given, we cannot multiply, therefore we must divide.

The question now is, shall we divide by $1 + \text{the rate}$, or by $1 - \text{the rate}$? Is our result to be larger or smaller than the \$4.80? Larger. Then we must divide by less than one, or $1 - \text{the rate}$.

The equation must read, therefore:

$$\$4.80 = 1 - \frac{1}{5} \text{ or } \frac{4}{5}$$

$$\$1.20 = \frac{1}{5}$$

$$\$6.00 = \frac{5}{5} \text{ or the Marked Price.}$$

Dividing as indicated above, we have— $\$4.80 \div .80 = \$6.00$ the Marked Price.

(80)

DRILL IN THE USE OF THE EQUATION

NOTE: In solving these problems test the three methods given on preceding page.

1. What will $7\frac{1}{3}$ bushels of apples cost if $2\frac{1}{2}$ bushels cost \$1.20?
2. If 4 girls earn \$42.20, what will 7 girls earn?
3. If 240 lbs. of wheat produce 180 lbs. of flour, what will 6000 lbs. produce?
4. How many lbs. of wheat would it take to produce 50 lbs. of flour? (See No. 3 above.)
5. If four hundredths of my money is invested in land costing \$24000, what amount did I first have?
6. $\frac{3}{7}$ of my corn crop sold at 80c a bushel, brought \$960. What is the balance worth at 90c a bushel?
7. Three tenths of a man's taxes amounting to \$42.60 was for school purposes. Two tenths was for police and fire protection. What was the amount of tax for fire and police?
8. What was his total tax?
9. If it took me $5\frac{1}{2}$ hours to make 45 spools, how long will it take me to make a dozen? A hundred?
10. How many could I make in 8 hours?

In working the following problems, keep in mind the use of aliquot parts. You should again study this table in case you have not memorized these parts.

ALIQUOT PARTS OF \$1.00

\$1	1	2	3	4	5	6	7	8	9	10	11
2	.50										
3	.33 $\frac{1}{3}$	.66 $\frac{2}{3}$									
4	.25	($\frac{1}{2}$)	.75								
5	.20	.40	.60	.80							
6	.16 $\frac{2}{3}$	($\frac{1}{3}$)	($\frac{1}{2}$)	($\frac{2}{3}$)	.83 $\frac{1}{3}$						
7	.14 $\frac{2}{7}$		...			.85 $\frac{5}{7}$					
8	.12 $\frac{1}{2}$	($\frac{1}{4}$)	.37 $\frac{1}{2}$	($\frac{1}{2}$)	.62 $\frac{1}{2}$	($\frac{3}{4}$)	.87 $\frac{1}{2}$				
9	.11 $\frac{1}{9}$	.22 $\frac{2}{9}$	($\frac{1}{3}$)	.44 $\frac{4}{9}$	.55 $\frac{5}{9}$	($\frac{2}{3}$)	.77 $\frac{7}{9}$	.88 $\frac{8}{9}$			
10	.10	($\frac{1}{5}$)	.30	($\frac{2}{5}$)	($\frac{1}{2}$)	($\frac{3}{5}$)	.70	($\frac{4}{5}$)	.90		
11	.09 $\frac{1}{11}$		...			...		...	...	.90 $\frac{10}{11}$	
12	.08 $\frac{1}{3}$	($\frac{1}{6}$)	($\frac{1}{4}$)	($\frac{1}{3}$)	.41 $\frac{2}{3}$	($\frac{1}{2}$)	.58 $\frac{1}{3}$	($\frac{2}{3}$)	($\frac{3}{4}$)	($\frac{5}{6}$)	.91 $\frac{2}{3}$

Those less frequently used:

1. $\frac{1}{15} = .06\frac{2}{3}$

4. $\frac{1}{20} = .05$

7. $\frac{1}{30} = .03\frac{1}{3}$

2. $\frac{1}{16} = .06\frac{1}{4}$

5. $\frac{1}{24} = .04\frac{1}{6}$

8. $\frac{1}{40} = .02\frac{1}{2}$

3. $\frac{1}{18} = .05\frac{5}{9}$

6. $\frac{1}{25} = .04$

9. $\frac{1}{50} = .02$

DRILL IN ALIQUOT PARTS

Compare carefully:

$$\$480 \times \$0.33\frac{1}{3} = \$480 \div 3 = \$160$$

$$\$480 \div \$0.33\frac{1}{3} = \$480 \times 3 = \$1440$$

$$\$480 \times \$1.33\frac{1}{3} = \$480 + \frac{1}{3} \text{ of itself} = (480 + 160) = \$640$$

$$\$480 \div \$1.33\frac{1}{3} = \$480 - \frac{1}{3} \text{ of itself} = (480 - 120) = \$360$$

$$\$480 \times \$0.66\frac{2}{3} = \$480 - \frac{1}{3} \text{ of itself} = (480 - 160) = \$320$$

$$\$480 \div \$0.66\frac{2}{3} = \$480 + \frac{1}{3} \text{ of itself} = (480 + 240) = \$720$$

(81)

ORAL DRILL

Give the answers to the following, rapidly, using the fractions as indicated in the table. See the drill above.

- 90 bushels of oats at $33\frac{1}{3}\text{c}$ per bushel.
- 120 yds. of ribbon at $8\frac{1}{3}\text{c}$ per yd.
- 30 yds. of tape at $3\frac{1}{3}\text{c}$ per yd.
- 12 quarts of plums at 25c per quart.
- 24 lbs. of butter at $37\frac{1}{2}\text{c}$ per lb.
- 160 bushels of barley at $62\frac{1}{2}\text{c}$ per bushel.
- 820 pounds of tobacco at $12\frac{1}{2}\text{c}$ per lb.
- 40 bushels of corn at $87\frac{1}{2}\text{c}$ per bushel.
- How many yards at 75c can be bought for \$8.40?
- How many yards at $16\frac{2}{3}\text{c}$ can be bought for \$7.20?
- How many yards at $\$1.16\frac{2}{3}$ can be bought for \$22.40?
- How many yards at $\$1.66\frac{2}{3}$ can be bought for \$20.00?

By inspection determine which of the following are aliquot parts of \$1.00.

Examples:

$$.56\frac{2}{3} = \frac{56\frac{2}{3}}{100} \text{ (Multiply both terms by 3)} = \frac{170}{300} = \frac{17}{30}, \text{ Ans.}$$

$$.37\frac{1}{2} = \frac{75}{200} = \frac{3}{8}, \text{ Ans.}$$

$$1. \quad \$0.66\frac{2}{3}$$

$$6. \quad \$0.944\frac{4}{9}$$

$$11. \quad \$0.02\frac{1}{5}$$

$$16. \quad \$0.74\frac{1}{3}$$

$$2. \quad .43\frac{1}{3}$$

$$7. \quad .73\frac{1}{4}$$

$$12. \quad .725$$

$$17. \quad .9\frac{2}{3}$$

$$3. \quad .4375$$

$$8. \quad .5416\frac{2}{3}$$

$$13. \quad .0625$$

$$18. \quad .087\frac{1}{2}$$

$$4. \quad .562\frac{1}{2}$$

$$9. \quad .44$$

$$14. \quad .033\frac{1}{3}$$

$$19. \quad .041\frac{2}{3}$$

$$5. \quad .04\frac{1}{8}$$

$$10. \quad .006\frac{2}{3}$$

$$15. \quad .08\frac{1}{2}$$

$$20. \quad .04\frac{1}{6}$$

CHAPTER VI

COMMON FRACTIONS

GENERAL PROBLEMS

(82)

ORAL

1. At $8\frac{1}{3}$ c a pound, how many pounds of sugar can you buy for a dollar?

2. If a messenger boy earns $\$1\frac{4}{5}$ a day, six days a week, how much does he earn in a week?

3. You and your school chum form a partnership for selling papers after school hours. The two of you together sell 800 papers for the week at 3c each. You make a profit of $1\frac{1}{4}$ cents on each paper sold. On Saturday night, when profits are divided, how much does each one have?

4. You have kept an accurate account of your school expenses for the month, including lunch, school supplies, carfare, etc. The total for the month amounts to $\$12\frac{4}{5}$. Counting 20 days to the school month, what were your expenses for the day?

5. You buy tickets for your carfare at the rate of 6 for 51c. You use 2 tickets per day for a school month of 20 days, what is your carfare per month? If your school runs ten months, what will your carfare cost for the year?

*(83)

WRITTEN

NOTE: Business men as a rule count $\frac{1}{2}$ or more on the final result and drop the fraction if under $\frac{1}{2}$. For practice the pupil will be required to give the exact result.

1. In a test an automobile ran 28 miles in 30 minutes. At this rate, how many miles would it travel in $6\frac{1}{2}$ hours?

*You may not be able to give the exact result to the written problems mentally, but should approximate the answer before solving the problem.

2. A farm containing one section of land has 200 acres planted in corn, $\frac{1}{4}$ of the remainder in oats, $\frac{1}{3}$ in barley, $\frac{2}{3}$ in orchard and the balance in pasture. Give the acreage in each.

3. A man and wife started on a vacation tour and traveled 1860 miles, which they estimated to be $\frac{4}{5}$ of their journey. What would be the mileage at the end of the journey?

At 28c a gallon, and an average mileage of $12\frac{1}{2}$ miles to the gallon, what would be the cost of gasoline for the trip?

4. Wheat was quoted at the opening of the market on Monday at \$1.87; on Tuesday \$1.57 $\frac{1}{2}$; on Wednesday \$1.56 $\frac{3}{4}$; on Thursday \$1.56 $\frac{1}{4}$; on Friday \$1.55 $\frac{1}{2}$; on Saturday \$1.55 $\frac{3}{4}$. What was the average quotation for the week?

5. You pay \$40.00 a month for rent; your grocery bill averaged \$37.25. If this is $\frac{1}{3}$ of your monthly salary, what is your income for the year?

6. A bookkeeper spends $\frac{1}{4}$ of his monthly salary for board and room; $\frac{1}{3}$ for clothing; and $\frac{1}{6}$ for amusements and incidentals. If he has left at the end of the month \$60.00, what is his income and the amount expended for each?

7. Sugar is retailing at two pounds for 15c. At this rate what will 100 pounds cost?

8. If the housewife pays the huckster 20c for 3 pounds of potatoes, at this rate what is the cost per peck? Per bushel?

9. The retail merchant buys beans at \$4.50 per hundred pounds wholesale. He retails these at an average of 3 pounds for 25c. What is his gain per pound? Per hundred pounds?

10. Divide \$90.00 between two boys so that the first boy will have $\frac{2}{3}$ as much as the second boy; $\frac{2}{3}$ less than the second boy; $\frac{2}{3}$ more than the second boy.

(84)

ORAL

1. By selling eggs at 45c a dozen you gain $\frac{2}{3}$ of a cent apiece. How much would you have gained apiece by selling at 48c per dozen?

2. At 36c a dozen what would be the cost of 224 eggs?

3. At $16\frac{2}{3}$ c a pound, what will a 25 pound box of prunes cost?

4. At $16\frac{1}{2}$ c a pound what will $12\frac{1}{2}$ pounds of evaporated peaches cost?

5. I gain $1\frac{1}{3}$ c each by selling apples at 5c apiece; what would I lose by selling them 3 for 10c?

(85)

WRITTEN

1. An office building was insured for \$60,000 and the policies were written by three companies, as follows: Company A policy \$15,000; Company B policy \$20,000; Company C policy \$25,000. The building was damaged by fire to the extent of \$18,000. What part of the loss should each company stand?

2. At $\frac{3}{4}$ of a dollar per 100 pounds, how many tons of hay can you purchase for \$186.00?

3. You go to a store with \$3.00. You buy 3 pounds of coffee at $33\frac{1}{3}$ c a pound; 3 pounds of onions at $4\frac{1}{2}$ c a pound; 10 pounds of flour at $4\frac{1}{2}$ c a pound; $\frac{1}{2}$ pound of pepper at 75c a pound. How much change should you receive?

4. 36,630 pounds of wheat were put into sacks averaging $2\frac{1}{2}$ bushels to the sack. How many sacks were there?

5. By rearranging and installing new machinery, the factory increased production by $\frac{1}{3}$. At the same time the working hours were reduced from 10 to 8 hours. Was there an increase or decrease in production, and how much?

6. Find the cost of $18\frac{1}{2}$ pounds of raisins at $13\frac{1}{2}$ c a pound.

7. You spend \$7.00 for dress goods, paying $87\frac{1}{2}$ c a yard. How many yards did you buy?

8. I spent $\frac{1}{3}$ of my money and lent $\frac{5}{6}$ of the remainder. What part do I have left?

9. $\frac{1}{8}$ of a dollar is what part of $\frac{1}{12}$ of a dollar?

10. A dealer paid $\$3\frac{3}{4}$ a ton for coal. He retailed this coal at $18\frac{1}{2}$ c a bushel. What does he gain per bushel?

(86)

ORAL

1. A merchant's debts amount to \$2500.00, which is $\frac{1}{7}$ of his assets. What is the value of his assets and how much is he worth?
Note, assets — debts = worth.

2. A stock of a bankrupt merchant was sold for \$24,000, which was $\frac{2}{3}$ the cost. What was the cost of the stock? What was the loss?

3. What will $3\frac{1}{4}$ yards of silk cost at $\$5\frac{1}{4}$ per yard?

4. Bananas are priced at 40c per dozen. How many can you buy for 10c?

5. A merchant sells apples 3 for 10c. If the box contains 120 apples, what will he get for the entire box? If he bought them at the rate of 4 for 10c, what will be his gross profit on the box?

(87)

WRITTEN

(Try some of these mentally)

1. If $4\frac{1}{2}$ subtracted from a certain number gives $\frac{1}{30}$ as a remainder, what is the fraction?

2. A clerk spends \$42.00 a week, which is $\frac{3}{4}$ of his salary. A bookkeeper of the same firm spends \$32.00 a week or $\frac{4}{7}$ of his salary. What is the weekly salary of each?

3. If you earn \$48.00 per week, and spend $\frac{1}{5}$ for board and room, $\frac{1}{8}$ for clothes, $\frac{1}{12}$ for amusements, clubs, etc., $\frac{3}{20}$ for travel and miscellaneous, $\frac{3}{16}$ in Liberty Bonds, and the remainder in the savings bank, how much was expended for each, and what was your bank account at end of the year?

4. A farmer took to market 8 dozen eggs at $31\frac{1}{2}$ cents per dozen and $6\frac{1}{2}$ pounds of butter at 38c. He exchanged the eggs and butter for sugar at $8\frac{1}{3}$ cents a pound. How many pounds of sugar did he get?

5. $\frac{7}{9}$ added to a certain fraction gives $1\frac{5}{6}$. Find the fraction.
6. If the sum of two numbers is $95\frac{1}{2}$ and one of the numbers is $67\frac{1}{5}$, what is the other number?
7. What number divided by $2\frac{3}{8}$ will equal $12\frac{1}{8}$?
8. A. B. Jones and T. G. Smith contract to do a piece of work for \$625. Jones works 65 days and Smith 35 days. How should the money be divided?
9. Your wages are \$4 $\frac{1}{2}$ per day. If your expenses are \$2 $\frac{3}{4}$ per day, how long will it take you to save \$250.00?
10. If you run your automobile 75 miles on 7 gallons of gasoline, how many gallons will be required to make a trip of 350 miles?

(88)

ORAL

(Problems with unnecessary facts given.)

1. If a tank $12\frac{1}{2}$ ft. wide and $16\frac{1}{8}$ ft. long contains $820\frac{1}{3}$ gallons when $\frac{2}{3}$ full, how many gallons will it hold when full?
2. A cube is $7\frac{1}{3}$ inches each way and weighs $125\frac{2}{3}$ lbs. What will it cost to put a brass edging on all edges at $4\frac{1}{2}^c$ a foot, and a brass ball on each corner at $7\frac{1}{2}^c$ each?
3. A square garden $4\frac{1}{3}$ rods each way and a square garden $8\frac{2}{3}$ rods each way were planted to cabbage. The first produced 2175 lbs. of cabbage; the second 4350 lbs. The cabbage sold at $3\frac{1}{3}^c$ a lb. The area of the first garden is what fractional part of the area of the second?
4. The cabbage produced on the first garden is what fractional part of that produced on the second?
5. The value of the cabbage produced by garden No. 1 is what fractional part of the value of the cabbage produced by garden No. 2?

NOTE: If the teacher sees the need of additional problems of this nature they should be supplied.

(89)

WRITTEN

1. Your neighbor A. Smith bought 6 tons of coal at $\$6\frac{3}{8}$ per ton. What will your estimated supply of $12\frac{1}{2}$ tons cost?

2. What will be the cost of 12 gross of note books at $3\frac{1}{2}$ cents each?

3. A farmer sold $\frac{3}{4}$ bushels of seed potatoes for \$2.50. At the same rate what would his crop of 75 bushels produce?

4. Two brothers bought a farm for \$2400.00, each paying half. They each sold $\frac{1}{4}$ of their interest to a brother at cost. What part does each brother own and what is the amount invested?

5. A certain young man spent an average of $\$ \frac{1}{8}$ per day for movies, $\$ \frac{1}{4}$ for cigars and $\$ \frac{1}{12}$ for candy. How long will it take him to spend \$100.00.

6. You pay $8\frac{1}{2}$ c per yard for $10\frac{1}{2}$ yards of ribbon, $16\frac{3}{4}$ c per yard for 9 yards of gingham. What change would you receive from a \$5.00 bill? What would be the most convenient change for the cashier to make?

7. A box of apples containing 12 dozen were sold at the rate of $2\frac{1}{2}$ c each. If they sold for $\frac{6}{5}$ of cost, what was the cost?

8. You earn $\$3\frac{1}{4}$ per day, six days per week. If your expenses are $\$10\frac{1}{8}$ per week, how long will it take you to save \$100.00?

9. Three railroads carry goods a distance of 1560 miles for \$7.50. If the distance carried by the first road is 780 miles, the second 520 miles and the remainder by the third road, how would the charge of \$7.50 be divided?

10. A box of 10 dozen oranges cost \$4.00. If one dozen freeze and the remainder are sold at $4\frac{1}{6}$ c each, what was the profit?

(90)

ORAL

1. A family saves \$960.00 per year, which is $\frac{2}{3}$ of its income. What is the income?

2. If you perform $\frac{1}{12}$ of a certain piece of work in the first week, working 6 days per week, how many days will it take to complete the task?

3. If you earn $3\frac{1}{2}$ dollars per day, how much will you earn in a month of 26 work days?

4. The athletic association of a school collected \$820.00 for the year. $\frac{1}{8}$ was for basket ball, $\frac{1}{4}$ for baseball, $\frac{1}{2}$ for football and the remainder for hockey. How much was collected for each?

5. A farmer sold $\frac{4}{9}$ of his apples for \$120.00. What is the value of entire crop?

(91)

WRITTEN

1. Sugar is sold in packages of 5 pounds for 38c and in 10-pound packages for 72c. How much per pound is saved by buying 10 pounds?

2. Sugar increased in retail price from $6\frac{1}{4}$ c per pound in 1910 to 3 pounds for \$1.00 in 1920. What fractional part of the increase is the 1910 price?

3. Eggs are sold at 28c per dozen with a profit of 7c per dozen. The gain is what fractional part of cost? Of selling price?

4. You owned a lot containing $2\frac{3}{4}$ acres. You sold one lot of $\frac{1}{3}$ of an acre, another of $\frac{5}{8}$ of an acre. How much remained in your lot?

5. Five hogs dressed, weighed respectively $150\frac{1}{4}$ lbs., $180\frac{2}{3}$ lbs., $165\frac{1}{2}$ lbs., $138\frac{3}{4}$ lbs., and $186\frac{1}{3}$ lbs. What was the average weight and the cost at $16\frac{2}{3}$ c per pound?

6. A certain lot is $50\frac{1}{4}$ feet wide and $180\frac{2}{3}$ feet deep. How many rods of wire will be required to fence the lot?

7. Another lot is $180\frac{2}{3}$ feet by $212\frac{1}{4}$ feet. How many rods of wire are required to fence it?

8. If you buy $6\frac{3}{4}$ acres of land for \$5650.50 and sell lots of $\frac{1}{4}$ acre each at \$500.00, what was the cost per acre and the total profit?

9. Find the cost of the butter in 5 tubs weighing $33\frac{1}{2}$ lbs., $34\frac{1}{4}$ lbs., $30\frac{3}{4}$ lbs., $31\frac{1}{8}$ lbs., and $31\frac{1}{3}$ lbs., if the tubs average $4\frac{1}{2}$ lbs. each, and the butter is billed at 40c.

10. Multiply the sum of $\frac{7}{8}$ and $\frac{2}{3}$ by their difference.

(92)

ORAL

1. A plant employed 350 men. In January, 1921, 150 were laid off. What part remained at work?

2. I spent $\frac{2}{5}$ of my bank account for an automobile costing \$2500. What was my bank account? What was left in the bank?

3. A clerk sold $6\frac{1}{2}$ yards of cloth from a piece containing $20\frac{1}{4}$ yds. How many yards remained?

4. Multiply the sum of $\frac{1}{2}$ and $\frac{1}{4}$ by their difference.

5. If 3 boys can prepare the ground and plant a garden in 5 days, how long will it take 5 boys? If it takes 5 boys 5 days, how long will it take 3 boys?

(93)

WRITTEN

1. A boy rides a bicycle 7 miles in $\frac{3}{4}$ of an hour. At the same rate how long would it take him to ride $20\frac{3}{4}$ miles?

2. A merchant owned $\frac{3}{5}$ of an interest in a store and sold $\frac{1}{2}$ his interest for \$6000.00. What was the value of the store, the value of his remaining interest and the interest sold?

3. A typist gets $\$3\frac{1}{4}$ per thousand for addressing envelopes. If she addresses 950 a day, what will be her income for the week of $\frac{1}{2}$ work days?

4. A merchant asked $\frac{1}{2}$ more for an article than it cost. During a dull season he reduced the price $\frac{1}{5}$. If he gains \$2.40, what was the cost and selling price?

5. One merchant offered $\frac{5}{4}$ the cost of a certain lot while another offered $\frac{8}{7}$ the cost. If their offers differed to the amount of \$960.00, what was the cost of the property?

6. An agent bought a section of land at $\$62\frac{1}{2}$ per acre and sold $85\frac{1}{2}$ acres to A at $\$92\frac{1}{2}$ per acre, $40\frac{1}{2}$ acres to B for $\$97\frac{1}{4}$ per acre, and $36\frac{1}{2}$ acres to C at $\$90\frac{3}{4}$ per acre, and the remainder to D at $\$67\frac{1}{4}$ per acre. If his expenses in disposing of the land averaged $\$1\frac{1}{8}$ per acre, what did he gain?

7. I bought 25 pounds of sugar at $\$ \frac{1}{8}$ per pound, 10 pounds of coffee at $\$ \frac{3}{8}$ per pound, 8 pounds of bacon at $\$ \frac{1}{6}$ per pound, one ham weighing $11 \frac{1}{4}$ pounds at $\$ \frac{1}{3}$ per pound. What was my bill?

8. Find the total cost of the following:

$3 \frac{1}{2}$ thousand feet of lumber at $\$ 27 \frac{1}{2}$ per thousand

1530 pounds of wheat at $\$ 1 \frac{7}{8}$ per bushel

2828 pounds of shelled corn at $\$ 1 \frac{1}{12}$ per bushel (use 56 lbs.)

$25 \frac{1}{4}$ tons of coal at $\$ 7 \frac{1}{2}$ per ton.

9. Find the sum of the results:

$$(16 \frac{2}{3} + 27 \frac{1}{3} - 12 \frac{1}{5}) \times 3 \frac{5}{8}$$

$$90 \frac{1}{2} \div 10 \frac{1}{2} + 125 \frac{1}{7} - 81 \frac{1}{3}$$

$$(12 \frac{1}{6} + 50 \frac{5}{6} + 40 \frac{1}{3} - 76 \frac{2}{3}) \times 3 \frac{1}{4}$$

10. Find the total of the sum, difference, product and quotient of $16 \frac{1}{2}$ and $3 \frac{1}{4}$.

(94)

ORAL

1. If a yard of cloth costs $\$ \frac{3}{4}$, how much will $\frac{1}{2}$ yard cost? $\frac{3}{4}$ yards? $1 \frac{1}{2}$ yards? $8 \frac{1}{2}$ yards?

2. If one dozen oranges costs $\$ \frac{3}{5}$, how much will one orange cost? How much will $\frac{1}{4}$ dozen cost? $\frac{1}{2}$ dozen?

3. At $\$ \frac{7}{8}$ per yard, what will 4 yards of cloth cost? At $\$ 1 \frac{1}{4}$ per yard? At $\$ 1 \frac{3}{4}$ per yard?

4. What is the cost of $6 \frac{1}{2}$ pounds of flour at $6 \frac{1}{2}c$ per pound?

5. Multiply: $2 \frac{1}{2} \times 2 \frac{1}{2}$; $6 \frac{1}{4} \times 6 \frac{1}{4}$; $8 \frac{1}{2} \times 8 \frac{1}{2}$; $3 \frac{1}{3} \times 3 \frac{1}{3}$; $8 \frac{1}{4} \times 8 \frac{1}{4}$; $12 \frac{1}{2} \times 12 \frac{1}{2}$.

(95)

WRITTEN

1. A merchant bought 12 pieces of denim as follows: $40 \frac{1}{2}$, $43 \frac{1}{4}$, $41 \frac{1}{4}$, $41 \frac{1}{2}$, $44 \frac{1}{4}$, $43 \frac{3}{4}$, $40 \frac{1}{2}$, $40 \frac{1}{4}$, $42 \frac{1}{2}$, $41 \frac{3}{4}$, $42 \frac{1}{4}$, $40 \frac{1}{2}$ yards at $\$ \frac{1}{6}$ per yard. What was the cost?

2. A merchant bought a piano and marked it at $\frac{1}{2}$ above cost. To assure quick sale he reduced the price $\frac{1}{8}$. If the profit was \$50.00, what was the cost, marked price and selling price?

3. A dealer bought some grain at $\frac{9}{10}$ of market price and sold it later for $\frac{1\frac{3}{2}}$ of former market price. If he gained \$66.00, how much did he pay for the grain?

4. In a certain shipbuilding yard employes are allowed time and a half for all time over eight hours per day, with double time for Sundays and holidays. Employee Jones averages $9\frac{3}{4}$ hours a day for six days and works seven hours on Sunday. What were his weekly wages at 65c per hour?

5. Find the value of $(25\frac{1}{5} \div \frac{6}{7}) - (36\frac{1}{2} \div 3\frac{1}{3})$.

6. What is the number whose $\frac{4}{5}$ exceeds its $\frac{2}{3}$ by 60?

7. I spend $\frac{1}{3}$ of my salary for board, and deposit $\frac{3}{4}$ of the remainder. How much do I earn if I have \$25.00 in my pocket, and how much deposited?

8. By what number must $\frac{5}{6}$ be multiplied to produce 1704?

9. You start on your vacation in an automobile. You cover $\frac{3}{8}$ of the distance the first day, $\frac{1}{8}$ the second day and $\frac{1}{5}$ the third day. The fourth day you reach your destination, driving 150 miles. How many miles covered each day? How many the four days?

10. An estate of \$12,500 was left to a son and daughter. The son received $2\frac{3}{4}$ times the amount left to the daughter. What was the value of the daughter's estate?

(96)

ORAL

1. A merchant bought a piece of cloth containing $30\frac{1}{2}$ yards, from which he has cut two pieces of $8\frac{3}{4}$ and $12\frac{1}{2}$ yards. How many yards remaining?

2. From the number 34 continue to subtract $4\frac{1}{4}$ until nothing remains.

3. You buy $14\frac{1}{2}$ yards of cloth and make two dresses from the same pattern. If you have $\frac{3}{4}$ yard left and allow $\frac{1}{2}$ yard for waste, how many yards in each dress?

4. Add 6 to each term of $\frac{2}{3}$ and state how much the fraction is increased or decreased. Multiply each term by 6 and state how much the number is increased or decreased.

(97)

WRITTEN

1. I buy 25 turkeys for \$145.50. If the average weight is $10\frac{4}{5}$ pounds, how much did I pay per pound? If I desire a profit of $\frac{1}{5}$ above cost, what must I sell at per pound?

2. Four men are partners in the general merchandise business and gain the first year \$6250.00. If they share the profits $\frac{2}{5}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{3}{20}$ respectively, how much will each get?

3. At $\$37\frac{1}{2}$ per acre, how many acres of land can be bought for $\$3018\frac{3}{4}$?

4. What is the price of hay per ton when 8250 pounds were bought for \$133.65?

5. A man's liabilities are equal to $\frac{3}{16}$ of his assets. If his assets amount to \$48,000.00, how much is he worth?

6. A real estate dealer bought two pieces of land containing $\frac{6}{7}$ acres and $3\frac{5}{8}$ acres each, for \$4,200.00. He divided these into lots of $\frac{1}{4}$ acre each and sold them for an average of \$324.00. If his expense of dividing and selling amounted to $\$20\frac{1}{2}$ per lot, what was his profit?

7. If you spend $\frac{7}{15}$ of your capital for a farm, $\frac{1}{12}$ for repairs and $\frac{1}{8}$ for an automobile and have \$3,900 left, what was your original capital and the amount expended for each?

8. I built a house on a lot which cost \$1,200. The cost of the lot was $\frac{2}{3}$ the cost of the house. What was the cost of both?

9. A speculator gained $\frac{1}{20}$ on stock he sold at \$84.00. Had he sold 10 days later when they were quoted at 90, what would have been his gain? What would he have lost if sold at \$75.00?

10. A commission merchant received \$20,000 to invest in grain. He invested $\frac{1}{5}$ in corn at $\$2\frac{2}{3}$ per bushel, $\frac{2}{5}$ in wheat at $\$1\frac{1}{5}$ per bushel, and $\frac{1}{4}$ in oats at $\$2\frac{2}{5}$ per bushel. He invested the remainder in barley at $\$3\frac{3}{8}$ per bushel after deducting \$225.00 for commission. How many bushels of each did he purchase?

(98)

ORAL

1. From a piece of cloth containing $43\frac{3}{4}$ yards there were cut 9 dresses of $4\frac{1}{2}$ yards each. How many yards remained?

2. If $3\frac{1}{4}$ yards of cloth are required for one suit of clothes, how many suits can be cut from 39 yards?

3. You go to market with butter and eggs. The eggs bring you \$4.00, which is $\frac{2}{3}$ the value of the butter. What did you get for the butter?

4. A woman bought two chickens weighing $3\frac{1}{4}$ and $3\frac{1}{2}$ pounds each, at 28c per pound. How much change will she get from a \$5.00 bill?

5. A tract of land was sold at \$110.00 per acre which was $\frac{2}{9}$ above cost. What was the cost?

(99)

WRITTEN

1. A merchant bought a box of 120 apples at the rate of 3 for 5c. 18 were bruised and unfit for sale. He sold the others at the rate of 3 for 10c. What was his profit?

2. I buy 10 bags of wheat containing $120\frac{1}{2}$, $122\frac{1}{4}$, $119\frac{3}{4}$, $121\frac{1}{2}$, $122\frac{3}{4}$, $119\frac{1}{2}$, $122\frac{1}{2}$, $120\frac{1}{4}$, 121, $123\frac{1}{4}$ pounds each. If I pay $\$1\frac{1}{16}$ per bushel, allowing 1 pound each for the weight of the bags, what is the cost of the wheat?

3. A speculator bought 25,500 pounds of wool during high prices of 1918 at $\$ \frac{3}{8}$ per pound. During the falling prices of 1920 he was forced into bankruptcy and the wool was sold at an average of $16\frac{2}{3}$ c per pound. What was the loss?

4. I sold $\frac{3}{5}$ of my property for \$7,500. The part I have remaining at the same rate is worth $\frac{4}{5}$ the original cost of the entire property. What was the cost?

5. My income for January, 1922, was $\$250\frac{3}{4}$. Commission on increased sales raised my income an average of $\$15\frac{1}{4}$ per month. What was my salary for December and my total income for the year 1922?

6. I bought 10,800 shingles billed at $\$5\frac{3}{4}$ per thousand. Paid freight and cartage \$8.10. I sold the shingles at $\$7\frac{3}{4}$ per thousand. What was the total gain?

7. A, B and C are partners in a business owning $\frac{1}{3}$ each. A sells his interest to B and C, $\frac{1}{3}$ to B and the remainder to C. What part do B and C now own?

8. I speculated in grain last year and lost $\frac{1}{3}$ of my investment. This year I gained \$600.00 and now have \$2600.00. What was my original investment?

9. The sum of two numbers is $48\frac{1}{4}$. Their difference is $6\frac{3}{4}$. What are the numbers?

10. A, B, C and D invest \$1200, \$1500, \$2500, and \$1000 respectively. They are to share gains and losses in proportion to capital invested. What will each get?

(100)

ORAL

1. Reduce $\frac{3}{8}$ to thirds, $\frac{5}{8}$ to thirds, $\frac{3}{4}$ to twelfths, $\frac{2}{3}$ to fourths, $\frac{4}{5}$ to sevenths.

2. A boy has in his pocket $\$ \frac{3}{5}$ and earns $\$ \frac{3}{4}$. How much does he have?

3. You buy a lot and pay $\frac{1}{5}$ cash. You pay $\frac{1}{4}$ the second year and $\frac{1}{3}$ the third year. What was the cost of the lot if you paid the balance, \$260, the fourth year?

4. By selling some cattle at $\frac{9}{7}$ of cost I gained \$120.00. What was cost and selling price?

5. What number increased by $\frac{1}{6}$ of itself equals to 42, 56, 84, 35?

(101)

WRITTEN

1. Four brothers earn respectively, $\$6\frac{1}{2}$, $\$7\frac{1}{4}$, $\$6\frac{4}{5}$, and $\$6\frac{1}{4}$ per day. What is the average daily earning? If they work on an average of $5\frac{1}{2}$ days per week, how much will they earn in a year?

2. What will be the wholesale cost of 1506 eggs at $27\frac{1}{2}$ c per dozen?

3. If you earn \$150 per month and spend \$75 for clothing and board and \$45 for miscellaneous expenses, what fractional part do you save?

4. A barrel of flour weighs 200 lbs., gross. If the flour weighs 196 lbs., what is the fractional weight of both the barrel and the flour?

5. I buy flour at \$8.60 per barrel. At what price must I sell to gain $\frac{1}{5}$ of the cost?

6. If you get \$4.50 per day for 8 hours and time and a half for overtime, how much do you earn the first week working 6 days if you put in $60\frac{1}{2}$ hours?

7. Find the sum of the Sum, Difference and Product of $42\frac{7}{8}$ and $15\frac{1}{3}$.

8. A merchant buys 3650 yds. of silk at $\$1.12\frac{1}{2}$ per yard and sold $\frac{4}{5}$ of it at $\frac{3}{8}$ above cost. He had a special sale and reduced the price on the remainder $\frac{1}{3}$. It was all sold. What was the profit?

9. A yard is what fractional part of a rod? A rod is what part of a mile?

10. I can do a piece of work in 6 days that requires 8 days for you to complete. If we work together, how long will it take us to finish the task?

11. A contractor bought a lot and built a house thereon. The two cost him \$7500. If the lot cost $\frac{1}{4}$ as much as the house, what did each cost?

12. In a manufacturing plant $\frac{2}{5}$ of the space is given to machinery, $\frac{3}{10}$ to stores, $\frac{1}{5}$ to assembling and shipping and the remainder, 1200 square feet, for offices. How many square feet is occupied by each?

13. A dealer bought a herd of cattle for \$3600. He sold $\frac{3}{5}$ of the herd at an advance of $\frac{1}{6}$ above cost. He held the others to feed for better market. He was, however, forced to sell the remainder at $\frac{1}{8}$ less than cost. Did he gain or lose, and how much?

14. If you walk to school $1\frac{1}{2}$ miles in $\frac{2}{5}$ of an hour, how long will it take you to walk to the city, $4\frac{1}{2}$ miles at the same gait?

15. A grocer bought eggs at 56c a dozen wholesale. He retailed these at $5\frac{1}{2}$ c each. What was his profit on the dozen?

16. Your father has $\frac{1}{2}$ of his money in a checking account in a certain bank; $\frac{1}{3}$ in a savings account and the remainder, \$300.00, in another savings bank. How much did he have in each bank, and the total amount on deposit?

17. You have 10 acres of land in the suburbs. You divide this into lots of $\frac{5}{8}$ of an acre each. You sold these at \$600.00 per lot, how much did you get per acre?

18. During vacation of 10 weeks you earned \$2 $\frac{3}{8}$ per day, six days per week. How much did you earn during vacation?

19. Two fields are the same width, but one is $\frac{1}{5}$ longer than the other. If the smaller contains 35 acres, how many acres are there in the larger field?

20. If a field 20 x 120 rods is to be increased $\frac{1}{8}$ in width, what fractional part will the area of the field be increased?

21. Multiply $145\frac{2}{3}$ by $12\frac{1}{4}$ and divide the product by $25\frac{1}{2}$.

22. A stock of merchandise is marked $\frac{1}{4}$ above cost, but on account of damage by water was reduced $\frac{1}{3}$ of marked price. If the loss was \$600.00, what was the cost and selling price?

23. What will $31\frac{1}{2}$ gallons of vinegar cost if $2\frac{1}{2}$ gallons cost 94c?

24. I gave a \$20.00 bill in payment of 6 pounds of nuts at $16\frac{2}{3}$ c per pound, 10 pounds of coffee at $37\frac{1}{2}$ c per pound and $12\frac{1}{2}$ pounds of butter at 50c per pound. What change should I receive?

QUESTIONS FOR REVIEW

1. What is a fraction?
2. In adding fractions, what is the first step to take? Why?
3. In subtracting fractions what is the first step to take? Why?
4. Name the different kinds of fractions and give examples of each.
5. What value is gained by approximating answers before solving?
6. What is the numerator of a fraction? Denominator?
7. How are fractions reduced to lower terms?
8. What effect will it have upon a fraction if you divide the numerator by two? The denominator?
9. How do you change fractions to higher terms?
10. Multiply both terms of a fraction by the same number. What is the result?
11. If the denominators are the same, how are fractions added? Subtracted? Divided?
12. What is meant by finding the L. C. M.?
13. When is it necessary to use the G. C. D.?
14. How do you multiply fractions? Mixed numbers?
15. How would you change an improper fraction to a mixed number?
16. Change $4\frac{1}{2}$ to an improper fraction.
17. State a simple rule for the division of fractions.
18. If a whole number is multiplied by a fraction, is the product larger or smaller than the multiplicand? Explain.
19. If a whole number is divided by a fraction is the quotient larger or smaller than the dividend? Explain.
20. What is the chief objection to using improper fractions?
21. What effect will it have upon a fraction if you multiply the numerator by two? The denominator?
22. State how you would multiply mixed numbers having like fractions.
23. What is the meaning of the word "of" when used in connection with arithmetic?
24. How would you divide a mixed number by a mixed number?

CHAPTER VII

DECIMAL FRACTIONS

In Common Fractions the denominator is expressed in figures, and may be any number from 2 to 200,000 or even higher. In Decimal Fractions the denominator may be expressed in figures or by the position of a period called the *decimal point*, and is always a multiple of ten—generally tenths, hundredths or thousandths, as $\frac{18}{100}$ (a decimal fraction) or .18 (a decimal).

The use of Common Fractions will always produce an exact answer, but very often at the expense of much time and labor. The use of decimals is often inexact, but sufficiently near for general business purposes.

Two great advantages are found in the use of decimals. Decimal fractions afford a much easier reduction to higher or lower terms, and are therefore much easier to add or subtract, multiply or divide. Decimal fractions, as found in business, are generally expressed in *tenths*, *hundredths* or *thousandths*, although in cost figuring the decimal may be carried to the fifth place or hundred-thousandths. This greatly facilitates the comparison of numbers and number relationships. To illustrate: It is easier to compare .7 and .9 than it is to compare $\frac{7}{8}$ and $\frac{9}{10}$. If we reduce $\frac{7}{8}$ and $\frac{9}{10}$ to a common denominator we have $\frac{70}{80}$ and $\frac{72}{80}$. Comparison is now possible, but the denominator must still be written in figures. If we change $\frac{7}{8}$ and $\frac{9}{10}$ to thousandths we have .875 and .900. Here we discover that the fractions can be much more easily used and an easier, finer comparison has been made possible. This is the reason why baseball records are recorded in thousandths.

If a business man gains \$2463.73 on a total sales of \$17,524.87 the relation between these two facts can be expressed accurately

only in the common fraction form $\frac{\$2463.73}{17525.87}$. This is exact, but a very difficult fraction to use in figuring, so the business man changes it to the decimal form of .45+. This is not exact, but near enough, and much more easily used. If he shortens this to 14% or 15% the relation between Selling Expense and Sales is only approximately expressed.

If a manufacturer finds that it cost \$754.81 to produce 64,250 bolts, the cost of each may be expressed as $\frac{\$754.81}{64250}$, but this is too cumbersome for use, so he divides the numerator by the denominator and carries it to four or five places, \$.011747. He does not shorten this to .0117 or to .012, for to do so would throw his estimate on 10,000 bolts off as follows: 10,000 bolts at .0117 = \$117; at .012 = \$120; at .011747 = \$117.47 the correct result.

In figuring dollars and cents final results are changed to even cents; a half of a cent or more is counted as a whole cent; less than half cent is dropped. $72\frac{1}{4}$ dozen eggs at $33\frac{1}{2}$ c would not be figured as 72×33 . These fractions cannot be changed or dropped, but the final result $\$24.08\frac{1}{3}$ is changed to \$24.08. If the eggs were exchanged for sugar at $6\frac{1}{4}$ c a pound, we would receive $385\frac{7}{8}$ pounds or $385\frac{1}{4}$ pounds nearly. Figured exact by common fractions we should receive $385\frac{1}{3}$ pounds.

HOW TO USE DECIMALS

1. To reduce a common fraction to decimal form divide the numerator by the denominator.

$$\frac{7}{12} = 7 \div 12 = .58333 +$$

NOTE: $\frac{7}{12} = .58333 +$ an inexact decimal

$$\frac{7}{8} = .875 \text{ an exact decimal}$$

2. Reduce a decimal fraction to common form, rewrite the fraction expressing the denominator in figures, and reduce to lowest terms.

$$.0625 = \frac{625}{10000} = \frac{25}{400} = \frac{5}{80} = \frac{1}{16}$$

TO EXTEND OR SHORTEN THE FORM OF A DECIMAL

3. To extend an exact decimal, add ciphers; $.875 = .87500$.

To extend an inexact or circulating decimal; by inspection discover its value and add figures accordingly. $.222\frac{2}{9} = .222222\frac{2}{9}$ and $254.2342 = 254.234234 +$.

To extend a mixed decimal, change the appended fraction to its decimal equivalent and append the resulting figures instead.

$$\frac{1}{30} = .033\frac{1}{3} \text{ therefore } .427\frac{1}{30} = .427033\frac{1}{3}$$

To shorten a decimal the part you wish to abbreviate should be read as an independent number, written in common fraction form and reduced to its lowest terms.

Given: $7.20853\frac{1}{3}$ to shorten one, two, three, four, or five places.

$$\frac{3\frac{1}{3}}{10} = \frac{1}{3}, \text{ therefore } 7.2085\frac{1}{3} = 7.20853\frac{1}{3}$$

$$\frac{53\frac{1}{3}}{100} = \frac{160}{300} = \frac{8}{15}, \text{ therefore } 7.208\frac{8}{15} = 7.20853\frac{1}{3}$$

$$\frac{853\frac{1}{3}}{1000} = \frac{2560}{3000} = \frac{64}{75}, \text{ therefore } 7.20\frac{64}{75} = 7.20853\frac{1}{3}$$

NOTE: See No. 2 under Axioms on Decimals, when reading the above end decimals.

4. To add or subtract decimals, arrange in columns, keeping the decimal points in a perpendicular line and add as whole numbers are added. (The decimal point in the answer will be in line with the decimal points above.) If mixed decimals occur, extend or shorten the decimals until the common fractions at the end are in a perpendicular line.

$$4\frac{1}{8} + 7.28575 + 3.33\frac{1}{3} + 7\frac{1}{24} + .47 =$$

4.125		4.125
7.285 $\frac{3}{4}$		7.28575
3.333 $\frac{1}{3}$	or	3.33333 $\frac{1}{3}$
7.041 $\frac{2}{3}$		7.04166 $\frac{2}{3}$
.47		.47
22.255 $\frac{3}{4}$		22.25575

5. To multiply decimals, multiply as whole or mixed numbers are multiplied and point off places in the product (counting from the right) equal to the sum of the decimal places found in the numbers multiplied.

$$\begin{array}{r}
 .47525 \\
 \underline{.152} \\
 95050 \\
 237625 \\
 47525 \\
 \hline
 .07223800
 \end{array}
 \qquad
 \begin{array}{r}
 .072\frac{5}{8} \\
 .024\frac{3}{7} \\
 \hline
 288 \\
 144 \\
 15 = (\frac{5}{8} \text{ of } 24) \\
 30\frac{6}{7} = (\frac{3}{7} \text{ of } 72) \\
 \frac{15}{56} = (\frac{3}{7} \text{ of } \frac{5}{8}) \\
 \hline
 .001774 (\frac{7}{56}) = \frac{1}{8}
 \end{array}$$

6. To divide decimals, divide as whole or mixed numbers are divided, and point off as many places in the quotient as the decimal places in the dividend exceed those in the divisor, or place the quotient above the dividend and predetermine the position of the point.

No. 1. Without
avoiding decimal
points:

$$\begin{array}{r}
 .7825 \div .15 \\
 5.21\frac{2}{3} \\
 .15) \underline{.7825} \\
 \underline{75} \\
 32 \\
 \underline{30} \\
 25 \\
 \underline{15} \\
 10 \\
 \frac{10}{15} = \frac{2}{3}
 \end{array}$$

No. 2. Avoiding
decimal points
in the decimal:

$$\begin{array}{r}
 .7825 \div .15 \\
 5.21\frac{2}{3} \\
 15) \underline{78.25} \\
 \underline{75} \\
 32 \\
 \underline{30} \\
 25 \\
 \underline{15} \\
 10 \\
 \frac{10}{15} = \frac{2}{3}
 \end{array}$$

No. 3. Avoiding
decimal points
in both terms:

$$\begin{array}{r}
 .7825 \div .15 \\
 5\frac{13}{60} \text{ or } 5.21\frac{2}{3} \\
 1500) \underline{7825} \\
 \underline{7500} \\
 325 \\
 \frac{325}{1500} = \\
 \frac{65}{300} = \frac{13}{60}
 \end{array}$$

NOTE: In No. 2 the decimal point in the divisor has been eliminated by multiplying both terms by 100; i. e., moving the points two places to the right; in No. 3 the decimal point has been eliminated in both divisor and dividend by multiplying both terms by 10000; i. e., moving the point four places to the right. It is better to use form No. 2 or No. 3, either placing the decimal point before dividing or eliminating it entirely.

ILLUSTRATING DIVISION OF MIXED DECIMALS

$$7.083\frac{1}{3} \div 16\frac{2}{3} =$$

$.16\frac{2}{3}$	$7.083\frac{1}{3}$	(Both multi-	$.16\frac{2}{3}$	$7.083\frac{1}{3}$
3	3	plied by 3)	300	300
$.50$	21.250	(42.5, Ans.)		42.5 = Quotient -
	20 0		50.	2125.0
	1 25	*(3-2 = 1 place	200	(Multiplying
	1 00	to point off) or	125	both sides by 300
	250		100	(3 × 100) and set-
	250		250	ting quotient
			250	above.)

A LONG WAY TO WORK THE PROBLEM

$$7.083\frac{1}{3} \div 16\frac{2}{3} = \frac{21.250}{3} \div \frac{.50}{3} = \frac{21.250}{3} \times \frac{3}{.50} =$$

$$\frac{63.750}{1.50} = 1.50)63.750(42.5$$

$$\begin{array}{r} 60 \ 0 \\ 3 \ 75 \\ 3 \ 00 \\ \hline 750 \\ 750 \\ \hline \end{array}$$

This method generally leads to confusion and errors in determining the decimal point.

A SHORT WAY TO WORK THE PROBLEM

$$.16\frac{2}{3} = \frac{1}{6} \text{ and } 7.083\frac{1}{3} \times 6 = 42.5, \text{ the Answer.}$$

NOTE: Here is another illustration of the convenience of aliquot parts. If you do not know readily the aliquot parts of 10, 100 and 1000 you should learn them at once. (See Tables, Chapter V.)

*The rule usually given for placing the decimal point is as follows: The quotient must contain as many decimal places as those in the dividend exceed those in the divisor. Note that there are three places here in the dividend and two in the divisor and hence one place in the quotient.

AXIOMS ON DECIMALS

1. Moving the decimal point:

One place to the left divides by 10; to the right multiplies by 10.

Two places to the left divides by 100; to the right multiplies by 100.

Three places to the left divides by 1000; to the right multiplies by 1000.

Four places to the left divides by 10,000; to the right multiplies by 10,000.

2. Common fractions are always written one place to the right of decimal position; i. e., they are not counted in enumerating:

$.07\frac{1}{3}$ is read seven and one-third hundredths, not thousandths.

$.000\frac{2}{3}$ is read two-thirds thousandths, not ten-thousandths.

3. Any common fraction in its lowest terms whose denominator is not divisible by 2 or 5 will give an inexact or interminate decimal. This is because 2 and 5 are factors of 10, and decimals are written only in multiples of 10. However, not all fractions whose denominators are divisible by 2 are simple or exact decimals.

To illustrate: $\frac{5}{6} = .8333+$

4. Ciphers may be added or dropped at the right of a decimal, without changing its value. To illustrate: $.725 = .725000$.

5. To read a decimal, read the number as a whole number is read (disregarding ciphers at the left), and give the denominator indicated by the position of the decimal point. The decimal point is read as *and*.

To illustrate:

.0072 is read seventy-two ten-thousandths.

$.08\frac{1}{3}$ is read eight and one-third hundredths.

200.004 is read two hundred *and* four thousandths.

.204 is read two hundred four thousandths.

REVIEW QUESTIONS ON FRACTIONS

1. If you add any number to both the numerator and denominator of a fraction, will the value of the fraction be increased or decreased? Give several examples to illustrate your answer.

2. Is the value of a fraction altered by dividing the numerator and denominator by the same number? Illustrate.

3. Why do we invert the terms of the divisor and multiply in dividing fractions? When is this practice justifiable? When not?

4. If you divide the numerator by the denominator what form does the fraction take?

5. If all the denominators of a series of fractions are prime numbers, how do you find the L. C. M. (or denominator)?

6. If you were dividing 7846 by $18\frac{2}{3}$, how would you avoid the use of fractions?

7. In dividing \$7846 by $\$1.06\frac{2}{3}$ why is it wrong to begin thus:
 $1.0666)7846.0000($ _____

8. Why would it be unwise to use improper fractions thus:

$$\frac{7846}{1} \div \frac{320}{3} =$$

9. How would you avoid both difficulties?

10. Why are decimals used rather than common fractions in showing the relation existing between large numbers?

11. Why does the annexing of a cipher to a decimal not alter its value? Illustrate.

12. What figure in a decimal is given the denominator?

13. Give an example of the decimal form being more convenient than the common fraction.

14. Explain in your own words how decimals are arranged for adding. For subtracting.

15. Do you think it essential that you be able to use the decimal and common fraction interchangeably? Why?

16. Can you compare as to value $\frac{1}{2}$ ton and $\frac{1}{4}$ acre? Why?
17. Take any two fractions and explain how you can determine which is the larger.
18. You know the sum of two fractions. You also know one of the fractions. Explain how you would find the other fraction.
19. Explain why you cannot add $\frac{3}{4} + .6$.
20. What is an *aliquot part* of a number?
21. What different definitions may be given for a fraction?
22. What is the purpose or use of a fraction?
23. In how many ways may it be expressed?
24. What is a common fraction? A decimal fraction?
25. How is a mixed number reduced to an improper fraction? How reduced back again?
26. Why should improper fractions be avoided?
27. Explain how you can avoid them in addition. In subtraction. In division.
28. What is meant by *reducing to higher or lower terms*?
29. State five axioms concerning fractions.
30. How may the fractions be eliminated from dividend or divisor, in division problems?
31. What is the rule for arithmetical reasoning?
32. In how many ways may the comparison of numbers be expressed? Illustrate.
33. Give two axioms concerning the equation.
34. Explain how $.84263\frac{1}{3}$ may be shortened one place. Two places.
35. Is $93\frac{3}{4}$ an aliquot of 100?

EQUIVALENT FRACTIONS

The list of common and decimal fraction equivalents given below may be used to good advantage in various ways and will be reviewed again later in the text. The future operations will be much easier* if these are studied carefully and understood.

Common Fraction	Decimal Fraction	Common Fraction	Decimal Fraction
$\frac{1}{2}$	.5	$\frac{5}{12}$	$.41\frac{2}{3}$
$\frac{1}{3}$	$.33\frac{1}{3}$	$\frac{7}{12}$	$.58\frac{1}{3}$
$\frac{2}{3}$	$.66\frac{2}{3}$	$\frac{11}{12}$	$.91\frac{2}{3}$
$\frac{1}{4}$	.25	$\frac{1}{16}$	.0625
$\frac{3}{4}$	.75	$\frac{3}{16}$	.1875
$\frac{1}{6}$	$.16\frac{2}{3}$	$\frac{5}{16}$	.3125
$\frac{5}{6}$	$.83\frac{1}{3}$	$\frac{7}{16}$	.4375
$\frac{1}{8}$	.125	$\frac{9}{16}$	.5625
$\frac{3}{8}$	.375	$\frac{11}{16}$	.6875
$\frac{5}{8}$	.625	$\frac{13}{16}$	.8125
$\frac{7}{8}$	.875	$\frac{15}{16}$	.9375
$\frac{1}{12}$	$.08\frac{1}{3}$		

To again illustrate the value of the above table let us find the value of 600 pounds of nuts at $16\frac{2}{3}$ cents per pound. You can multiply and find the result, but since you know that 600 pounds at \$1.00 would cost \$600.00, and since $16\frac{2}{3}$ c is $\frac{1}{6}$ of \$1.00, then $\frac{1}{6}$ of \$600.00 will give \$100.00, the cost at $16\frac{2}{3}$ c. Apply the others in the same manner.

*Suppose you want to find $16\frac{2}{3}\%$ of \$240.00. It is much easier to say $16\frac{2}{3} = \frac{1}{6}$ of \$240, or \$40.00, than it is to multiply. Or suppose you have 2,000 yards of cloth at \$.43 $\frac{3}{4}$ per yard. \$.43 $\frac{3}{4} = \$\frac{7}{16}$. You readily see that $\frac{7}{16}$ is $\frac{1}{16}$ less than $\frac{8}{16}$ or $\frac{1}{2}$ of $\frac{1}{8}$ less. Then $\frac{1}{2}$ less $\frac{1}{8}$ of $\frac{1}{2}$ will give the desired result.

Operation: $\frac{1}{2}$ of 2,000=\$1,000 or cost at 50c per yd.
 $\frac{1}{8}$ of 1,000= $\frac{125}{}$ or $6\frac{1}{4}$ (Deduct).
\$ 875 cost at $43\frac{3}{4}$ c per yd., Ans.

CHAPTER VIII

DECIMAL FRACTIONS

(102)

ORAL

1. How many rods in .5 of a mile?
2. Reduce $\frac{7\frac{1}{2}}{30}$ to decimal form.
3. Reduce $.0666\frac{2}{3}$ to simple form.
4. A ton of ground feed contains 250 lbs. flaxseed, 1250 lbs. corn, and the remainder is barley. What decimal part of the whole is each?
5. If we add 500 lbs. of ground oats to the above, what decimal part of the whole amount will be oats?

(103)

WRITTEN

1. A party touring New England, traveled 142.7 miles on Monday, 173.4 miles on Tuesday, 180.7 miles on Wednesday, 101.1 miles on Thursday, 27.7 miles on Friday and camped over Saturday and Sunday. What was the average daily mileage for the week?
2. Multiply $408.06\frac{2}{3}$ by 75.125 and divide result by 4.
3. Multiply $79.304\frac{1}{3}$ by 600 and divide result by $\frac{1}{6}$.
4. The gross cost of manufacturing an order of 5240 knives was \$642.58. Find the exact cost of each to four decimal places.
5. The cost of manufacturing 6500 spools was \$4.28 $\frac{1}{2}$ c. Find the exact cost of each to five decimal places.

6. On July 19, the Pittsburgh "Pirates" had won 55 games and lost 29. The New York "Giants" had won 52 games and lost 30. What were the League standings of the two Clubs? (Express in thousandths.)

7. If the "Giants" defeat the "Pirates" in the next two games, which will be ahead?

8. From $7.03\frac{1}{3}$ subtract $4\frac{3}{2}$.

9. In a certain city there are 11,500 Jews, 500 Chinese, 8250 negroes, 750 Greeks, and 82,500 of other ancestry. What decimal part of the whole population is Greek?

10. If 980 people died in the above city in 1918, what was the death rate per M?

(104)

ORAL

1. If $\frac{4}{9}$ is reduced to decimal form what do we have? $\frac{5}{9}$? $\frac{7}{9}$?

2. Is there any difference between *two hundred fifteen thousandths* and *two hundred and fifteen thousandths*?

3. How do you multiply a decimal by 1000? by 100? by 10?

4. What is the difference between 4.8 and 4.8000?

5. What is the fractional equivalent of .750? .075? .0075? $.007\frac{1}{2}$? $.7\frac{1}{2}$? .000075?

(105)

WRITTEN

1. From *three hundred forty-six and two hundred five ten-thousandths*, take *three hundred forty-six ten-thousandths*.

2. To $7.0852\frac{1}{3}$ add $\frac{1}{8}$ of it.

3. Reduce $.0416\frac{2}{3}$ to simple fractional form.

4. From $16.2480\frac{1}{7}$ take $\frac{1}{3}$ of it.

5. Reduce 4.027 miles to feet.

6. Change .245 of a rod to the decimal part of a mile.

7. The circumference of a wheel is 3.1416 times the diameter. Find the circumference in inches of a wheel .72 of a foot in diameter.

8. 24 hundredths of a farm was planted to corn. The yield was 48.4 bushels per acre and sold at 75c a bushel. If the crop sold for \$1742.40, how many acres were in the farm?

9. Ten thousand soldiers were reduced in battle to 8424. What was the casualty rate per hundred?

10. Find the cost of $27\frac{1}{2}$ lbs. of coal at \$8.50 per ton.

(106)

ORAL

1. Which is larger, $\frac{7}{20}$ of a pie or .345 of it?

2. What is the cost of 550 lbs. of lime at \$3.50 per cwt.?

3. .01 is what part of 10?

4. .45 is .005 of what number?

5. Give the approximate whole numbers for the following products. Write your approximate answer and then arrive at exact answer by using pencil and compare:

(a) $7.255 \times 8.33\frac{1}{3}$

(f) 254.095×10.01

(b) $.925 \times 12.625$

(g) $68.66\frac{2}{3} \times .245$

(c) $47.115 \times .525$

(h) 7.9825×400.5

(d) $8.437 \times .7525$

(i) $.0215 \times 25.80$

(e) 2.48×15.75

(j) $250.75 \times .0125$

(107)

WRITTEN

1. Divide $72.046\frac{2}{3}$ by $1.4409\frac{1}{3}$.

2. From 64.273 take $\frac{1}{80}$ of it.

3. Divide $4.275\frac{2}{3}$ by $\frac{1}{5}$.

4. Divide $54.09216\frac{2}{3}$ by 5.

5. Reduce $.0093\frac{1}{3}$ to a simple fraction.

6. The present standing of the Champion Baseball Club is .875; of the Challengers .625. At the present time each club has played 48 games. If the Champions win only 4 out of the next 12 games, and the Challengers win eight out of the next 12 games, how will the clubs stand?

7. The tariff rate on a certain commodity was fixed at .00225 of the import value. If the tariff (tax) amounts to \$75,240, what was the total value of the product imported?

8. I sold 3725 lbs. of hay at \$37.50 per ton and with the proceeds bought lumber at \$45 a M ft. How many feet did I buy?

9. In a class of 42 pupils, 3 quit, 5 failed, 6 made *D*, 20 made *C*, 7 made *B* and the others made *A*. Show by thousandths what part of the class was in each group—express decimally. (NOTE: Your answers must total 1,000.)

10. The "Simplex" Auto makes $255\frac{1}{2}$ miles on 5 gallons of oil and the "Complex" makes $175\frac{1}{4}$ miles on $3\frac{1}{2}$ gallons. What is the difference in distance per gallon expressed in thousandths of a mile?

(108)

ORAL

1. Give the simple fraction equivalents of: .75, .075, .70, .07, .625, $.62\frac{1}{2}$, $.6\frac{1}{4}$, $.7\frac{1}{2}$. (Reduce your answer to lowest terms.)

2. .25 of \$840 is .5 of what number?

3. Read the quotients:

$$(a) 5 \div .5 = \quad (e) 15 \div 500 = \quad (i) 12 \times .05 =$$

$$(b) 5 \div .05 = \quad (f) 15 \div .05 = \quad (j) 12 \times .005 =$$

$$(c) 50 \div .5 = \quad (g) 15 \div .005 = \quad (k) 12 \div .00\frac{1}{3} =$$

$$(d) 50 \div .05 = \quad (h) 15 \div 5000 = \quad (l) 12 \times .00\frac{1}{3} =$$

4. If $.00\frac{1}{3}$ of a farm cost \$50, what will the whole farm cost?

5. From $8\frac{1}{8}$ take 7.625.

(109)

WRITTEN

1. Add $7\frac{1}{8}$, $14\frac{1}{36}$, $275.0\frac{1}{16}$, and 33.24725.

2. From 72.896205 take $13.01\frac{1}{48}$.

3. $7\frac{1}{2}$ sq. in. is what decimal part of a sq. foot. Show answer correct to five places.

4. A girl was given 12 problems, one of which required 3 answers. If she gave the correct answers for $7\frac{2}{3}$ problems, what grade should she receive? Express in hundredths.

5. The insurance on three frame houses, valued at \$2250, \$6400 and \$1350, respectively, cost \$72.45. What amount is chargeable to each house?

6. A *kilogram* is 2.20462 lbs. How many kilograms in a ton?

7. A *liter* is 1.05625 quarts. How many quarts in $2\frac{1}{2}$ liters?

8. A *meter*—39.37079 inches—is how many yards expressed decimally?

9. How many bbls., containing 31.5 gallons each, will it take to ship the yearly output of a well flowing 1984.5 gallons daily?

10. What will the freight be on 7245 lbs. of merchandise if the rate is \$1.05 per cwt.? (Figure exact.)

(110)

ORAL

- | | | | |
|----------------------|------------------------|----------------------------------|---|
| 1. $5 \times .5 =$ | 6. $.06 \times 2 =$ | 11. $.12\frac{1}{2} \times 10 =$ | 16. $1\frac{1}{2} \times .1\frac{1}{2} =$ |
| 2. $.5 \times .5 =$ | 7. $.16 \times 4 =$ | 12. $8 \times .08 =$ | 17. $15 \times .08 =$ |
| 3. $.05 \times .5 =$ | 8. $.6 \times .4 =$ | 13. $12 \times 1.5 =$ | 18. $.06\frac{2}{3} \times .03 =$ |
| 4. $.05 \times 5 =$ | 9. $25 \times .25 =$ | 14. $7.5 \times .5 =$ | 19. $.10 \times 9.5 =$ |
| 5. $.01 \times .2 =$ | 10. $.15 \times 1.5 =$ | 15. $16\frac{2}{3} \times 1.2 =$ | 20. $.8\frac{1}{3} \times 12 =$ |

(111)

WRITTEN

1. Add four and fifty-five ten thousandths, seventy-five one hundred thousandths, one and sixteen hundredths, five thousand and one millionths.

2. From the sum of No. 1 take the sum of the following: sixteen and two-thirds ten thousandths, one and nine hundred fifteen ten thousandths and seven eighths one hundred thousandths.

3. Add 75 units, 75 tenths, 75 hundredths, 75 thousandths and 75 ten thousandths.

4. Add $\frac{3}{4} + \frac{7}{8} + \frac{2}{3} + 1.6\frac{2}{3} + 10\frac{1}{4} + 37\frac{3}{8} + 1.9 + \frac{5}{64} + 12\frac{3}{16} + 2\frac{1}{8}$.

5. From 49 take $15.99\frac{1}{9}$.

6. From $71.4\frac{2}{3}$ take $65.87\frac{3}{8}$.

7. From $3.45\frac{1}{3}$ take $.99\frac{8}{9}$.

8. How much greater is $\frac{1}{2}$ tenth than $\frac{2}{3}$ hundredths?

9. How much less is *nine and one-third* hundredths than *six and two-third* tenths?

10. From 300 take 3 hundredths.

(112)

ORAL

- | | | |
|------------------------|--------------------------|-------------------------|
| 1. $75 \times .01 =$ | 6. $.75 \times 10 =$ | 11. $500 \times .015 =$ |
| 2. $75 \times 1.1 =$ | 7. $.75 \times .01 =$ | 12. $.0008 \times 75 =$ |
| 3. $75 \times 10.1 =$ | 8. $.125 \times .50 =$ | 13. $.75 \times 80 =$ |
| 4. $75 \times .005 =$ | 9. $.105 \times 5 =$ | 14. $800 \times .45 =$ |
| 5. $125 \times .008 =$ | 10. $.195 \times 2000 =$ | 15. $15.9 \times 20 =$ |

(113)

WRITTEN

1. $9.45 + .06\frac{1}{3} + 5.6 + 4.16\frac{2}{3} + 7.8\frac{1}{6} = ?$
2. $.24\frac{4}{9} + .1745\frac{2}{9} + 46.0\frac{5}{9} + 38.1876\frac{2}{3} = ?$
3. $.3708 + \frac{1}{2} + 2.06\frac{2}{3} + \frac{3}{4} + .17\frac{5}{6} + .0\frac{1}{3} = ?$
4. $81.26\frac{1}{2} + .046\frac{1}{9} + 9.6\frac{2}{3} + 76.7895\frac{5}{9} = ?$
5. From $\frac{2}{3}$ of a tenth take $\frac{1}{2}$ of a hundredth.
6. To 19 hundredths add $\frac{2}{3}$ tenths.
7. What is the result in decimals, of $3\frac{7}{8}$, $1\frac{2}{3}$, $5\frac{1}{3}$, $2\frac{3}{4}$?
8. Subtract $16\frac{1}{2}$ tenths from 4609 thousandths.
9. From $\frac{1}{8}$ of a tenth take $\frac{5}{9}$ of a hundredth.
10. From $\frac{1}{3}$ of $\frac{1}{3}$ take .0925.

(114)

ORAL

- | | | |
|------------------------|--------------------------|--------------------------|
| 1. $25 \div 25 =$ | 12. $750 \div .15 =$ | 23. $2800 \div 14000 =$ |
| 2. $25 \div 2.5 =$ | 13. $36.9 \div 1.23 =$ | 24. $280.4 \div .04 =$ |
| 3. $25 \div .25 =$ | 14. $19.65 \div .15 =$ | 25. $1600 \div .004 =$ |
| 4. $25 \div .025 =$ | 15. $7.5 \div 25 =$ | 26. $2200 \div .11 =$ |
| 5. $2.5 \div 25 =$ | 16. $125 \div .00025 =$ | 27. $136.35 \div 13.5 =$ |
| 6. $.25 \div 25 =$ | 17. $460 \div 2300 =$ | 28. $.6096 \div 1.016 =$ |
| 7. $.025 \div 25 =$ | 18. $.75 \div .0025 =$ | 29. $.999 \div .009 =$ |
| 8. $.025 \div 2.5 =$ | 19. $.000065 \div 13 =$ | 30. $7.75 \div .005 =$ |
| 9. $.025 \div .25 =$ | 20. $.00072 \div .002 =$ | 31. $999 \div .0099 =$ |
| 10. $4500 \div .9 =$ | 21. $36 \div 1200 =$ | 32. $1100 \div .022 =$ |
| 11. $9000 \div .045 =$ | 22. $.36 \div 120 =$ | 33. $162 \div 3000 =$ |

(115)

WRITTEN

1. A merchant paid \$67.50 for eggs at \$.375 per dozen. How many crates did he buy if there were 30 dozen to the crate?

2. A capitalist built an apartment house of 8 apartments at a cost of \$60,000. His yearly income from rent is .092 of the cost of the property. How much monthly rent does he get from each apartment?

3. Divide one tenth by one thousandth.

4. Divide 45 hundredths by 15 ten thousandths.

5. If sugar is selling at 7.5c per pound, how many pounds can you buy for \$75.75?

6. Corn meal is advertised as a special sale 12.25 pounds for \$.62. What is the cost per pound?

7. The sum of three numbers is 8.2. The smallest number is 1.9 and the largest is 4.25. Give the product of the three numbers.

8. I bought one gross of steel pens at a cost of .5 of a cent each. These I sold at a rate of 8 for 5 cents. How much will I gain on the gross? How much if I sell at 6 for 5c?

9. Three freight cars loaded with coal weighed respectively 19.75 tons, 22.285 tons, 20.72 tons. What is the total weight and value at \$6.75 per ton?

10. Reduce to decimals and add: $\frac{5}{12} + \frac{5}{9} + \frac{3}{4} + \frac{1}{64} + \frac{2}{3} + \frac{7}{50} + \frac{1}{6} + 1\frac{5}{8}$. Carry to four places exact.

GENERAL REVIEW PROBLEMS IN FRACTIONS

(116)

WRITTEN

1. An automobile makes $\frac{3}{4}$ of a mile on a circular track in $\frac{2}{3}$ of a minute. How many miles per hour does it run?

2. You buy $\frac{3}{4}$ yd. of cloth for 72c. At this rate what would $8\frac{1}{2}$ yds. cost?

3. If flour is selling at \$5.50 per cwt., what is the price per lb.?

4. A merchant bought a lot of eggs which he sold for \$33.60. If he got 28c per dozen, how many dozen did he sell? If he gained \$4.20, what did he pay per dozen for them?

5. A merchant bought tape at .5c per yard which he retailed at 4 yds. for \$.05. What was his gain per yard?

6. A certain farmer grows 76.5 bushels of wheat on $3\frac{1}{4}$ acres. At the same rate what will be the yield of $35\frac{3}{4}$ acres?

7. At \$47 $\frac{1}{2}$ per M, how many feet of lumber can you buy for \$22 $\frac{1}{2}$?

8. What will 650 lbs. of hay cost at \$18.00 per ton?

9. A speculator bought $26\frac{2}{3}$ acres of land which he divided into lots of $\frac{2}{3}$ acres each. These he sold at \$350.00 each. What did he get for the entire tract?

10. At \$2.25 per cwt. what will be the cost of 425 lbs. of nails?

(117)

WRITTEN

1. A farmer sold 3 calves weighing $195\frac{1}{2}$ lbs, $204\frac{3}{4}$ lbs., and $199\frac{3}{4}$ lbs. respectively and received in payment 3 tons of feed at 85c per cwt. How much did he get per pound for the calves?

2. A farmer sold 180.25 acres of land for \$15,000. What was the price per acre?

3. A big farm in Kentucky yields the following crops for the year: 5.5 acres of strawberries produced 550 crates at an average of $\$3\frac{1}{2}$ per crate; 10.2 acres of tobacco produced 13,600 pounds, sold at an average of 15c per lb.; 80 acres of corn produced 40 bushels per acre at $\$1\frac{1}{2}$ per bu. What was the value of the crop?

4. At \$.95 per cwt., how many tons of hay can be had in exchange for 7 head of cattle weighing 6440 lbs. if the market quotation is \$.0825 per lb.?

5. A farmer put 3025 pounds of fertilizer on a field of 20.5 acres. If the average cost was \$.5.25 per C what was the total cost and the cost per acre?

6. The above field is planted in corn and yields 1230 bushels. This was sold at \$1.20 per bushel. It was estimated by other acreage not fertilized that the increased yield by fertilization was 12.5 bu. per acre. Did it pay to fertilize? How much per acre?

7. Add the sum, difference and product of 12.825 and 9.0725.

8. $\frac{2}{3}$ is what part of $\frac{3}{4}$?

9. By what number must the sum of $.01\frac{2}{3}$, $12\frac{1}{2}$, .145, and 3.375 be multiplied to produce $200.45833\frac{1}{3}$?

10. Divide the sum of 1.275 and .08175 by the product of 3.15 and .48625.

(118)

WRITTEN

1. Divide 7 and 45 hundredths by 25 thousandths.

2. Add to the product of 78.125 and 11.25 their difference.

3. A farmer received a return of \$940.00 from a piece of land this year which was .05 greater than last year and $.12\frac{1}{2}$ greater than the year before. What was the return each year?

4. $16\frac{1}{2}$ is what part of 24?

5. What is the value of $16\frac{1}{2}$ acres of land if an adjoining .75 of an acre sold for \$150.00?

6. Divide four hundred fifteen and twenty-five thousandths by three thousand and multiply the quotient by four and ninety-five hundredths.

7. Subtract from the product of 34.625 and 15.6 the product of 9.575 and 5.9.

8. Change to decimals (four places) and add: $20\frac{1}{3}$, $16\frac{1}{2}$, $419\frac{1}{4}$, $37\frac{3}{8}$, $415\frac{4}{5}$, $905\frac{1}{8}$, and $76\frac{7}{8}$.

9. Find the sum of the quotients of:

$$\begin{array}{rcl} 65.37\frac{1}{2} \div 12.5 & = & 38.12\frac{1}{2} \div 12\frac{1}{2} = \\ 150.12\frac{1}{2} \div 5.25 & = & 145\frac{3}{4} \div .075 = \end{array}$$

10. A field of $60\frac{5}{8}$ acres produced an average of 62.4 bushels per acre. A dealer bought the crop for 44.5 cents per bu. and gave a certified check for $\frac{1}{2}$, a promissory note for $\frac{1}{4}$ and currency and coin for the remainder. What was the face of the check, note, amount of money received?

(119)

WRITTEN

1. An accountant spends $\frac{1}{3}$ of his money for board and clothes, $\frac{1}{15}$ for insurance, $\frac{1}{12}$ for entertainment and recreation, $\frac{1}{20}$ for traveling, $\frac{1}{16}$ for incidentals and the remainder is placed in his savings account. What fractional part is saved?

2. Four brothers earn, respectively, $\$7\frac{1}{2}$, $\$7\frac{1}{4}$, $\$6\frac{4}{5}$ and $\$6\frac{1}{4}$ per day. What is the average daily earning? If they work on an average $5\frac{1}{2}$ days per week, how much will they earn in a year?

3. What will be the wholesale cost of 1506 eggs at 27.5c per dozen?

4. If you earn \$150 per month and spend \$75 for clothing and board and \$45 for miscellaneous expenses, what fractional part do you save?

5. A barrel of flour weighs 200 lbs. gross. If the flour weighs 196 lbs., what is the fractional weight of both the barrel and the flour? (Express decimally.)

6. I buy flour at \$8.60 per barrel. At what price must I sell to gain .2 the cost?

7. You get \$4.50 per day for 8 hours, and time and a half for overtime. How much do you earn the first week working 6 days if you put in 60.5 hrs?

8. Divide each of the following by 3.5 and find the sum of their quotients: 75.04, .0252, 6.076, 10.0955 and .056.

9. A merchant bought 3650 yds. of silk at \$1.125 per yard and sold .8 of it at $.37\frac{1}{2}$ above cost. He had a special sale and reduced the price on the remainder $.33\frac{1}{3}$. It was all sold. What was the profit?

10. Reduce to simplest form $\frac{12\frac{1}{2}}{36\frac{1}{2}}$, $\frac{\frac{8}{16}}{\frac{2}{3}}$, $\frac{27\frac{1}{2}}{50}$, $\frac{\frac{4}{5}}{5\frac{1}{2}}$, $\frac{.3\frac{1}{2}}{.6\frac{1}{4}}$

NOTE: Compare Nos. 5 and 9 above to Nos. 4 and 8 on page 118.

(120)

WRITTEN

1. A foot is what fractional part of a rod? A rod is what part of a mile? (Express Decimally.)

2. During high prices of 1919 bacon sold in bulk retail for 36c per lb., ham 45c per lb., flour \$15.00 per bbl., sugar 32c per pound potatoes \$5.00 per bushel. Inquire the prices of these commodities at your local store and show the fractional cost with those of 1919.

3. In a certain high school .25 of the pupils are enrolled in the commercial dept., .3 in technical art, and the remainder 639 are taking college preparatory. How many in each dept., and how many all together?

4. A contractor bought a lot and built a house thereon. The two cost him \$7500. If the lot cost $\frac{1}{4}$ as much as the house, what did each cost?

5. In a manufacturing plant .4 of the space is given to machinery .3 to stores, .2 to assembling and shipping and the remainder 1200 square feet, for offices. How many square feet occupied by each?

6. A dealer bought a herd of cattle for \$3600. He sold .6 of the herd at an advance of $.16\frac{2}{3}$ above cost. He held the others to feed for a better market. He was, however, forced to sell the remainder at $.12\frac{1}{2}$ less than cost. Did he gain or lose and how much?

7. If you walk to school 1.5 miles in .4 of an hour, how long will it take you to walk to the city 4.5 miles at the same gait?

8. A dealer bought 85,250 feet of lumber for \$2429.625. At how much per M feet must he sell it to gain .2 of cost?

9. A farmer sold 125.5 bushels of wheat at \$1.12 $\frac{1}{2}$ per bu., 75.75 bus. of oats at \$.50 per bu., and 12.5 bus. of turnips at \$.75 per bushel. He bought 2 plows at \$16.25 each, three bushels of clover seed at \$18.25 per bu. and 6 lbs. of coffee at 32.5c per pound. How much money had he left?

NOTE: Compare 5, 6, 7 above to 12, 13, 14 on pages 118 and 119.

CHAPTER IX

PERCENTAGE

Per cent. is a contraction of the Latin phrase per centum and means by or on the hundred.

Percentage is not difficult if considered as a form of fraction, as a continued study of decimal fractions wherein all decimal relationships are expressed in hundredths. As decimal fractions have advantages over common fractions for certain purposes (see discussion, Chapter V), so percentage has its advantages for certain calculations in business. (If possible have the class read Chapter 33 of Bookkeeping and Accounting, by McKinsey.)

Let us compare $\frac{5}{100}$, .05 and 5%.* 5% means $\frac{5}{100}$. In other words when we speak of percentage, 100 is always the denomination.

To further illustrate:

$$20\% = \frac{20}{100} \text{ or } .20 \text{ or } \frac{1}{5}$$
$$\frac{2}{3} = \frac{66\frac{2}{3}}{100} \text{ or } .66\frac{2}{3} \text{ or } 66\frac{2}{3}\%$$

Again:

$$\frac{1}{4} = \frac{25}{100} \text{ or } .25 \text{ or } 25\%$$
$$\frac{1}{2} = \frac{50}{100} \text{ or } .50 \text{ or } 50\%$$
$$\frac{3}{4} = \frac{75}{100} \text{ or } .75 \text{ or } 75\%$$
$$\frac{4}{4} = \frac{100}{100} \text{ or } 1 \text{ or } 100\%$$

It is therefore evident that 100% is the whole of anything as indicated by the $\frac{100}{100}$ and $\frac{4}{4}$. Also it is evident that $\frac{1}{4}$ of anything is 25% of it, as $\frac{1}{2}$ is 50% and so on.

*The commercial symbol (%) to express per cent or hundredths.

ITS APPLICATION EXPLAINED

There are three primary elements in the application of percentage, also two secondary elements. These are Base, Rate and Percentage; Amount and Difference. The Base and Percentage are the two quantities compared. The Base being the quantity by which the percentage is measured. The Rate is the relation existing between these two quantities when expressed in hundredths.

The Amount is the base plus the percentage.

The Difference is the base minus the percentage.

The following formulas may be used to good advantage.

$$1. \text{ Base} \times \text{Rate} = \text{Percentage}$$

$$2. \text{ Percentage} \div \text{Base} = \text{Rate}$$

$$3. \text{ Percentage} \div \text{Rate} = \text{Base}$$

$$4. \text{ Amount} \div (1 + \text{Rate}) = \text{Base}$$

$$5. \text{ Difference} \div (1 - \text{Rate}) = \text{Base}$$

TO THE PUPIL: Notice that four of the five operations are division, and that we multiply only where the base is given.

To illustrate formula (1): $B. \times R. = P.$

What is 5% of 60? 60 is the base and 5% is the rate.

$$B. \times R. = P. \text{ or } 60 \times .05 = 3$$

This formula should be discussed until the pupil sees clearly the relation of fractions to percentage. Let us take the above illustration. Is it not easier to say that $5\% = \frac{1}{20}$ of 60 which equals 3, the answer? Notice, also, the close relation between this equation and those of the multiplication table. $7 \times 8 = 56$ and $.07 \times \$800 = \$56$. Also that having any two of the three given we easily find the third. $56 \div 7 = 8$ and $56 \div 8 = 7$.

To illustrate formula (2): $P. \div B. = R.$

What per cent. of 100 is 25?

It is clear that 25 is per cent. and 100 the base; that 25 is to be measured in its relation to 100.

$$25 \div 100 = .25 \text{ or } 25\%$$

Is it easier to say, $\frac{25}{100} = \frac{1}{4}$ or 25%?

To illustrate formula (3): $P. \div R. = B.$

75 is 20% of what number?

Again to simplify:

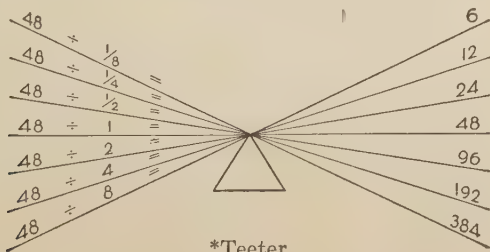
$$20\% \text{ of the number} = \frac{1}{5}$$

$$\frac{1}{5} \text{ of the number} = 75$$

$$\frac{5}{5} \text{ of the number} = 5 \times 75 \text{ or } 375 \text{ or } 75 \div .20 = 375$$

To illustrate formula (4): $\text{Amount} \div (1 + \text{Rate}) = \text{Base}.$ It has been made clear that dividing any number by unity (1) gives the number divided ($48 \div 1 = 48$); that as we increase the divisor we diminish the quotient, and as we diminish the divisor we increase the quotient, always in the same measure or proportion, but in opposite direction.

Any change in the divisor makes an exactly opposite change in the quotient. We may explain this on the principle of the teeter*; the lighter or smaller the divisor the higher it goes and the higher, the smaller the resulting quotient. At unity it stands level. If, therefore, we are to have an answer larger than the working-fact given, it is evident that we must divide by less than one; i. e., by 100%—the rate given. If the answer is to be smaller than the working-fact given, it is evident that we must divide by more than one; i. e., by 100%+the rate given.



PROBLEM TO ILLUSTRATE: After an increase in price of 5%, corn sold for \$2.10. What was the original price? First we must decide whether to multiply \$2.10 by 5% and add or subtract the result, or divide by .95 or 1.05. If \$2.10 is the base we can multiply; if it is not the base, we must divide. As 5% of the original price was added and no original price is given, the \$2.10 cannot be the base, and we cannot multiply the \$2.10 by 5% and subtract or add to get the result. We must divide. As 5% of the original price was added and no original price is given, the \$2.10 cannot be the base, and we cannot multiply the \$2.10 by 5% and subtract or add to get the result. We must divide. As our result must be smaller than \$2.10 it becomes clear that we must divide by $(100\%+5\%)$ or 1.05; $\$2.10 \div \$1.05 = \$2.00$, the original cost.

To illustrate formula (5): $\text{Difference} \div (1 - \text{Rate}) = \text{Base}$. If the problem had read, "After a decrease in price of 5%, corn sold for \$1.90, what was the original cost?" We would, by the same reasoning, have found that we couldn't multiply, and inasmuch as our answer would then have had to be larger than \$1.90 we would have divided by $(100\%-5\%)$ or .95; $\$1.90 \div .95 = \$2.00$, the original cost.

In connection with dividing by $(1 + \text{the Rate})$ or $(1 - \text{the Rate})$, special drills on the aliquot parts are necessary. It must be made clear that dividing \$1.90 by .95 is but a short method of changing the equation, $95\% = \$1.90$, to a 100% basis. To illustrate:

$$\begin{aligned} 95\% \text{ of price} &= \$1.90 \\ 1\% \text{ of price} &= .02 \\ 100\% &= \$2.00, \text{ or the original cost.} \end{aligned}$$

It should be made clear, also, that adding $\frac{1}{19}$ of each term to that respective term would produce the same result. $.95 + (\frac{1}{19} \text{ of } .95) = 1.00$ and $\$1.90 + (\frac{1}{19} \text{ of } 1.90) = \$2.00$. Begin with simple problems, however; showing how dividing by .75 is equivalent to adding a third, and dividing by 1.25 equivalent to deducting a fifth, etc.

DO NOT SLIGHT THESE EQUATION DRILLS.

THE ALIUOT PARTS OF 100 %

It is very necessary that the pupil know the equivalent per cent. rate of those common and decimal fractions (expressed in their lowest terms) that are even or aliquot parts of 100%.

The following should be studied and practiced until the equivalents can be written without hesitation:

$\frac{1}{2}\% = \frac{1}{200} = .005$	$20\% = \frac{1}{5} = .2$	$50\% = \frac{1}{2} = .5$
$1\% = \frac{1}{100} = .01$	$25\% = \frac{1}{4} = .25$	$60\% = \frac{3}{5} = .6$
$2\% = \frac{1}{50} = .02$	$33\frac{1}{3}\% = \frac{1}{3} = .33\frac{1}{3}$	$62\frac{1}{2}\% = \frac{5}{8} = .625$
$2\frac{1}{2}\% = \frac{1}{40} = .025$	$37\frac{1}{2}\% = \frac{3}{8} = .375$	$66\frac{2}{3}\% = \frac{2}{3} = .66\frac{2}{3}$
$10\% = \frac{1}{10} = .1$	$40\% = \frac{2}{5} = .4$	$75\% = \frac{3}{4} = .75$
$11\frac{1}{9}\% = \frac{1}{9} = .11\frac{1}{9}$	$4\% = \frac{1}{25} = .04$	$80\% = \frac{4}{5} = .8$
$12\frac{1}{2}\% = \frac{1}{8} = .125$	$5\% = \frac{1}{20} = .05$	$87\frac{1}{2}\% = \frac{7}{8} = .875$
$14\frac{2}{7}\% = \frac{1}{7} = .14\frac{2}{7}$	$6\frac{1}{4}\% = \frac{1}{16} = .0625$	$112\frac{1}{2}\% = \frac{9}{8} = 1.125$
$16\frac{2}{3}\% = \frac{1}{6} = .16\frac{2}{3}$	$6\frac{2}{3}\% = \frac{1}{15} = .06\frac{2}{3}$	$125\% = \frac{5}{4} = 1.25$

More fractions could be given, such as $46\frac{2}{3}\%$, etc., but the above are the most useful equivalents and should be remembered. The student should also know how to determine the equivalents of those not memorized.

For example:

$$37\frac{1}{2}\% = .37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{75\frac{1}{2}}{200} = \frac{75}{200} = \frac{3}{8}$$

Suggestion:

Take the table given and work through all as suggested in the above illustrations.

(121)

ORAL DRILL

What per cent. of anything is:

1. $\frac{1}{2}$ of it? $\frac{1}{8}$? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{3}{4}$? $\frac{2}{3}$? $\frac{3}{8}$? $\frac{4}{5}$? $\frac{2}{5}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{5}{8}$?
2. What per cent. of 25 bushels is $2\frac{1}{2}$ bushels?
3. What per cent. of \$10 is \$ $2\frac{1}{2}$?
4. What per cent. of 25% is 5%?

(122)

WRITTEN DRILL

Express decimally:

1. $37\frac{1}{2}\%$, 5% , 50% , $33\frac{1}{3}\%$, 10% , 75% , $87\frac{1}{2}\%$, 15% , 20% , $66\frac{2}{3}\%$, $16\frac{2}{3}\%$, $2\frac{1}{2}\%$, 25% , $12\frac{1}{2}\%$, $11\frac{1}{9}\%$.

2. Write the above in common fraction form—lowest terms.

CHAPTER X

PROBLEMS IN PERCENTAGE

(123)

ORAL

1. C. W. Curry has \$500.00 on January 1, 1921. He spends 50% of this amount during the first three months of the year. How much did he spend and how much is left in the bank?

NOTE: Since 50% is $\frac{1}{2}$ he expends $\frac{1}{2}$ of \$500 or \$250. He therefore has $\frac{1}{2}$ in the bank or \$250.00. Show that the same result can be had by using decimal form .50.

2. A stenographer spends during the month, \$60.00 for clothes which is 75% of her monthly salary. What is her salary.

Analysis:

100% of Salary = Salary	$\frac{4}{4}$ = Salary	3) \$60 = $\frac{3}{4}$
75% = \$60.00	$\frac{3}{4}$ = \$60.00	<u>20</u> = $\frac{1}{4}$
1% = .80	or $\frac{1}{4}$ = 20.00	or \$80 = $\frac{4}{4}$
100% = 80.00	or $\frac{4}{4}$ = 80.00	

3. I have \$60.00 and spend \$20.00 for an overcoat. What per cent. of my cash did I expend?

4. You buy cloth for \$15.00 and pay \$25.00 to the tailor for making the suit. What per cent. of the cost is each?

5. Early in 1919 sugar was selling at 30c per pound. Several months later it was selling at 10c. What was the per cent. decrease in price?

6. Within two years hogs on foot dropped from \$24.00 per cwt. to \$8.00. What was the per cent. decrease?

7. You buy Corporation shares at \$35.00 and two years later sell at \$50.00. What was your per cent. of profit?

8. You bought bonds at \$80.00. Later you sold at $12\frac{1}{2}\%$ increase in price. What did you get for the bonds?

9. What per cent. of \$60.00 is \$20.00? \$15.00? \$30.00? \$40.00?

10. What per cent. of $\frac{3}{4}$ is $\frac{1}{2}$?

(124)

WRITTEN

1. Find the sum of $6\frac{2}{3}\%$, $8\frac{1}{3}\%$, $12\frac{1}{2}\%$, $14\frac{2}{7}\%$, $16\frac{2}{3}\%$, $33\frac{1}{3}\%$, 50% and $66\frac{2}{3}\%$ of \$900.00.

2. You own a $\frac{2}{3}$ interest in a certain business and sell $\frac{1}{2}$ your share for \$6000.00. The profits for the year are equivalent to $12\frac{1}{2}\%$ of entire capital. What are the profits?

3. A farmer bought a truck for \$1500 and sold it a year later for \$1200. What per cent. was this below cost?

4. You buy a horse in the fall for \$125.00. In the spring you sell him for \$225.00. If his feed during the winter was \$50.00 what was your per cent. profit?

5. A farmer bought 180 pigs to feed and lost 25% of them during an epidemic of cholera. How many did he lose and how many has he remaining?

6. You owned $\frac{3}{5}$ interest in a corporation and sold $33\frac{1}{3}\%$ of your interest. What part of the whole did you sell? What part retain?

7. Your father has a farm of 600 acres. He sells 25% at one time and $66\frac{2}{3}\%$ of the remainder at a later date. How many acres has he left?

8. In a certain high school in Pennsylvania there were enrolled 2500 pupils. Of this number 49% were boys and the remainder girls. How many of each are enrolled?

9. What is 25% of \$795.20?

10. What is $.5\%$ of \$2000.00?

(125)

ORAL

1. You and George Smith invest \$2000.00 and \$3000.00, respectively, in a grocery business. What per cent. of the total capital is invested by each?

2. The above partners conduct a very profitable business and at the end of the year drew out the net profits which amounted to \$200.00 for you and \$300.00 for Smith. What per cent. did you each receive on your investment?

3. A farmer harvested 200 bushels of apples and sold 75 bushels at one time and 20% of the remainder at another time. What per cent. remained unsold? How many bushels?

4. Your father buys a lot for the purpose of building a home, paying \$1200.00. He is offered a profit of $66\frac{2}{3}\%$ on the cost. He decides to sell and buy again. What did he get for the lot? His profit is what per cent. of the selling price?

5. 20c is what per cent. of \$1.60?

$$20 \div 1.60 =$$

6. 18 inches is what per cent. of 2 yards?

7. You sold your bicycle for \$24.00, thereby losing 25% of cost. What did it cost you?

8. I have \$150.00 in my pocket which is 5% of what I just deposited in the savings bank. How much did I deposit?

9. Your father has just had his property appraised and the appraisals show as follows: Land \$3000.00, Buildings \$2000.00, Live Stock \$4000, and Machinery \$5000.00. He has Liberty Bonds costing \$6000.00. What per cent. is invested in each?

10. A merchant bought cotton cloth at a high price of 30c per yard and was forced to sell during declining prices at 20c a yard. What per cent. did he lose on investment?

(126)

WRITTEN

1. The Federal Bureau of Education statistics show that an average of one high school per day has been added to our educational system during the last 28 years. It shows that there are now over 17,000 such schools; an increase of 452% over 1890. Based on 17,250, how many such schools were there in 1890? What was the increase in number?

2. The number of high school graduates increased from 21,880 in 1890, to 224,367 in 1918. What was the per cent. increase? 202 4
125

SUGGESTION: You have been cautioned to approximate the result. Look at the above problem and approximate the result. You can see that the number has increased almost 10 times, hence almost 1000% increase. 224 30

3. In 1918 there were 81,034 high school teachers as compared to 9120 in 1890. What was the per cent. increase? (First, what is the approximate answer?) 218 8
2481

4. If 34% of the teachers in 1918 were men and the remainder women, how many of each were engaged? (It is evident that 34% was not the exact rate.)

5. 14,000 high schools listed, showed an enrollment of 1,736,000 students. 10% of the schools were city schools with 52% of the total enrollment. What was the number of city and rural schools and the enrollment of each?

6. Our national debt was \$1,028,564,000 in 1913, \$17,005,431,000 in 1918, and \$24,974,936,000 in 1920. What was the per cent. of increase for 1918 over 1913, 1920 over 1918 and 1920 over 1913? (NOTE: Approximate these answers mentally.)

7. The per capita tax of France increased from \$160.00 in 1913 to \$1150.00 in 1920. What was the per cent. increase?

8. Australia's per capita tax increased from \$63.00 to \$525.00 during the above period. Is this a greater or less increase than that of France, and how much?

9. In 1861 the world debt was \$1,500,000,000.00. In 1913 it had reached only \$8,400,000,000.00, but in 1920 it had jumped to \$255,000,000,000.00. Give the approximate per cent. of increase between all dates and the increase from 1861 to 1920.

10. The Cincinnati health department shows by the following report the death rate per 1000 population for the last 10 years:

	Population	Total Deaths	Death Rate per 1000 Pop.
1910.....	363,891	6,330	17.39
1911.....	376,031	6,225	
1912.....	383,159	6,453	
1913.....	391,133	6,734	
1914.....	392,571	6,429	
1915.....	394,608	6,357	
1916.....	396,122	6,734	
1917.....	397,561	6,833	
1918.....	399,000	8,650	
1919.....	400,439	6,365	
1920.....	401,247	6,072	

Calculate and fill in the rate for each year.

(127)

ORAL

1. What is $8\frac{1}{3}\%$ of 12? 48? 132? 252? 72?

SUGGESTION: $8\frac{1}{3}\% = \frac{1}{12}$; $\frac{1}{12}$ of 12 = 1; of 48 = 4; of 132 = 11; etc.

2. What is $33\frac{1}{3}\%$ of 330? 96? 27? 147? 72?

3. What is $16\frac{2}{3}\%$ of 18? 90? 66? 198? 54?

4. What is 120% of 25? 75? 35? 125? 250?

5. What is $\frac{3}{4}\%$ of \$1760.00?

6. What is $12\frac{1}{2}\%$ of 136? 96? 88? 40?

7. What is $66\frac{2}{3}\%$ of \$240? \$90? \$48? \$81?

8. What per cent. of \$75.00 is \$50, \$25, \$15, \$75, \$100, \$150?

9. What per cent. of 60 bushels is 10 bushels? 20 bu? 25 bu?
30 bu? 40 bu? 50 bu?

10. 100 is 5% of what number? 10%, 25%, $16\frac{2}{3}\%$, 50%?

11. \$1.25 is what percent of \$4.00? \$7.50? \$10.00? \$25.00?

12. \$2.40 is 5% of what amount? $12\frac{1}{2}\%$? $33\frac{1}{3}\%$?

13. What is 200% of \$75.00? \$750.00? \$12.50?

14. What part of a number is $16\frac{2}{3}\%$ of it? 50%?

15. $\frac{1}{2}$ is 50% of what number? 75%?

(128)

WRITTEN

1. The report problem 10, previous page, also shows the deaths from tuberculosis for 1920 to be 625 as compared to 950 for the past decade. What was the per cent. decrease?

2. The 1920 census shows 51.4% of the United States population in the cities and towns and 48.6 on farms and in rural villages. What is the number of each, the total population being 105,683,108?

NOTE: Forty years ago 70.5 of the population was classed rural. What are some of the reasons for the 48% producing the largest crop in the history of the country in 1920.

3. In 1913 a count of the cash and securities held in the National Treasury revealed a total of \$1,426,422,051.48. Of this amount \$199,231,911.90 was cash. In January, 1921, the total was \$13,883,819,826.36, of which \$97,410,283.02 was cash. What was the per cent. of increase in securities and decrease in cash?

NOTE: This increase in securities represents evidence of the indebtedness of the allied nations for money borrowed during the world war.

4. The total number of immigrants arriving between 1820 and 1919 were 33,200,103. The average yearly total for the 5 years, 1915 to 1919, inclusive, was 270,000; for the 34 years from 1881 to 1914, inclusive, it was 650,000. What per cent. of the total entered during the 60 years from 1820 to 1880 inclusive?

5. The New York Commissioner's reports show the following figures on those sent to workhouse for intoxication during the past 5 years.

1915.....	4926
1916.....	2361
1917.....	1567
1918.....	664
1919.....	176

What was the per cent. decrease between each period and the per cent. decrease from 1915 to 1920.

6. A certain city shows an increase in birth rate of 2475 in 1920 over 1919. This is approximately $1\frac{7}{8}\%$ increase. Find the birth rate for 1920.

7. There was a general decline in prices of foods in February, 1921, over the previous month. Eggs showed the greatest decline, approximately 40%. If they were selling at 36c per dozen wholesale in February, what were they per dozen in January?

8. A star ball player batted for the week as follows: Monday at bat 5 times and made 3 hits; Tuesday, 4 times and 1 hit; Wednesday, 4 times and 4 hits; Thursday, 3 times and two hits; Saturday, up 7 times in a "double header" and made 5 hits. What was his batting average for the week?

9. From 1913 wheat rose 150% and flour 145%. If the quotations in 1918 were \$2.25 per bushel, and \$15.68, respectively, what were the 1913 quotations?

10. The following table shows the par value of the currency of the leading nations, and exchange rate in November 1920.

1920	Unit Value	Rate in Cents Nov. 27	Change from Par	Depre- ciation
Canada—\$.	1.00	.8930	.1070	11%
Germany—Mark	.2382	.0145	.2237	94%
Italy—Lire	.1930	.0370		
Belgium—Franc	.1930	.0644		
France—Franc	.1930	.0608		
England—Pound	4.8665	3.4950		
Switzerland—Franc	.1930	.1570		
Holland—Florin	.4020	.3056		
Denmark—Krone	.2680	.1356		
Norway—Krone	.2680	.1356		
Sweden—Krona	.2680	.1932		
Spain—Peso	.1930	.1318		
Argentina—Peso	.9648	.7575		
Japan—Yen	.4985	.5060		

Copy this table and fill in the remainder of the last two columns left blank. The first and second items are figured.

NOTE: The French franc with a par of .1930; that is, approximately 19c, is only worth .0608. Japan, who did not suffer from the war, is the only nation whose currency is above par.

(129)

ORAL

1. What per cent. of 30 is 3? 15? 6? 25?
2. What per cent. of $\frac{3}{4}$ is $\frac{1}{2}$? 2?
3. \$500.00 is what per cent. of \$6000.00?
4. What is $12\frac{1}{2}\%$ of \$1648.16?
5. 25 is $16\frac{2}{3}\%$ of what number?
6. What number increased by $33\frac{1}{3}\%$ of itself will equal 120?

SUGGESTION:

100% = Number		1 = Number
$33\frac{1}{3}\%$ = Increase		$\frac{1}{3}$ = Increase
$133\frac{1}{3}\%$ = 120	or	$\frac{4}{3}$ = 120
1% = .90		$\frac{1}{3}$ = 30
100% = 90 the number		$\frac{2}{3}$ = 90 the number

7. What number decreased by $33\frac{1}{3}\%$ of itself will equal 120?
8. What number increased by $8\frac{1}{3}\%$ of itself will equal 91, 130, 273, 26, 403?

9. In March 1921, a certain bookkeeper's salary was reduced 20% below his 1920 salary. If this monthly salary is now \$160.00 what did he formerly earn? (Compare this with Problem 7.)

10. A secretary received \$3500.00 per year in 1921. This is an increase of $16\frac{2}{3}\%$ over 1920. What was his 1920 salary?

(130)

WRITTEN

1. $\frac{3}{4}$ is 50% greater than what fraction?
2. In a bookkeeping class of 36 students, 26 passed the final examination. What per cent. passed and what per cent. failed?
3. A corporation increases its Capital Stock 50%. At the end of the first business year a dividend of \$9000.00 was paid stockholders, which was 6% of the present capital. What was the capital stock before the increase?
4. A manufacturer purchased 500 tons of coal at the first of the year. Six months later there remained 125 tons. What per cent. was consumed?

5. What per cent. of a number is $\frac{1}{2}$ of $\frac{2}{3}$ of it?
6. A dealer sold $37\frac{1}{2}\%$ of the 840 hogs he had bought during the week. How many did he have left and what per cent?
7. What per cent. of a gallon is 3 pints?
8. The Geographical Survey placed our oil supply at 11,300,000 barrels. 4,600,000,000 had been extracted at the end of 1920. What percent of the supply remained?
9. If the consumption is 400,000,000 barrels annually, what percent of the supply is consumed each year and how long will the supply last?
10. Herbert Hoover reported during July, 1921, that 10,000,000 people were sustained from 4 to 6 years at a cost of \$1,300,000.00 the cost of administration being .42 of 1 per cent. What was cost of administration?

(131)

ORAL

1. 24 is $\frac{1}{6}$ or $16\frac{2}{3}\%$ of what number?
2. What per cent of 30 bu. is 24 bu.?
3. What is $12\frac{1}{2}\%$ of \$300.00?
4. What is $66\frac{2}{3}\%$ of 360 men?
5. What per cent of 52 weeks is 13 weeks?
6. What part of 25 is 5? How many hundredths? What per cent?
7. What per cent of 24 is 4? What part? How many hundredths?
8. What per cent greater is 25% than 20% ?
9. What is $.5\%$ of \$1000.00?
10. What is $\frac{3}{4}\%$ of \$1000.00?
11. What per cent is $\frac{1}{3}$ of $1\frac{2}{3}$?
12. What number is 20% greater than 60? 20% less than 60?
13. $6\frac{2}{3}\%$ of \$360.00 is what per cent of \$600?

14. $\frac{1}{2}$ is 75% of what number?
15. 35 is $2\frac{1}{2}\%$ of what number?
16. \$6.30 is $16\frac{2}{3}\%$ more than what amount?
17. What is 25% of $\frac{1}{2}$?
18. What per cent of 1 yard is 12 inches?
19. What per cent greater is 1 yard than 1 foot?
20. What per cent less than a yard is 1 foot?

(NOTE: Compare 19 and 20.)

(132)

WRITTEN

1. About 8,000 persons are mentioned in "Who's who in America." Of this number, 5,768 have a college education; 1,245 have only a high school education, and 808, an elementary education. What per cent. of those mentioned in "Who's who" are college graduates; what per cent. high school and what per cent. only an elementary education?

2. One out of every 188 college graduates is on the list of "Who's who" referred to above. One out of every 1,725 high school graduates and one out of every 37,500 with only an elementary education. By how much will you multiply your chance of success by completing your high school and college courses?

NOTE: Less than 1% of American men are college graduates, yet this 1% has furnished 55% of our presidents, 36% of the members of Congress, and 47% of the speakers of the House. As a rule, the college men will be the leaders.

3. These agricultural staples declined as follows during the latter part of 1920:

	Oct. 30	Nov. 29
Wheat per bushel (December).....	\$2.10 $\frac{3}{4}$	\$1.56 $\frac{1}{2}$
Corn per bushel (December).....	.85 $\frac{3}{8}$	.66 $\frac{5}{8}$
Oats per bushel (December).....	.55 $\frac{1}{2}$	.45 $\frac{5}{8}$
Cotton per pound (December).....	21.6	15.2

Calculate the per cent. decrease of each.

NOTE: The farmers lost heavily on declining prices because the crops were produced at an enormous cost and were slow in marketing.

(133)

ORAL

1. \$ 25.00 is what per cent. of \$150.00?
2. 85.00 is what per cent. of 170.00?
3. 35.00 is what per cent. of 105.00?
4. 25.00 is what per cent. of 300.00?
5. 1.00 is what per cent. of 200.00?
6. .50 is what per cent. of 5.00?
7. 48.00 is what per cent. of 128.00?
8. 12.00 is what per cent. of 144.00?
9. 125.00 is what per cent. of 100.00?
10. 75.00 is what per cent. of 125.00?

(134)

WRITTEN

From the data given in the following Profit and Loss Statements, figure these percentages to the nearest $\frac{1}{4}\%$.

1. What per cent. of the goods sold was allowed in Rebates and Returns?
2. What per cent. of the total sales was allowed for Sales Discount?
3. The Selling Expense is what per cent. of the Net Sales? Gross Trading Profit?
4. The General Administrative Expense is what per cent. of the Net Sales?
5. The Net Cost of Sales is what per cent. of the Net Sales?
6. Freight In is what per cent. of the April purchases?
7. Freight In is what per cent. of the total cost of goods bought during April?
8. The new inventory is what per cent. increase over the old inventory?
9. What per cent. Net Profit was made on the capital invested?
10. The Accounts Receivable total is \$2706.40. What per cent. was written off for Bad Debts?

PROFIT AND LOSS STATEMENT, C. W. KEELAND & CO.,

April 30, 1921

RETURNS					
Sales.....		4436	87		
Less Rebates and Returns.....	36.54				
Sales Discount.....	116.27	152	81		
Net Returns from merchandise sold.....				4284	06
COSTS					
Inventory, Mdse. on hand April 1.....	1121.74				
Add Purchases.....	4115.09				
Freight In.....	1241.86	6478	69		
Deduct Rebates and Returns.....	109.12				
Deduct Purchases Discount.....	46.75	155	87		
Net Cost of merchandise purchased.....		6322	82		
Less Inventory, April 30.....		3045	94		
Net Cost of merchandise sold.....				3276	88
Gross Trading Profit.....				1007	18
OTHER PROFITS					
Interest.....		8	17		
Commission.....		20	56	28	73
Total Profits.....				1035	91
OPERATING EXPENSES					
General Administrative Expense.....	453.45				
Selling Expense.....	165.75	619	20		
OTHER LOSSES					
Loss on Bad Accounts.....	67.66				
Selling Expense and Revenue.....	32.34	100	00		
Total Losses.....				719	20
Net Gain.....				316	71
C. W. Keeland, invested.....				1500	00
A. D. Munson, invested.....				1500	00
C. W. Keeland, one-half net gain.....	158.36				
C. W. Keeland, invested.....	1500.00				
C. W. Keeland's Present Capital.....		1658	36		
A. D. Munson, one-half net gain.....	158.35				
A. D. Munson, invested.....	1500.00				
A. D. Munson's Present Capital.....		1658	35		
		3316	71	3316	71

PERCENTAGE AND ITS APPLICATIONS COMPARED

These tables are given side by side, that you may more clearly see how the principles followed in percentage are the same throughout all its applications. Remember that formulas 1, 2 and 3 are based on the principles of your multiplication table.

PERCENTAGE

1. $\text{Base} \times \text{Rate} = \text{Percentage}$.
2. $\text{Percentage} \div \text{Rate} = \text{Base}$.
3. $\text{Percentage} \div \text{Base} = \text{Rate}$.
4. $\text{Amount} \div (1 + \text{Rate}) = \text{Base}$
5. $\text{Difference} \div (1 - \text{Rate}) = \text{Base}$.

ILLUSTRATION

1. $\$600 \times .04 = \$24$ Percentage
2. $24 \div .04 = 600$ Base
3. $24 \div 600 = .04$ Rate
4. $624 \div 1.04 = 600$ Base
5. $576 \div .96 = 600$ Base

TRADE DISCOUNT

1. $\text{List Price} \times \text{Rate of Discount} = \text{Discount}$.
2. $\text{Discount} \div \text{Rate of Discount} = \text{List Price}$.
3. $\text{*Discount} \div \text{List Price} = \text{Rate of Discount}$.
4.
5. $\text{Net Cost} \div (1 - \text{Rate of Discount}) = \text{List Price}$.

*Where a series of discounts are given, reduce to an equivalent single discount.

PROFIT AND LOSS

Old Method (On Cost)*

1. $\text{Cost of Sales} \times \text{Rate of Profit or Loss} = \text{Profit or Loss}$.
2. $\text{Profit or Loss} \div \text{Rate of Profit or Loss} = \text{Cost of Sales}$.
3. $\text{Profit or Loss} \div \text{Cost of Sales} = \text{Rate of Profit or Loss}$.
4. $\text{Selling Price} \div (1 + \text{Rate of Profit}) = \text{Cost of Sales}$.
5. $\text{Selling Price} \div (1 - \text{Rate of Loss}) = \text{Cost of Sales}$.

*Note top next page.

PROFIT AND LOSS

New Method (On Sales)

1. $\text{Selling Price} \times (\text{Rate of Profit or Loss}) = \text{Profit or Loss}$.
2. $\text{Profit or Loss} \div (\text{Rate of Profit or Loss}) = \text{Selling Price}$.
3. $\text{Profit or Loss} \div \text{Selling Price} = (\text{Rate of Profit or Loss})$.
4. $\text{*Cost} \div (1 + \text{Rate of Loss}) = \text{Selling Price}$.
5. $\text{Cost} \div (1 - \text{Rate of Profit}) = \text{Selling Price}$.

*Not often used.

*NOTE: To find the Selling price, where the cost of doing business is given as a per cent. of Sales: Under the old plan, divide the Net Selling Price (Cost+Profit) by 100%—Rate of Selling expense. Under the new plan, divide the cost by (100% minus the Rate of Profit and the Rate of Selling Expense added together). The new method is used by more than half of our business houses and should be taught in preference to the old. See Some of the Problems of a Merchant, page 171.

COMMISSION

$$1. \left. \begin{array}{l} \text{Prime Cost} \\ \text{Total Sales} \end{array} \right\} \times \text{Rate of Commission} = \text{Commission}$$

$$3. \text{Commission} \div \text{Rate of Commission} = \left\{ \begin{array}{l} \text{Prime Cost} \\ \text{Total Sales} \end{array} \right.$$

$$3. \text{Commission} \div \left\{ \begin{array}{l} \text{Prime Cost} \\ \text{Total Sales} \end{array} \right\} = \text{Rate of Commission}$$

$$4. \text{Gross Cost} \div (1 + \text{Rate of Commission}) = \text{Prime Cost}$$

$$5. \text{Net Proceeds} \div (1 - \text{Rate of Commission}) = \text{Total Sales}$$

NOTE: Under No. 4, before dividing, subtract all charges other than commission.

Under No. 5, before dividing, add all charges other than commission.

STOCKS

$$1. \left\{ \begin{array}{l} \text{Par Value} \times \text{Rate of Dividend or Brokerage} = \text{Dividend or} \\ \text{Brokerage} \\ \text{Cost} \times \text{Rate of Income on Investment} = \text{Income} \end{array} \right.$$

$$2. \left\{ \begin{array}{l} \text{Dividend or Brokerage} \div \text{Rate of Dividend or Brokerage} = \\ \text{Par Value} \\ \text{Income} \div \text{Rate of Income} = \text{Cost} \end{array} \right.$$

$$3. \left\{ \begin{array}{l} \text{Dividend or Brokerage} \div \text{Par Value} = \text{Rate of Dividend or} \\ \text{Brokerage} \\ \text{Income} \div \text{Cost} = \text{Rate of Income on Investment} \end{array} \right.$$

4 and 5 (Same as Profit and Loss)

SUGGESTION: Make up a similar formula to apply to Bonds and Interest and Discount.

NOTE: For problems illustrating the application of these formulas, see subsequent chapters under the same heading.

TRADE DISCOUNT

Trade discount is a deduction from catalog or list price allowed to the purchaser. The rate of discount is generally stated in the terms of the sale; as on a/c less 10% and 5%; or 2/10, n/30. The first is known as a straight or trade discount, the second, as a conditional or cash discount. The first never appears on the books, the sale being entered for the net amount. The cash discount does appear on the books, and is known to the purchaser as a Purchase Discount, to the seller as a Sales Discount.

Why are these Discounts allowed?

The straight or trade discount is given as an adjustment of catalog price, necessary because of market conditions, or it may be merely a matter of sales policy. The customer who buys less often, in small amounts and makes frequent complaint, may receive no discount. The customer who buys often in fair amounts and makes no complaint gets 5% off, perhaps, and the customer who buys frequently in larger amounts and is a good booster for our goods may get 10% and 5%.

The conditional or cash discount is offered as an incentive for prompt payment. If the buyer pays within ten days, he may be allowed 2% discount.

In many sales we find both straight and cash discounts.

PROBLEMS ILLUSTRATING TRADE DISCOUNT

1. What will 420 dozen brooms cost at \$7.15 a dozen. Terms, on a/c less 5% and 2%.

2. If the freight on these brooms is \$8.40, what is the cost per dozen?

3. If the terms had been 2/10, n/30, what would each broom have cost, provided we sent a check in full within ten days?

The discount our competitor offers may determine the rate we must allow, or our price may be placed high enough to allow a very tempting discount rate to catch the buyer.

METHOD OF FIGURING COMPARED

Find cost of 4 dozen stoves at \$17.50 less 20% and 5% and a further discount of 2% for cash payment.

1. \$17.50	2. \$17.50	3. \$1.00
48	48	20 Deduct
<u>14000</u>	<u>840.00</u>	<u>.80</u>
7000	80	<u>.95</u>
<u>\$840.00</u>	<u>672.00</u>	<u>.7600</u>
.20	95	<u>98</u>
<u>\$168.00 = 1st Dis.</u>	<u>3360</u>	<u>608</u>
\$840	6048	<u>684</u>
<u>168</u>	<u>638.40</u>	*.7448 = rate
672	.98	840 = Gross
<u>.05</u>	<u>510720</u>	Cost
<u>\$33.60 = 2d Dis.</u>	<u>574560</u>	<u>287920</u>
\$672	\$625.6320 = Net	<u>59584</u>
33.60	Cost	\$625.6320 = Net
<u>\$638.40</u>		Cost
.02	4. \$17.50	
<u>\$12.7680 = 3d Dis.</u>	48	
\$638.40	5)840 ($20\% = \frac{1}{5}$)	
12.77	<u>168</u>	
<u>\$625.63 = Net Cost</u>	20)672 ($5\% = \frac{1}{20}$)	
	<u>33.60</u>	
	50)638.40 ($2\% = \frac{1}{50}$)	
	<u>12.77</u>	
	<u>\$625.63 = Net Cost</u>	

1. Multiply by given rate, deduct, and repeat operation.

2. Multiply by $(100\% - \text{given rate})$ and repeat operation.

3. Same as No. 1, using \$1 as a base, and multiplying resultant rate by gross cost.

4. By deducting aliquot parts from gross cost.

BILLING

SPECIAL APPLICATIONS OF FRACTIONS AND PERCENTAGE

Too much importance cannot be attached to the correct billing of out-going goods. John Wanamaker, the prince of merchants, is quoted as saying that the inaccuracies and errors made by his clerks cost him \$30,000 a year. He says he knows this because the mistakes corrected for those who return and make complaint amount to that sum each year, and calculating on a fifty-fifty basis he concludes that as many mistakes were made against the firm, but these he says are seldom reported. A loss is thus entailed of \$30,000.

Bill and Invoice are fairly synonymous terms, both being used interchangeably in connection with the statement of goods sold by one merchant to another. At times, however, the following distinction is made: Invoice is referred to strictly as a list of merchandise or material purchased, while a bill is a statement of services rendered. Bills are rendered by telephone companies, gas companies, physicians, attorneys, etc. This voucher should give an exact description of the services, goods, correct prices, terms of credit, discounts deducted, address of purchaser, and directions for shipping. When invoices are received they should be carefully checked with the purchase order to determine as to whether the order was filled as given; also if the prices and terms are in accordance with previous orders or agreement. A receipted bill is one marked paid at the time of payment, showing the signature of the firm receiving the payment. This receipted bill should be filed as evidence of payment, which also supports the entry on the books showing such payment.

The following are reproduced invoices. Study carefully the contents of each. If blank invoices are convenient the following problems may be copied on such blanks and extended. If blanks are not available extend each invoice on practice paper or rule forms as directed by the teacher.

(135)

PROBLEMS IN TRADE DISCOUNT

1. Find the amount owed on the following bills if paid June 12:



THE PICKERING BUILDING

THE PICKERING HARDWARE CO.

HARDWARE, CUTLERY, MECHANICS' TOOLS.
FACTORY SUPPLIES, KITCHEN UTENSILS

PRIVATE
BRANCH EXCHANGE
MAIN 3565

Cincinnati
My Happy Home June 5, 1921.

James W. Baker,
Cincinnati, Ohio.

Terms: 2/10, n/30. F 34902

	24½ 120 1 5/6	Doz. X L. Garden Hoes 16" Steel Rakes Doz. Lawn Mowers Less 10, 5 and 2%	@ \$5.50 each .06 2/3 " 4.20	XXX XX XXX XX XX XX		XXX XX XX XX XXX XX XXX XX
Freight Prepaid and charged						
A LITTLE BETTER QUALITY—A LITTLE LESS PRICE—"SAVE THE DIFFERENCE"						
Pkgs.	Terms: On a/c, Interest after 30 days.					
1, 2 & 3	17½	Doz. Miners Hoes Less 20% and 5%	\$6.40 per doz. 7.80 " "	XXX XX XX XX XXX XX		XX XX XXX XX
4-7	500	Coal Picks, special Less 5%, 2% and 1%	125.40 per C	XX XX XXXX XX		XXXX XX 140 27 XXXX XX
8&9	1420	Just Right Shovels Less 33 1/3% and 2½%		XXXX XX		
Freight Prepaid						

1. What was the exact cost per hoe in Bill No. 1 if freight on the 24½ dozen was \$4.20?

2. What would the lawn mowers have cost each, had the bill not been paid in ten days? (Freight on 1½ dozen \$7.50.)

3. In Bill No. 2 what is the cost per dozen of miners' hoes, if freight apportioned to the hoes was \$8.40?

4. What was the cost of Just Right shovels each, if freight apportioned was \$79.60 for the 1420?

5. If Bill No. 1 was paid within ten days, what was the discount?

(136)

6. Make extensions on the following candy bill:

The Nuss Confectionery Co. Butter Cream Specialties 311-313-315-317 WEST THIRD STREET. The Finest  TRADE MARK Cincinnati. November 20, 1921.																													
Claims for loss or damage should be made promptly on Transportation Company. Our responsibility ceases when we obtain shipping receipt signed "In Good Order." TERMS; 2% Discount in 10 Days - 30 Days Net. Accounts not paid when due subject to Sight Draft without notice.																													
Sold to THE JUNE DRUG CO., PITTSBURGH, PA. All goods shipped f o b. Cincinnati Subject to delay and prices on date of shipment																													
3 Cases Dolly Dimples Chocolates, per lb. $\\$.83 \frac{1}{3}$ (120-1$\frac{1}{2}$lb. boxes per case). 5 Pails Assorted Stick, per lb. $.24 \frac{1}{2}$ (42$\frac{1}{2}$#, 43$\frac{1}{2}$#, 39 7/8#, 44 1/8#, 41$\frac{1}{2}$#). 3 Bbls. Taffy Kisses, per lb. $.16 \frac{2}{3}$ (124$\frac{1}{2}$#, 132 3/4#, 124 7/8#). Less 5% and 2% 124 1/3Lbs. Assorted "Suckers", per lb. $.18 \frac{1}{2}$	<table border="1" style="width: 100%; text-align: center;"> <tr> <td>XXX</td> <td>XX</td> <td></td> <td></td> </tr> <tr> <td>XX</td> <td>XX</td> <td></td> <td></td> </tr> <tr> <td>XX</td> <td>XX</td> <td></td> <td></td> </tr> <tr> <td>XXX</td> <td>XX</td> <td></td> <td></td> </tr> <tr> <td>X</td> <td>XX</td> <td>XXX</td> <td>XX</td> </tr> <tr> <td></td> <td></td> <td>XX</td> <td>XX</td> </tr> <tr> <td></td> <td></td> <td>XXX</td> <td>XX</td> </tr> </table>	XXX	XX			XX	XX			XX	XX			XXX	XX			X	XX	XXX	XX			XX	XX			XXX	XX
XXX	XX																												
XX	XX																												
XX	XX																												
XXX	XX																												
X	XX	XXX	XX																										
		XX	XX																										
		XXX	XX																										
The following is complimentary: 6 3 lb. boxes Honeymoon Chocolates. (List price, 95¢ per lb.).																													
We hereby guarantee that the merchandise covered by this invoice has been produced or manufactured in accordance with the Federal Child Labor Act of September 1st, 1916.																													

7. In Bill No. 6, the Honeymoon Chocolates were given as a discount on the entire bill. To what rate per cent. was it equivalent? (If result shows a fraction, use nearest whole number; use entire bill as a basis.)

8. Find the exact cost per pound of the Dolly Dimples Chocolates; allow 2 cents per lb. for freight.

9. Suppose the above invoice had been paid on November 28 by check. What would have been the amount of such a check?

10. The payment could have been postponed until December 30. Show that it was good business policy to take the 2% discount.

(137)

Find the amount due on each of the five invoices below, assuming that they were paid on October 31. (Note the 2% discount.)

				The Cincinnati Cordage & Paper Co. Cincinnati — Dayton Cincinnati, Ohio Oct. 10, 1921.	
TERMS 2% - 30 days.		SOLD TO South-Western Publishing Co., 309 West Third St., City.			
<i>Accounts to be settled Monthly</i>		<i>No Claims allowed on this Bill unless made within Five Days after receipt of Goods</i> <i>Pay nothing to Agents unless by our written Authority</i>			
(1)	40	Reams 22"x34"-32lb. Cinto Writing	@ 8 $\frac{3}{4}$		
		Mark Form #13&14			
	15	Reams 17x28-20 $\frac{1}{2}$ lb. Gem R. R.	@ 6 $\frac{1}{2}$		
		Mark Form 11&15			
	1 $\frac{1}{2}$	Reams 17x28-30 $\frac{1}{2}$ lb. State Mills	@ 11 $\frac{1}{2}$		
		Mark Books #6&7			
(2)	110	Reams 22x34-28 lb. Gem R. R.	@ 9 $\frac{3}{4}$		
	126	6/20 Rms. 36x50-80 Crown M. F.	@ 8 $\frac{1}{2}$		
	76	19/20 Rms. 22x34-28 Gem R. R.	@ 8 $\frac{1}{2}$		
	140	Reams 22x34-32 Boone Bond	@ 11 $\frac{1}{2}$		
	233	Reams 22x34-32 Rivera Writing	@ 12 $\frac{1}{2}$		
(3)	27	2/20 Reams 28"x38"-136lb. Brown Cover	@ 16 $\frac{1}{2}$		
	61	13/20 Reams 24x38-116 lb. Brown Cover	@ 16 $\frac{1}{2}$		
	2	Reams 22x34-32 lb. Pink Oyster B. Bond	@ 23 $\frac{1}{2}$		
	38	Reams 22/34-32 lb. Rivera	@ 10 $\frac{1}{2}$		
	121	Reams 28x34-121 lb. Brown Special Cov.	@ 12 $\frac{1}{2}$		
	20	Reams 19x24-29 $\frac{1}{2}$ lb. State Mills	@ 15 $\frac{1}{2}$		
		Rule to Copy #307 Ruling	2 $\frac{1}{2}$		
(4)	28	4/20 Rms. 28x38-136 B.A. Spec. Cover	@ 16 $\frac{1}{2}$		
	2450	6x9 $\frac{1}{2}$ -32 Portfolio Envs. Printed, M.	@ 3.72		
	92	Reams 19x24-29 $\frac{1}{2}$ State Sub. 24	@ 15 $\frac{1}{2}$		
		Rule to Copy #309 Ruling	2 $\frac{1}{2}$		
(5)	105	M. #10-32# Mla. Envs. Printed	@ 2.61		
	104700	6x9 $\frac{1}{2}$ -32# Portfolio Mla. Envs.	@ 4.15		
	42	Reams 19x28-34 State Mills	@ 17 $\frac{1}{2}$		
		Rule to copy #310 Ruling	2 $\frac{1}{2}$		
	77	Reams 17x26-30 $\frac{1}{2}$ State Mills	@ 12 1/8		
		Rule to copy #108 Ruling	2 $\frac{1}{2}$		
	100	Reams 16x21-28 $\frac{1}{2}$ Traae Ledger	@ 27 $\frac{1}{2}$		

NOTE: The price on paper is per pound. The number of reams is multiplied by the number of pounds per ream and this product is multiplied by the price ($40 \times 32 \times 8\frac{3}{4} = \$112.00$).

(138)

ESTABLISHED 1864										
A.M. BYERS COMPANY										
MANUFACTURERS OF										
GENUINE WROUGHT IRON PIPE AND OIL COUNTRY TUBULAR PRODUCTS										
PITTSBURGH, PA.										
CONSIGNEE ADDRESS VIA.					INVOICE NO.					
SOLD TO: Maribona & Co., Sagua La Grande, Cuba.					DATE July 20, 1921.					
					TERMS Draft 30 days.					
CARNO. CNJ 88299					WEIGHT					
					P.O.B. Pgh.					
CUSTOMER'S ORDER NO.	QTY.	PIECES	SIZE	DESCRIPTION	FEET	IN	PRICE	AMOUNT	TOTAL AMOUNT	
		30	3	Blk Ran Pipe T&C 30 $\frac{1}{2}$ "	635	2	.76 $\frac{1}{2}$	486.67		
		30	4	DO 30 $\frac{1}{2}$ "	669	5	1.09	148.43	335.24	
		30	6	DO 30 $\frac{1}{2}$ "	639	0	1.92			
		20	10"	-35# DO 27 $\frac{1}{2}$ "	408	6	3.50			
		20	12"	-45# DO 27 $\frac{1}{2}$ "	397	0	4.50			
307-E		4	2	Ex Hvy Gal Ran Pipe Fe LW 7.5 & 5%	50	6	.50 $\frac{1}{2}$			
		2	1 $\frac{1}{2}$	Do Fe Bw 8.5 & 5%	114	9	.30			
		8	1	Do Fe Bw 6.5 & 5%	610	9	.22			
		3	3/4	Do Fe Bw 8.5 & 5%	416	5	.15			
				Marked "For - W. C. Merritt Co., San Diego, Calif. Collect B/L 22150, Weight 3100#.						
La-3		20	2 $\frac{1}{2}$	Blk Ran Pipe Y&C 23.5 & 5%	403	5	.58 $\frac{1}{2}$			
		10	3	Do 23.5 & 5%	204	6	.76 $\frac{1}{2}$			
		9	4	Do 23.5 & 5%	200	11	1.09			
La-2		153	2	Do LW 21.5 & 5%	3114	2	.37			
La-3		106	1/2	Gal Ran Pipe T&C Bw 2.5 & 5%	25211	11	.08 $\frac{1}{2}$			
		98	2	Do 7.5 & 5%	2011	0	.37			
				Collect B/L 21396. Weight 47900#.						

No Claims for Loss or Damage in Transit will be Considered Unless Accompanied by Paid Express Bill. Showing Notation on its Face of the Character of Loss or Damage Signed by Agent or Terminal Transportation Line. No Claim for Allowance on Returned Goods will be Considered Unless Permission to Return is First Secured.

UNDER NO CIRCUMSTANCES IS SELLER RESPONSIBLE FOR DAMAGES BEYOND PRICE OF GOODS.

The three Invoices above should be copied and completed as shown by the first extension in the first Invoice. The items are as they appeared on a sale by the above company and shows the various discounts that may be encountered in billing.

(139)

WRITTEN

1. *Farmers Elevator Co., bought of N. D. Farmer, at \$1.12½ per bushel, 10 loads of wheat as follows: 10 loads No. 1 Northern Wheat, 2740 lbs., 2963 lbs., 3123 lbs., 3045 lbs., 2976 lbs., 3003 lbs., 2997 lbs., 3112 lbs., 3527 lbs., 2749 lbs. Less storage, 3c per cwt. Terms: Cash, 1½ lbs. off per bu. for dirt. Therefore, 61½ lbs. to bushel. What did Farmer get for his wheat?

2. The Wearwell Mills Co., New York City, March 3, 1921, sold to Fitwell Tailoring Co., Cincinnati, Ohio, terms 3/10, n/30, the following: Case No. 1—12 pcs. Diagonal Suiting, 52¹, 53³, 48², 50¹, 52³, 49², 47³, 48, 47¹, 53³, 50², 46³, at \$1.24¼ per yd. Case No. 2—9 pcs. Scotch Worsted, 56³, 49¹, 50², 61¹, 49³, 70, 68³, 59¹, 48³, at \$1.33½. Case No. 3—3 pcs. Colonial Flannel, 84³, 79¹, 80², at \$.73½. Case No. 4—5 pcs. Metropolitan Suiting, 120³, 118¹, 115³, 119², 100³, at \$.83½. What was paid March 12, 1921?

NOTE: The small figures at the upper right hand represent fourths.

3. Find the cost of:

49 pieces 3" Wrought Iron Pipe 1018' and 8" at 76½c per foot, less 28.8 and 5 and 2½%.

12 pieces ½" Galvanized Pipe 2829' and 9" at 8½c per foot, less 18.8 and 5 and 2½%.

23 pieces 3" Galvanized Threaded Pipe 510' and 9" at 76½c per foot, less 15.8 and 5 and 2½%.

15 pieces ¾" Galvanized Threaded Pipe 2082' and 8" at 11½c per foot, less 6.8 and 5 and 2½%.

4. Find the cost of the following:

10 pieces 12" Extra Heavy Galvanized Pipe 80' at \$6.55 per foot, plus 4.2 less 5 and 2½%.

17 pieces 10" Extra Heavy Galvanized Pipe 136' at \$5.48 per foot, plus 4.2 less 5 and 2½%.

*If blank bills are convenient, make out in proper form and extend the items.

***COMMISSION**

Commission is a charge made for buying or selling goods. In selling it is a percentage of the total sales, in buying it is generally a percentage of the prime cost. A certain charge per bu., cwt., ton, or other unit of measure, is a better plan for figuring commission on buying than using a rate per cent. of cost.

If a commission of 2% on cost is allowed an agent for buying goods or property, there is a danger that he will buy at too high a price. If we were to allow a commission for selling, on the unit basis, the opposite would be true. If we allowed 15c a bbl. for selling apples, our agent would be interested in disposing of the lot quickly at any reasonable price, whereas if his commission were to be computed on sales, he would strive for the highest possible selling price.

Terms common to Commission are, Gross Sales, Net Proceeds, Prime Cost, Gross Cost, Account Sales, Guarantee or Insurance, Shipment and Consignment.

The Gross Sales is the total amount of the sales before any deductions are made; after Commission and other charges are deducted the result is Net Proceeds.

The first cost, without the addition of Commission or other charges is the Prime Cost; when these additions are made it becomes the Gross Cost.

An Account Sales is a record of sales made, the charges deducted and the proceeds to be remitted by the agent.

Guarantee is a charge made to protect the agent in making sales on credit or in guaranteeing quality of goods sold.

Shipment and Consignment are used interchangeably, but shipment generally refers to outgoing goods, and consignment to incoming goods.

Goods shipped for sale on commission—though listed on a Straight Bill of Lading—belong, while en route and after arrival, to the shipper. In a sale made on account, the goods are listed on a straight bill of lading also, but title generally passes to the

*See comparison of formulas, Chapter X.

purchaser as soon as the goods are delivered to the transportation company. If it is desired to reserve title until payment is made upon delivery, an Order Bill of Lading may be used. Here the shipper ships the goods to himself at the place designated. He then transfers title whenever he chooses by endorsing the order bill of lading and giving it to the purchaser, either directly or through the collecting agent. Goods will not be delivered by the transportation company until the consignee presents a bill of lading, pays the freight and acknowledges receipt of the goods. The Order Bill of Lading is generally sent to the consignee's bank, endorsed and ready for delivery upon payment of draft attached.

(140)

PROBLEMS IN COMMISSION

1. What will 5450 bbls. of apples cost at \$5.20 per bbl. if commission is $2\frac{1}{2}\%$? What will be the cost per bbl.?

2. Find the net proceeds of 420 baskets of peaches sold at \$1.25 a basket. Commission for selling 3% , and freight to be deducted (the agent paid the freight), \$6.25. What did the peaches bring the owner per basket?

3. My agent sold for me 18,450 lbs. of hay at \$21.50 per ton—commission for selling, $\frac{3}{4}\%$ —and with the proceeds purchased oak flooring at \$65 a M ft.; commission for buying, $\frac{1}{4}\%$. How many feet of flooring should I receive?

4. Glover and Grace, of Nashville, shipped a carload of 1240 watermelons to Detroit for sale on Commission. 1215 were sold at 95c each. (25 were broken) The commission was $1\frac{1}{2}\%$, the freight \$22.40, storage \$12.75, hauling \$18.20. What were the net proceeds?

5. O. Thornton, salesman, receives \$5 a day; 1% of daily sales between \$100 and \$250 and 2% of all daily sales over \$250. His daily sales record for the week is: \$125.50, \$240.85, \$75.10, \$464.90, \$252.10 and \$720.50. What is Thornton's salary for the week?

6. Make extensions on the following Account of Sales:

ACCOUNT OF SALES

October 14, 192..

Sold for CHARLES E. RUBERT
Forestburg, S. D.

By

The Enterprise Commission House,
Chicago, Ill.

192..						
Oct.	8	40 tubs Fancy Dairy Butter, 2456 lbs. at \$0.52½.....		..		
Oct.	9	3 tubs Clearview Lard, 191½ lbs. at \$0.32½..		..		
Oct.	9	1724½ doz. Eggs, 1922½ lbs., \$0.47½ per doz.		..		..
		Charges Deducted				
Oct.	14	Freight paid, 95 cents per cwt.....		..		
		Drayage.....	3	25		
		Commission, 2½%.....		..		..
		Net proceeds, by check No. 84.....				..

7. What price per pound did the butter net the shipper in problem No. 6?

8. What will 2120 bu. of clover seed cost at \$17.25 per bu. if commission for buying is 1½% and freight is \$22.40? What was the commission per bushel? What is the exact cost per bushel?

9. Sent B. Blake a \$4000 New York draft with instruction to purchase lumber. His commission charge is 2½%. How many feet of lumber will he purchase if the price is \$62.40 per M? (Each dollar's worth purchased will cost \$1.02½.)

10. Make up an Account Sales for the following: Sold June 24, 1921, by Conrad Compton, 246 Bay St., Boston, Mass., for Mrs. Winifred Allen, Bangor, Me.: 6 bbls. dressed spring chickens, 1324 lbs., at 62½ cents; 475¼ lbs. maple sugar at 12½ cents, and 430½ lbs. cheese at 22½ cents. Freight paid by Compton 62½ cents a cwt., storage \$4.80, drayage \$2.50, and commission 1½%.

CHAPTER XI

SOME OF THE PROBLEMS OF A MERCHANT

1. His first problem is that of selecting a good location, a place of business easy of access to many possible customers.

2. His next problem is to buy right; the best selling goods at the lowest possible price—a stock no larger than he can dispose of, at a profit, in small quantities, unless he can get worth while rates on larger orders.

3. A third problem is that of establishing right principles in the conduct of his business—a first-class service based on promptness, courtesy, good will and good values.

4. His next problem is to keep all expenses down to the minimum; not so low that the growth of the business is in any way hindered or the quality of service brought below standard. The cheaper he buys, however, and the lower he keeps the expense of doing business the greater his percentage of profits will be, with a resultant advantage over his competitors.

5. Still another problem, and this involves the first three mentioned above, is that of maintaining as rapid a turnover as possible, a proper relation between the total capital invested, the capital invested in merchandise, the gross sales for the year and the net profit or revenue on the sales made. It is apparent that a relatively large profit tends to decrease sales and a relatively small profit tends to increase sales. Many small profits may return a greater net profit than fewer but larger profits.

THE EFFECT OF A RAPID TURNOVER

It is apparent that if 20% is made on the money invested in goods, and our money returns to us six times during the year for reinvestment, that each \$1 of capital invested in goods turns \$1.20 profits for the year.

If a newsboy invests 50c in papers, and makes 25c each day, he has a 50% profit on his investment, not figuring his expenses, and $50\% \times 360 = 18000\%$ for the year.

If a piano dealer pays \$600 for a piano and holds it 6 months before selling for \$900, he too has made 50%, but in 6 months, not in a day, and twice $50\% = 100\%$, the piano-man's income rate for the year.

It is evident, therefore, that it is not so much what per cent. of profit you make, as how often you make it. The restaurant business, meat-shop, grocery, and ten-cent store are rapid turnover propositions; real estate, furniture, clothing and jewelry are generally slow turnover propositions.

The following average turnover rates are given by the Bureau of Business of A. W. Shaw & Company:

GENERAL LINES		DEPARTMENTAL LINES	
	Turns per Year		Turns per Year
Grocery Store	10	Notions	9
Dept. Store	7	Women's ready to wear	6
Variety Store	6	Wall Paper	4.2
Drug Store	4.5	Men's Furnishings	4.2
Dry Goods Store	4	Underwear	4.1
Hardware Store	3.5	Hosiery	4
Furniture Store	3	Gloves	3.5
Shoe Store	2.1	Dress Goods	3.2
Clothing Store	2	Silks	3.1
Jewelry Store	1.5	Carpets	1.5

6. No problem that the merchant faces is more perplexing nor more closely related to the success or failure of his business than that of marking his goods for sale.

To sell goods for what they cost plus the expense of doing business is not enough. The business to be really worth while must earn a fair rate of interest on the capital invested, pay the proprietors or managers their salaries, pay the expense of the business and turn a fair profit besides. Expressing it in an equa-

tion we have, $\text{Cost} + \text{Expense of Selling} + \text{General Administrative Expense} + \text{Profit} = \text{Selling Price}$.

How to mark the goods so that this is not only possible, but fairly certain, is the business man's big problem. He cannot mark by mere guesswork. His accounting system must be accurate. His bookkeepers must furnish dependable information as to the stock carried, the sales, the expense and the turnover rate for each line or department, and the probable expense of selling. He knows that if he marks too high, he will soon lose his trade; that if he marks too low he will soon lose a part or all of his capital. If he marks one line of goods at a higher rate than other lines he may sell the one line too quickly because sold at a loss and the other lines too slowly because sold at too high a profit. How to determine the exact relation between cost and selling price and then to mark correctly for selling, is his every-day-in-the-year problem.

WHY HE SHOULD FIGURE PROFITS ON SALES

1. The purchase price of the article represents only a part of the investment.* The Selling Price is the whole investment, the cost of the goods + the freight and storage + the cost of administration and selling + the profit. Unless the rate of profit is figured on this whole cost, a part of the capital invested in the business will return no income.

2. It is very difficult on a set of books to ascertain the exact cost of sales for a week or month. It involves the taking of an inventory, or the keeping of a going inventory, and either would be a heavy additional expense to the business. Without this knowledge of cost, under the old plan the profits for the day or month cannot be found.

Suppose both Smith and Jones sell \$4000 worth of goods on a certain day. Smith goes to his cash register at night and says, "\$4000—18% of that or \$727.27 is profit." Jones goes to his cash register and says, "Let's see, 40% of my cost is profit, I sold \$4000 worth. I wish I knew the cost, then I could figure my

*Salaries paid clerks represent an investment in services. The merchant expects to get back this salary paid, as well as the original cost.

profit," and he goes home wondering what his profits were. Both made the same profit; one knows how much and the other does not know. Not knowing what your business is doing is a dangerous matter. Without full and correct information furnished at the proper time, how is a man able to properly guide and control his business?

3. The rate per cent. of profit on cost may be anywhere from 1% to 1000% or more; on sales the profit can never be 100% unless the goods cost nothing, which is unlikely. An article bought for 40c and sold for \$2 turns only 80% profit on sales, but 400% profit on cost. Rates higher than 100% are not only confusing, but are looked upon with suspicion.

Or suppose an article costing \$1.00 sells for \$2.20 and the expense of doing business is 80c. If Merchant Jones figures his profit on cost he has 40%, while Merchant Smith who figures his profit on sales has only $18\frac{2}{11}\%$.

The 40c profit made by each of these two merchants is not fixed by any rate (rather, are the rates determined by profit in its relation to either cost or sales). Profit is an economic advantage resulting from the exchange of goods or service under favorable conditions of supply and demand. The more favorable the conditions the higher the rate of profit possible, and vice versa.

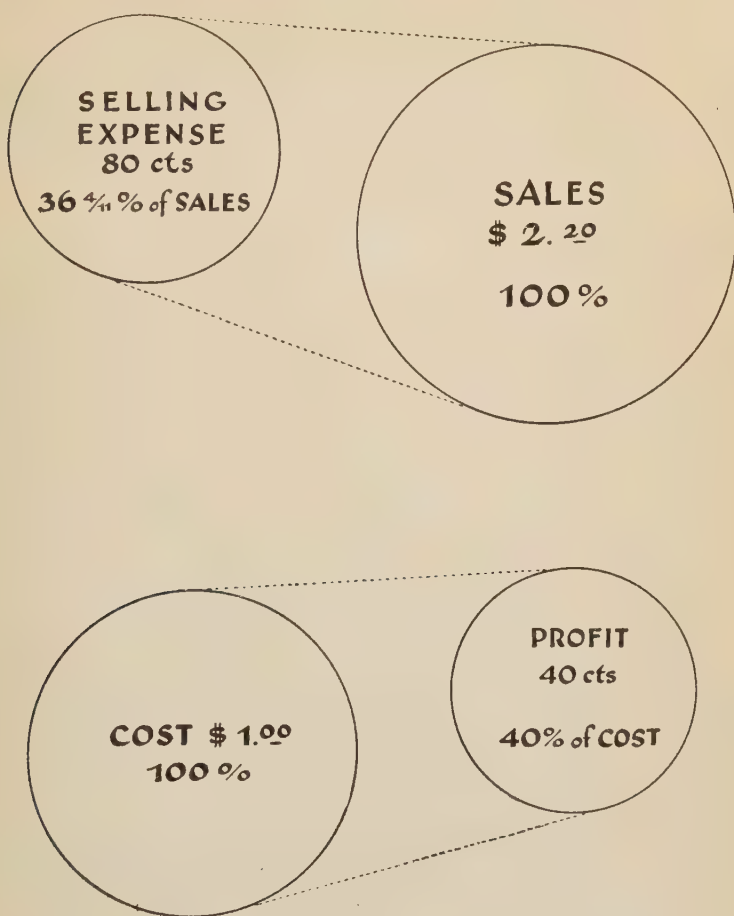
The possible profit being practically the same for merchants competing under like conditions, the difference is largely in having or not having a daily check on the course of business affairs.

4. Mercantile and other taxes levied by the Government are based on sales.

5. Commission, advertising and delivery expense are always thought of in relation to sales.

6. To use one base in measuring the profits and another in measuring taxes, commission or selling expense, leads to difficulty and confusion.

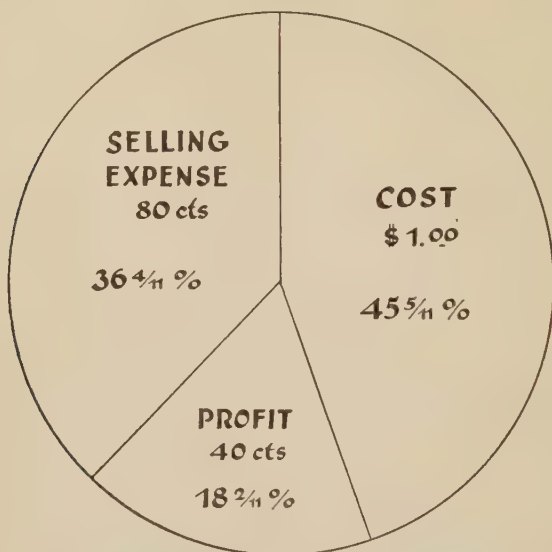
A double comparison—four facts compared in pairs.



JONES' CHART

Here the comparison is confusing. Selling Expense and Profit cannot be compared, as they are shown each in its relation to two different, unequal bases. Jones considers Profit as a thing separate from Cost instead of a part of Sales. He considers Selling Expense as a part separate from Sales rather than as a fact a part of the Sales.

A single comparison—three facts compared with each other and with the whole.



SMITH'S CHART

In this diagram the relation of all the facts is clearly shown. Sales Expense and Profit being shown in relation to the Sales are therefore shown in relation to each other. Smith considers cost, profit, and selling expense each to be a part of the Sales.

ILLUSTRATION OF PROFIT AND LOSS DEVELOPMENT

1. An article costs, including freight, \$12.60. At what price should it be sold to gain 20%?

OLD METHOD, PROFIT ON COST

$$\begin{aligned}
 5) \$12.60 &= \text{Cost, Base } 100\% \\
 \underline{2.52} &= 20\% \text{ Gain on Cost} \\
 15.12 &= \text{Selling Price}
 \end{aligned}$$

NEW METHOD, GAIN ON SALES

Sales = Base	100%
Profit on Sales	<u>20%</u>
Cost of Sales	80% = \$12.60

.80) \$12.60 (\$15.75 = Selling Price

<u>80</u>	or
460	4) \$12.60 = Cost = $\frac{4}{5}$ = 80% of Selling Price
<u>400</u>	3.15 = Profit = 20% of Selling Price
600	15.75 = Selling Price = $\frac{5}{5}$ = 100% or Selling Price
<u>560</u>	
400	

2. The same as above sold to gain 20% and allow 10% of the sales for selling expenses.

PROFIT ON COST; SELLING EXPENSE ON SALES

5) 12.60 = Cost 100%

2.52 = Gain 20% ($\frac{1}{5}$ on Cost)

.90) 15.12 = Net Receipts above Expenses

16.80 = Selling Price (Divide by 9 and move to left one place)

PROFIT AND SELLING EXPENSE ON SALES

100% = Sales

20% = Profit

+ 10% = 30% Expenses (100% - 30% = 70%)

70%

.70) 12.60 = Cost = 70% of Selling Price

18.00 Selling Price

(Divide by 7 and move to the left one place)

3. The same goods, sold to gain 20%, allowing 10% of Sales for expense and 2% of the goods to be unsalable.

GAIN AND SELLING EXPENSE ON SALES

.70) \$12.60 = Cost = 70% of Net Sales

.98) 18.00 = Profit + Expense + Cost

$$\begin{array}{r}
 98 \quad (18.37 = 100\% = \text{Selling Price on all of the goods}) \\
 \hline
 820 \\
 784 \\
 \hline
 360 \\
 294 \\
 \hline
 660 \\
 588 \\
 \hline
 720
 \end{array}$$

GAIN ON COST, EXPENSE ON SALES

5) \$12.60 = Cost = 100%

2.52 = Gain 20% on Cost

*.90) 15.12 to be rec'd Net

1.68 = 10% of Sales = Expenses

.98) 16.80 (17.14 = Selling Price = 100%)

$$\begin{array}{r}
 98 \\
 \hline
 700 \\
 686 \\
 \hline
 140 \\
 98 \\
 \hline
 420
 \end{array}$$

*NOTE: \$15.12 equals Sales less Expense and Shrinkage.

For these and other reasons there has developed during the last few years a strong movement among business men demanding that the old profit-on-cost method be abandoned, not only in business, but in our schools, and for that reason it is well to know the relative merit of the two methods and how to figure both. Under no circumstances should a business man use first one method and then the other. A business may be conducted safely by the old method, if the figuring is carefully and consistently done, but the profit on sales method has so many advantages and so strong a following that its general adoption in the near future seems practically assured.

A Table Showing Prices at Which You Must Sell an Article Costing \$1.00 In Order to Make Any Desired Net Profit.

15%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.19	1.21	1.22	1.23	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.54	1.67	1.82	2.00	2.22	2.86
	Sell for.....																					
16%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.21	1.22	1.24	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.56	1.70	1.85	2.04	2.27	2.94
	Sell for.....																					
17%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.22	1.24	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.59	1.72	1.89	2.08	2.33	3.03
	Sell for.....																					
18%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.24	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.61	1.75	1.92	2.13	2.38	3.13
	Sell for.....																					
19%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.25	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.64	1.79	1.96	2.17	2.44	3.23
	Sell for.....																					
20%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.27	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.67	1.82	2.00	2.22	2.50	3.33
	Sell for.....																					
21%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.28	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.56	1.70	1.85	2.04	2.27	2.54	3.45
	Sell for.....																					
22%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.30	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.56	1.59	1.72	1.89	2.08	2.33	2.63	3.57
	Sell for.....																					
23%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.32	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.56	1.59	1.61	1.75	1.92	2.13	2.38	2.70	3.67
	Sell for.....																					
24%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.33	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.56	1.59	1.61	1.64	1.79	1.96	2.17	2.44	2.81	3.85
	Sell for.....																					
25%	Cost of Doing Business	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	20%	25%	30%	35%	40%	50%
	Net Profit.....	1.35	1.37	1.39	1.41	1.43	1.45	1.47	1.49	1.52	1.54	1.56	1.59	1.61	1.64	1.67	1.82	2.00	2.22	2.50	2.83	4.00
	Sell for.....																					

These figures are computed on a basis of SALES, from Butler Bros. Way System Book.

MARKING GOODS

In marking goods for sale, a code or key is used to indicate the cost and generally the selling price or marked price also. The code is any word or phrase of ten letters, each letter of which represents, and is used instead of, one of the ten digits.

(1) Cornbasket, (2) M. H. Lockyear, (3) Victor Mead, (4) Pay-me-quick, (5) New Hampton and (6) Alice Thumbs, are illustrations. To prevent repetition an eleventh letter called a repeater is used. In the last key shown above, the s is the repeater; in the others we will use "j." The cost is generally written above and the selling price below, on the tag or label.

To ILLUSTRATE: Goods

mit
ace

 costing \$7.24 and selling at \$9.38 would be marked

mit
ace

 if we used the key Victor Mead.

(141)

PROBLEMS

Show the correct cost and selling marks for the following goods to make the profit and allow for the selling expense indicated.

No.	Kinds of Goods	Unit	Cost	Per Cent. Profit and Base	Per Cent. of Sales=S. Exp.	Key No.
1	Silk	yd.	\$ 1.25	20 % on Cost		1
2	Baskets	ea.	.96	10 % on Sales		4
3	Suits	ea.	24.40	25 % on Cost	10 %	5
4	Shoes	pr.	7.50	15 % on Sales	10 %	2
5	Rubber hose	ft.	.06 $\frac{1}{3}$	20 % on Cost	20 %	3
6	Clothes pins	doz.	.08 $\frac{1}{2}$	25 % on Sales	15 %	4
7	Tea	lb.	.33 $\frac{1}{3}$	10 % on Sales	5 %	5
8	Cheese	lb.	.26	40 % on Sales		1
9	Molasses	gal.	1.10	12 $\frac{1}{2}$ % on Cost		6
10	Stoves	ea.	45.80	8 $\frac{1}{3}$ % on Sales	12 $\frac{1}{2}$ %	3

11. 150 bu. of potatoes were bought at \$1.75 a bu. Freight was \$18.20 and hauling \$10.50. How should you mark them to

gain 20% on sales, if selling expense is $12\frac{1}{2}\%$ of the sales? Use key No. 2 for the cost, show selling price in figures.

12. How would you mark the above, using key No. 6 if you were to gain 25% on cost and allow $12\frac{1}{2}\%$ of sales for expense?

13. Bought 40 dozen straw hats at \$36.40 a doz., less 10%, 5%, and 2%. Paid freight \$19.10. How shall we mark them, using key No. 2, to gain $33\frac{1}{3}\%$ on sales and allow 10% of sales for expense of selling?

14. Goods marked

yek
mua

 by key No. 4, are sold at what per cent. on cost, if

yek
mua

 $12\frac{1}{2}\%$ of the sales are for selling expense? On sales?

15. Using key No. 6, show the tag changed so that a gain of 20% will be realized on sales and 5% of sales allowed for expense.

16. Bought 320 hammers, at \$16.20 a dozen less trade discounts of 5%, $2\frac{1}{2}\%$ and 1%; by paying cash I received a further discount of 2%. How shall I mark them each, using key No. 1, so that I may sell at a profit of 5% on cost and allow 10 cents each for selling expense; freight was \$7.80.

17. A ream of paper (500 sheets) costs \$1.20. How many sheets can I sell for a nickel and gain 20% on sales, allowing 5% of sales for expense of selling?

18. Bought 5000 cigars at \$45 a M and received 250 cigars extra as a discount. At what price must I sell to make a profit of 20% on sales and allow 5% of sales for expenses? Using key No. 4 show the marks I would place on a box containing 50 cigars. How many can I afford to sell for 35c?

19. 1820 lbs. of ham was bought at 54 cents a lb. It reduced 5% through shrinkage. How shall we sell it per lb. to gain 20% on sales and allow 10% of sales for expenses?

20. One hundred dolls were bought at \$21.84 a dozen. They were marked

meh
hkr

, using key No. 2. One of the dolls was broken after

meh
hkr

 59 of them were sold. What change shall we make in the selling mark to have the remaining dolls make good the cost of the one broken?

REVIEW QUESTIONS ON PERCENTAGE

1. Show three ways of expressing percentage.
2. Why is percentage so often used when fractions could be used instead?
3. What are the five terms used in percentage?
4. What is a straight or trade discount? Its purpose?
5. What is a conditional or cash discount? Its purpose?
6. Is it proper to figure freight as a part of the cost of goods bought?
7. How is the rate of selling expense ascertained?
8. In figuring gain per cent. on sales under what circumstances might we have a 100% profit?
9. What is a profit?
10. Give six reasons why business men fail.
11. How could a charge of \$96 for freight and \$4.80 for hauling be properly distributed on an invoice of 340 baskets and 480 18-lb. mauls, if the freight rate was 96c a cwt.
12. What is meant by turnover? Inventory?
13. What is the effect of a slow turnover, on a business?
14. Define Business, Trading, Manufacturing, Service.
15. What reasons can you give for figuring profit on sales rather than on cost?
16. In a series of discounts, which do you use first in finding the cost?
17. Distinguish between purchase discount and sales discount.
18. What should a business earn to be really worth while to the owner?
19. What is meant by terms of sale?
20. Name four applications of percentage.

CHAPTER XII

GRAPHS

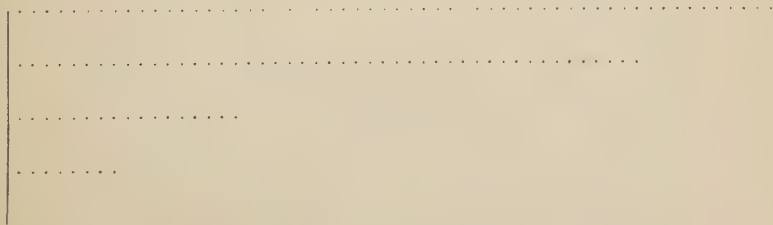
A graph is any drawing, the purpose of which is to set forth to the eyes in true proportions, the relation existing between certain statistical facts.

During recent years the use of graphs has become quite general. Pupils should therefore acquaint themselves well enough with the forms in daily use to be able not only to readily interpret any graph presented, but to construct accurately a graph well suited to any purpose.

A rehearsal of figures is confusing and seldom convincing. The correct comparison of data listed in parallel columns is generally difficult, but a graph properly executed sets forth all fact-differences, fact-relationships and fact-changes, in form that anyone can readily grasp and understand.

Group comparisons shown in hundredths or thousandths are not always clear, but a good graph is a bird's-eye view of the whole situation; it is interesting; it gets the message across, and for these reasons it is a first aid in any comparative study in figures.

UNSCALED GRAPH



NOTE: Does this give as much information as No. 1 on the following page?

THE STRAIGHT LINE OR BAR GRAPH

This form is most useful in making a comparison of facts for any particular date or period. The scale is generally expressed in halves, quarters, eighths and sixteenths of an inch or in tenths and hundredths of a decimeter, a metric measure equal to about $3\frac{1}{8}$ inches and divided into 100 centimeters.

The statement shown in Chapter X shows the following facts: To compare the first column figures is difficult; to express them in percentages as in column two offers an advantage, but the graph is necessary to a sure, quick grasp of conditions. See Illustration No. 1

Net Sales....	\$4284.06	100%
Cost of Sales...	3276.88	76%
Operating Exp..	619.20	15%
Losses-Profits..	71.27	2%
Net Profit...	316.71	7%

Using a decimeter as a measure and allowing a centimeter to each per cent., a straight line graph would take the form of illustration No. 1.

The straight line graph does not show what the bar graph and circle graph show; i. e., that the sum of items 2, 3, 4 and 5 is equal to item 1 and component parts of it. See illustrations 2 and 3.

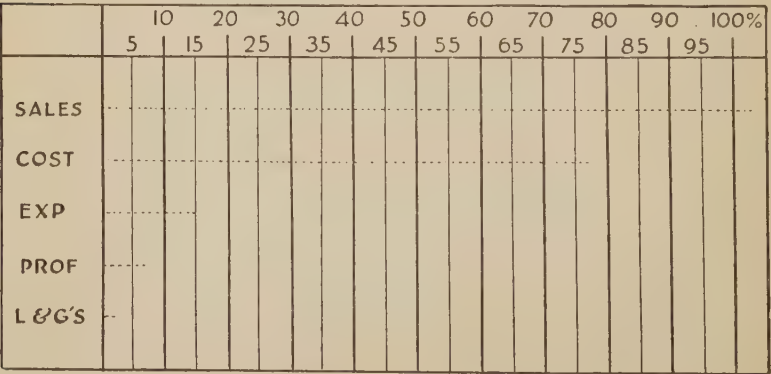


Illustration No. 1, Straight Line Graph.

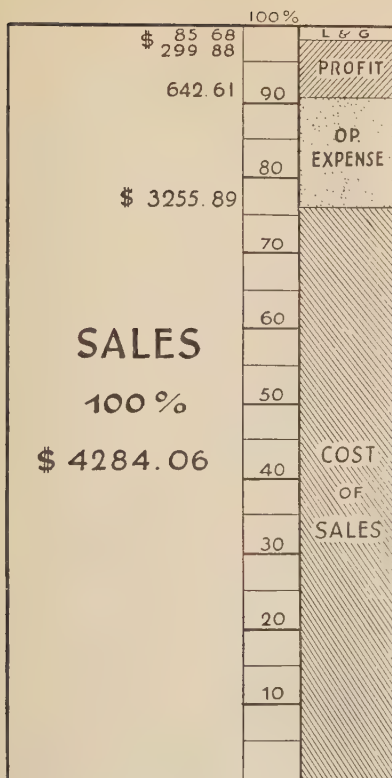
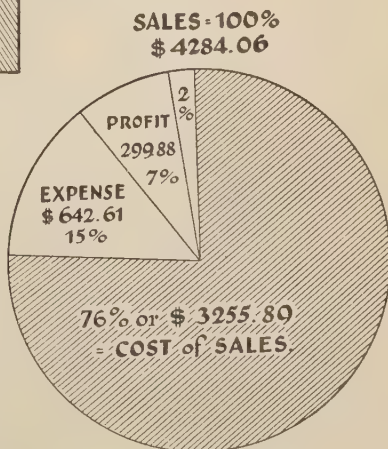


Illustration No. 2
Bar Graph

The comparison of one circle with another circle, or a square with another square, is more likely to be misleading than the comparison of the parts within a circle, or the parts within a rectangle or square. See illustrations 2 and 3.

Areas are more difficult to compare than straight lines, but where the component parts of a unit are to be studied in their relation to the unit and to each other, then the bar or circle graph can be used to advantage. The circle is a form familiar to all, and easily divided into fractional parts, as its circumference is 360° a multiple of 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18 and 20, making the conversion of these parts into 360ths very easy. The circle is not so well adapted to showing percentages.



Circle Graph No. 3

Problems to illustrate curved line graph:

Jan. Sales.....	\$27250	July Sales.....	15250
Feb. Sales.....	24500	Aug. Sales.....	17000
Mar. Sales.....	30000	Sept. Sales.....	19750
Apr. Sales.....	22500	Oct. Sales.....	25250
May Sales.....	17750	Nov. Sales.....	30000
June Sales.....	18500	Dec. Sales.....	37500

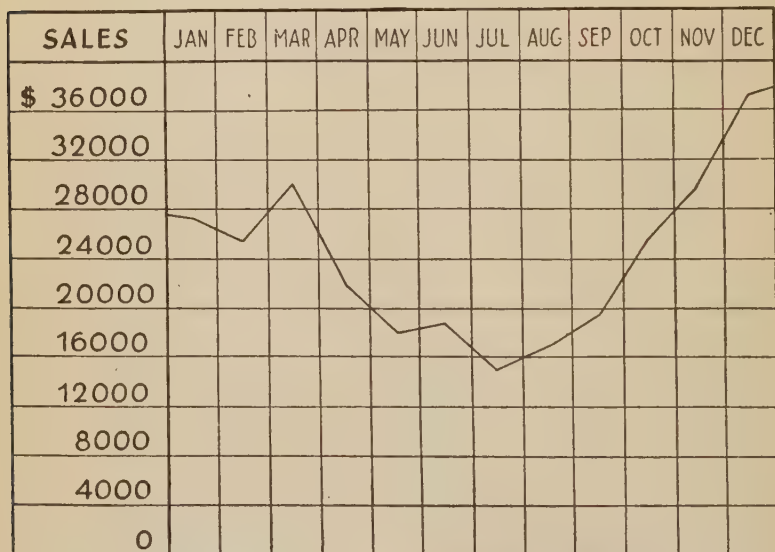


Illustration No. 4, Curved Line Two-Scaled Graph

By the use of a curved or zigzag line across a double-scaled area, the general trend of affairs for a period of time can be very vividly set forth, as shown in Illustration No. 4. The scale at the left represents the amount of sales and the scale at the top represents the time periods studied. The movement of the line from left to right indicates the passing of time, its movement up and down indicates the rise and fall in sales for the year. This is one of the most useful of all graph forms, and especially useful to the business man in recording and studying changes in business conditions.

Several related series of facts can be shown on the same chart. See Illustration No. 5. Here, five fact-series are indicated by the use of differently constructed lines. Different colored lines could have been used to advantage.

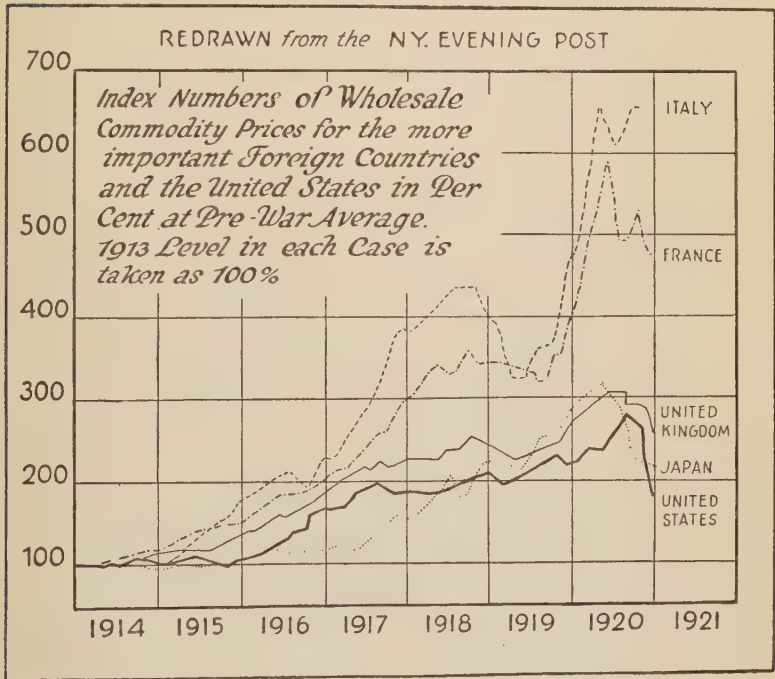
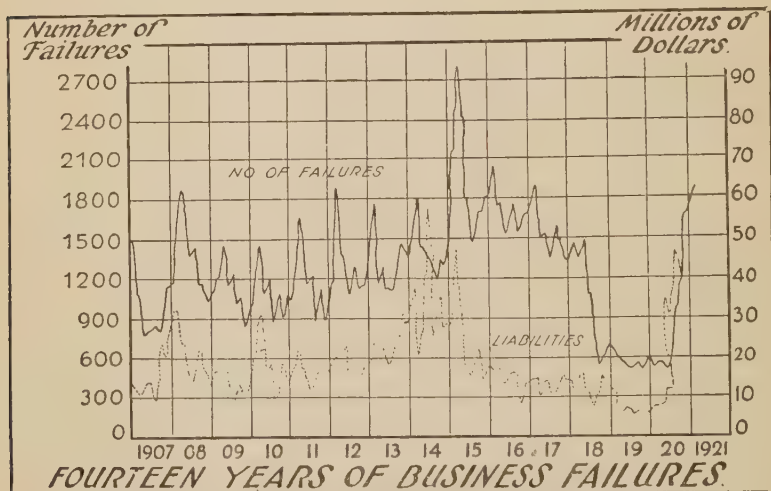


Illustration No. 5, Curved Line Two-Scaled Graph

The above chart shows very strikingly the rapid increase in prices during the war. Since early 1920 the prices are shown declining and are continuing throughout 1921. Assume that the prices at the end of 1921 are as follows compared to 100% for 1914: Italy 225%, France 210%, United Kingdom 180%, Japan 98% and U. S. 125%. Redraw the chart and show decline through 1921.

If a triple scale graph is used, see Illustration No. 6. Care must be taken that the scales shown on opposite sides of the area be in proper relation to each other, increasing and decreasing in like proportions, otherwise the fact-lines will not develop in proper relation to each other. Both scales must begin at zero, on the same base line.



Reprinted from the N. Y. Evening Post

Illustration No. 6, Triple-Scaled Graph

1. If possible obtain reports on failures for the subsequent years and extend the graph. If figures are available graph 1919 and 1920 by months as these vary widely and will give a good chance to compare months. Consult Dun or Bradstreet.

2. Why does the line representing liabilities rise and fall with the line of failures?

3. Can you suggest why the dotted line crossed the black line preceding the increased failures of 1914 and 1920?

When the opposing scales are expressed in different units great care must be taken that the two scales are set in proper relation to each other. In Illustration No. 7 two scales are shown at the right, and two lines drawn across the chart to indicate the change wrought in the appearance of the graph by following the different scales.

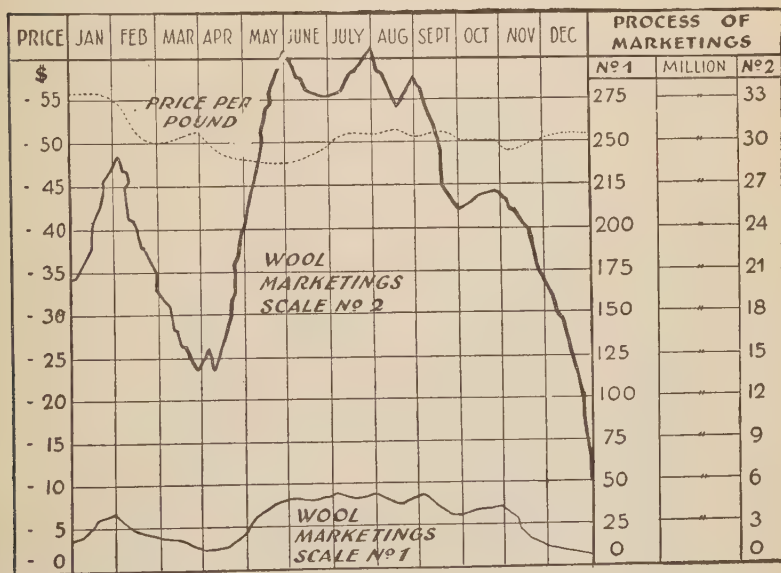


Illustration No. 7. A Three-Scaled Graph.

Showing what effect a change in opposing scales may have on the appearance of the Graph.

To avoid this difficulty, monthly averages should have been made for both price and production, and both lines drawn on one scale, using index figures to determine the course of the line. The average divided by the amount given will serve as a per cent. guide on the scale, assuming the average to be 100%.

Illustration No. 8 shows a similar graph drawn in correct form. Notice that the prices tend to rise as the marketings decrease and vice versa. Notice also that no data are furnished, only the relation of price and marketings on a 100% or average basis. This is an accurate treatment of two related fact-series, given in different units of measure, i. e., dollars and pounds.

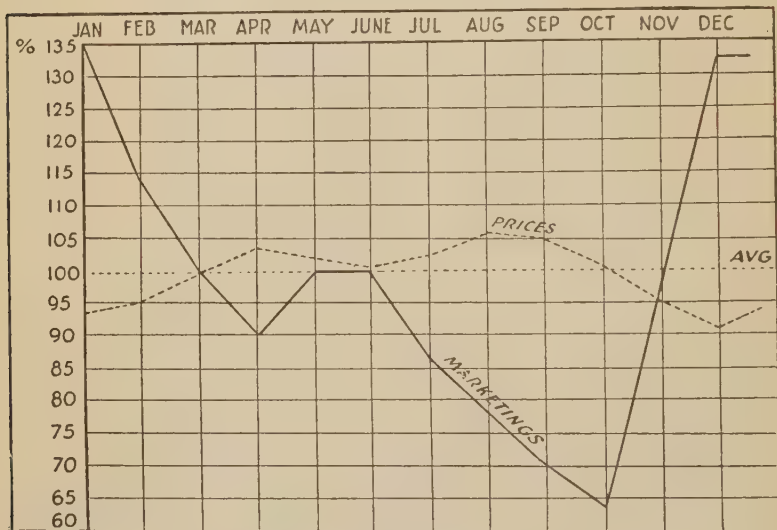


Illustration No. 8. Two-Scaled Graph.

Monthly trend of hog prices and marketings, based upon the average for ten years.

QUESTIONS

1. During which months did prices reach highest points in above?
2. When did prices reach lowest level?
3. When should the farmer sell, based on the above?
3. Can you suggest why prices increase when marketings decrease and vice versa?

Illustration No. 9 is a single scaled comparative bar graph constructed on the following statistics on a scale of $\frac{1}{8}$ inch to the billion dollars:

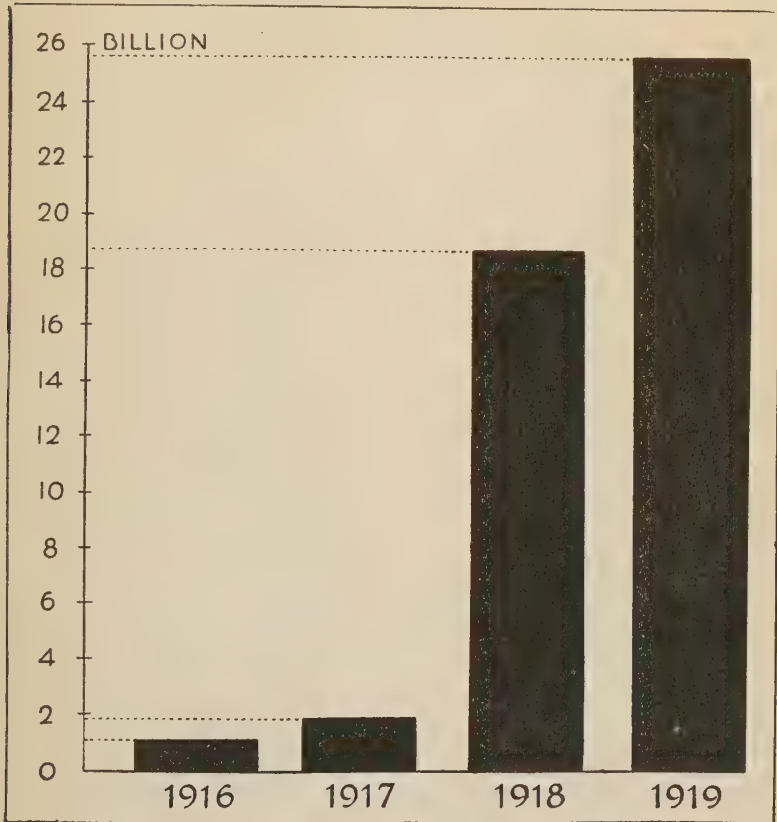


Illustration No. 9. Comparative Bar Graph.

Appropriations by Congress

in 1916 were \$678,677,848
 in 1917 were \$1,178,908,962
 in 1918 were \$18,144,861,745
 in 1919 were \$25,598,967,517

Illustration No. 10 is one of a variety of forms of pictorial graphs used in making approximate comparisons. It is especially interesting and impressive.

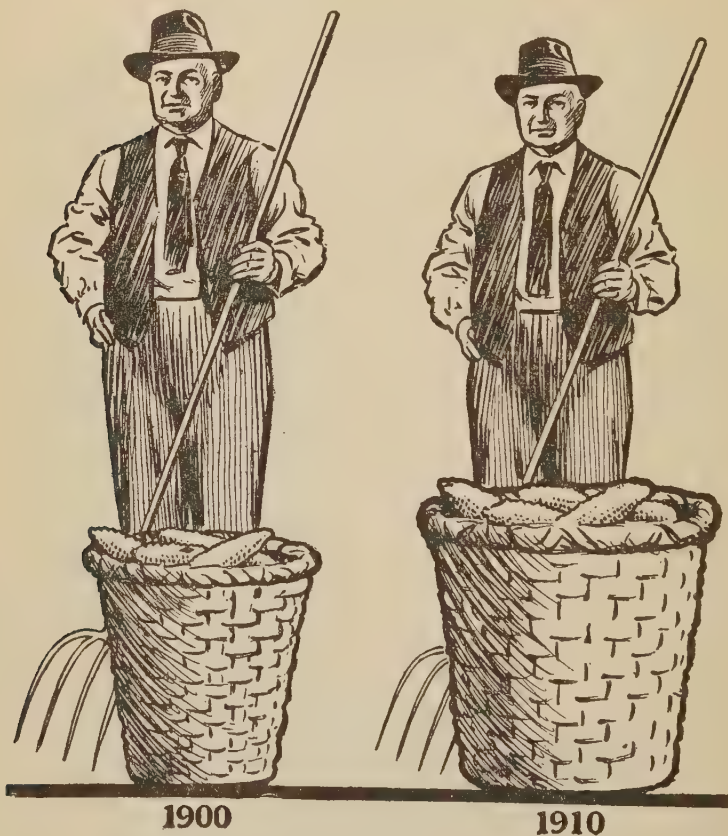


Illustration No. 10. Pictorial Graph.

The above shows that fewer farmers in three corn producing states were tilling the soil in 1910, yet this smaller number represented by the smaller man produced more corn (larger basket) than the larger man produced in 1900 (smaller basket). This increased production was brought about by improved machinery, fertilizer, etc.

HOW TO CONSTRUCT A GOOD GRAPH

1. Gather correct data.
2. Select the most suitable form of graph.
3. Determine your scales whether fractional or decimal.
4. Begin your scale at zero and show numbers for each dividing line.
5. Show dates and all other data possible in or near the graph.
6. Avoid the use of area or volume representations.
7. Place scales and figures at the bottom and at the left, the scale at the left should read from the bottom toward the top.
8. Build from left to right.
9. Make the drawing clear, and true to your scale.
10. Avoid misrepresentation of facts.
11. Show data to support your graph.
12. Show a key if one is necessary, for proper interpretation.
13. Use graph paper, if available, which is on sale at the leading stationery stores.

(142)

DRILLS IN THE CONSTRUCTION OF GRAPHS

From the statistics given construct neat, accurate graphs, following the forms indicated.

1. A girl's spelling grades for one month (the first day of which was Friday) were as follows: 80%, 75%, 90%, 70%, 65%, 90%, 95%, 100%, 95%, 80%, 85%, 75%, 95%, 90%, 95%, 100%, 85%, 95%, 95%, 90%, and 85%. Construct a two-scale graph showing dates of month (21 columns) at top and grade scale from 0 to 100% at the left. Let $\frac{1}{4}$ inch = 5%.

2. In a class of 45 pupils, 2 quit, 3 failed, 8 made a grade of D, 15 made C, 12 made B, and 5 made A. Construct a circular graph to show these facts. ($360^\circ \div 45 = 8$) therefore 8° should be allowed for each pupil. Use a protractor for exact measurement of circle.

3. In a school of 2000 pupils, daily absences for 10 months (September to June) average as follows:

	Boys	Girls
Sept.	150	100
Oct.	100	90
Nov.	120	85
Dec.	160	95
Jan.	125	100
Feb.	110	115
Mar.	115	110
Apr.	150	100
May	170	115
June	190	100

Construct a single line double-scale graph showing the rise and fall in total attendance. Base scale at left on 2000, marking the lines in multiples of 100.

4. On the data furnished in problem No. 3, construct a two-line, double-scaled graph, comparing the attendance of the boys and the attendance of the girls. There were 1200 boys enrolled and 800 girls. Base scale at the left on 100%, marking the lines in multiples of 10%.

5. The sources of incomes of the Hyde Park Country Club of Cincinnati for 1921 were as follows:

Initiation fees.....	\$ 9,650.00
Dues.....	33,252.98
Contributions.....	9,086.84
Green Fees.....	1,846.00
Locker Rents.....	884.50
Bowling Alley.....	66.33
Restaurant and Stand.....	298.10
Discount on Purchases.....	140.04
Interest on Sinking Fund.....	44.38
Discount on Share Stock.....	5.00
Total.....	<u>\$55,274.17</u>

Construct a bar graph 1 inch wide, on a scale of \$400, to the $\frac{1}{16}$ inch. Show the five small items as one, under the heading miscellaneous receipts.

6. Develop from the following data a double-scaled graph, using three lines, one for each year recorded, a left side scale of $\frac{1}{4}$ inch for each \$100,000,000.00 and a top scale of $\frac{1}{2}$ inch for each month.

Following are the comparative figures given by The Journal of Commerce and Commercial Bulletin showing companies incorporated in the Eastern States during the last three years with an authorized capital of \$1,000,000 or more:

	1919	1918	1917
January.....	\$ 380,865,000	\$ 209,285,000	\$ 244,450,000
February.....	248,975,000	120,650,000	283,815,000
March.....	252,950,000	128,620,000	281,000,000
April.....	394,450,000	169,375,000	361,510,000
May.....	548,000,000	213,372,000	388,481,000
June.....	1,208,000,000	144,400,000	352,584,000
July.....	1,230,000,000	90,725,000	416,350,000
August.....	677,000,000	77,975,000	382,100,000
September.....	1,402,843,500	132,925,000	202,500,000
October.....	2,192,607,500	57,665,000	308,513,700
November.....	1,192,250,000	94,260,000	285,590,000
December.....	919,014,000	68,200,000	186,350,000
Total.....	\$11,646,955,000	\$1,507,452,000	\$3,693,243,700

7. Construct a comparative eleven-bar graph from the data given, on a scale of $\frac{1}{8}$ inch to each \$100,000,000. Make bars $\frac{1}{4}$ inch wide and $\frac{1}{8}$ inch apart, arranged either horizontally or perpendicularly. (Take last three countries together.)

UNITED STATES LOANS TO FOREIGN COUNTRIES

(Aug. 15, 1920.)

Great Britain.....	\$4,277,000,000	Serbia.....	\$ 26,780,465
France.....	3,047,974,777	Roumania.....	25,000,000
Italy.....	1,666,260,179	Cuba.....	10,000,000
Belgium.....	350,428,793	Liberia.....	5,000,000
Russia.....	187,729,750		
Czecho-Slovakia.....	67,329,041	Total.....	\$9,711,739,636
Greece.....	48,236,629		

Some loans made by our Government during the war had been paid.

8. From the accompanying facts construct a double scaled graph (see Illustration No. 8), using the average Sales, Delivery Expense and Advertising as 100%.

	Sales	Delivery Exp.	Advertising
January.....	\$27500	\$1500	\$750
February.....	28500	1475	650
March.....	31000	1550	550
April.....	32500	1675	600
May.....	35750	1700	550
June.....	33000	1675	650
July.....	29750	1650	550
August.....	27250	1600	500
September.....	34500	1650	725
October.....	36750	1675	675
November.....	40250	1725	950
December.....	50000	1850	875

9. The following statistics were given out by the street railways of a leading city. Average number of passengers carried per day 445,000, distributed over the hours of the day as follows:

From 5 to 6 a. m..13,000	From 3 to 4 p. m..23,000
From 6 to 7 a. m..29,000	From 4 to 5 p. m..25,000
From 7 to 8 a. m..52,000	From 5 to 6 p. m..68,000
From 8 to 9 a. m..34,000	From 6 to 7 p. m..29,000
From 9 to 10 a. m..15,000	From 7 to 8 p. m..19,000
From 10 to 11 a. m..13,000	From 8 to 9 p. m..12,000
From 11 to 12 a. m..13,000	From 9 to 10 p. m..10,000
From 12 to 1 p. m..15,000	From 10 to 11 p. m..12,000
From 1 to 2 p. m..20,000	From 11 to 12 p. m.. 8,000
From 2 to 3 p. m..20,000	From 12 to 5 a. m.. 3,000 per hr.

Construct a graph, choosing the form you think best suited, to show the day's traffic in hour comparisons. What per cent. of the traffic falls between the hours of 5 and 6 p. m.? What per cent. between 7 and 8 a. m.? If tickets were sold at the rate of 6 for 51c what was the amount received daily by the Company?

10. From the following statistics construct two graphs, one to show the rise and fall of immigration and another to show a comparison of racial groups now in the United States:

Immigration for every fifth year 1820 to 1920.

1820.....8,385	1875....227,498
1825.....10,199	1880....457,257
1830.....23,322	1885....395,346
1835.....45,374	1890....455,302
1840.....84,066	1895....258,536
1845.....114,371	1900....448,572
1850.....369,980	1905...1,026,499
1855.....200,877	1910...1,041,570
1860.....133,143	1915....326,700
1865.....180,339	1920....400,001
1870.....387,203	

Number of people of each racial group in the United States, 1920.

British.....10,000,000	Lithuanian.....750,000
German.....9,250,000	Spanish.....700,000
Scandinavian....3,750,000	Japan-China.....550,000
Italian.....3,000,000	Hungarian.....450,000
Jewish.....3,000,000	Dutch.....400,000
Polish.....3,000,000	Greek.....300,000
French.....1,600,000	Finnish.....300,000
Cz. Slovak.....1,000,000	

SUGGESTION: Abundant material for use in the construction of graphs can be found in magazines and trade bulletins, also good graph forms for interpretation and general discussion. Select such material as will most likely interest the class. Facts relative to the school, home or city are especially good for this purpose.

11. The cost of manufacturing 1000 ladies' handkerchiefs was \$77, divided as follows: \$27.50 for materials, \$42.50 for labor, and \$7.00 for overhead expense. Construct a circular graph to show the relation of these facts.

12. The density of population per sq. mi. in the United States each census from 1790 to 1910 inclusive was 4.5; 6.1; 4.3; 5.5; 7.3; 9.7; 7.9; 10.6; 13.0; 16.9; 21.2; 25.6 and 30.9, respectively. Construct a double-scaled graph showing changes for the thirteen decades. Be sure your left-hand scale starts at zero. Let $\frac{1}{8}$ of an inch represent one person.

THRIFT AXIOMS

1. A dollar spent never returns; it has gone forever.
2. A dollar saved is equivalent to a dollar earned. You have gained the dollar and the strength to hold it.
3. A dollar invested will return again, bringing interest with it.
4. A dollar risked in a gamble is a great loss, even though many dollars are won, for character, reputation, and faith in one's self are worth more than much money.
5. According to the wisdom shown in placing your investment, so shall your rate per cent. of income be.
6. An idle dollar is a loss of income. Invest, do not hoard nor carry it.
7. "That which you need you will have to pay for, whether you get it or not."
8. Compound interest builds a fortune quickly.
9. Save, that you may have enough to give.
10. Measure the value of your time by the dollars you are able to earn per hour.
11. Dollars work 24 hours a day, Sundays and holidays included. How many are working for you? Interest is pay for their labor.
12. Despise not the nickels and dimes. Remember the height of the Woolworth Building—it was builded on nickels and dimes. A chew of gum is only a penny, but the gum bill for the U. S. for one year was \$50,000,000.

CHAPTER XIII

INTEREST

Interest has to do with money, or property values.

Money is the medium by which all exchanges of value are made possible; in other words it is the measure of all value, the common denominator for all commodities.

Here the question arises: What gives anything value? The economist says "the relation of Supply to Demand"; but this is not always true, as either may be controlled or affected by private interests, legislation or transportation conditions. Value may also be affected by the amount of money available. Money, too, is affected by the Law of Supply and Demand, and its value is indicated not only by what it will bring in commodities of trade, but by the rate of interest charged.

Interest is the relation between supply and demand in the money market.

A plentiful supply of money and a low demand=low rates of interest.

A scarcity of money and a great demand=high rates of interest.

Dollars and men are generally idle when interest and salaries are too high or when they cannot be employed to any advantage. The interest rate, therefore, becomes the measure by which investments and properties are evaluated.

Interest rates determine, too, in a large degree, the amount of money people will save. A few will save because of habits of thrift, whether the savings can be used to advantage or not; others will save if the interest or income on savings is made

attractive enough. The following classification is given for its suggestive value:

1. Some never earn or produce even a living. Why?
2. Many earn only a living. Why?
3. Many earn more than a living, but spend all they earn. Why?
4. A few earn more than a living, but their savings earn them only 4%. They don't care to risk losing it. Why?
5. The Banker gives them 4% and loans it to others at 8%. He takes very little risk, however.
6. The shrewd business man borrows this same money of the bank at 8%, takes all the risk incident to the business and makes 12% or very much more.

Not all capital is borrowed, but whether borrowed or not, all money invested is expected to earn a fair rate of interest, plus the expenses incident to management, operation, upkeep, depreciation, etc.

(143)

PROBLEMS TO ILLUSTRATE

If property can be rented for \$480 a year, over and above expenses, and money can be borrowed at 6%, then $\$480 \div .06 = \$8000$ the investment value of the property.

The net income per year divided by the current rate of Interest equals the investment value.

If property costs us \$17,500, and the net yearly income is \$940, we are earning $\$940 \div \$17500$ or $5\frac{1}{4}\%$. It is evident therefore that if our money is not earning what it should (6%), we will likely have to sell below \$17,500 because of the desire for a higher rate.

The net yearly income divided by the cost of the property investment equals the rate of net income earned.

A little cough-drop business represents only \$25,000 in invested property, but the yearly earnings are \$12,000 net. What would you offer for this business if money is worth 8%? 4%?

$$\$12,000 \div .08 = ?$$

$$\$12,000 \div .04 = ?$$

	Investment Cost	Gross Income	Upkeep, Depreciation and Expenses	Rate of Interest	Present Value
1.	40000	18000	12000	6%	?
2.	72000	24000	15000	?	
3.	145000	17500	12000	4%	?
4	200000	65200	50000	8%	?

5. Brown paid \$60,000 for a property last year. It earns him a net income of \$5400. He wants \$80,000 for it. Money is being loaned at 7%. Is Brown asking too much or not enough? Why?

6. A house cost us \$6400. At what price per month must we rent it to make 8% on the investment and allow \$500 a year for upkeep, taxes, insurance and depreciation?

7. A doctor spent \$10,000 for his education and \$5000 for his equipment. His expenses are \$300.00 per month. What must he earn per month before his investment in education returns him a profit, money being worth 8%.

8. If by learning to figure rapidly a boy increased his earnings \$5.00 a week, what amount of money on deposit at 6% would another boy have to have to receive the same increase through interest received?

9. *Mrs. Clark bought \$5000 worth of diamonds. Mrs. Frugal invested her \$5000 in property that earns 9% net. What do Mrs. Clark's diamonds cost her per month, without regard for original cost or taxes?

10. A shoe merchant carries an average stock of \$80,000 and accounts due from customers, \$24,000. If money is worth 7%, what is the monthly cost of carrying these items? Who has to pay this interest charge?

*Try and learn the cost of the highest priced automobile in your community and also the lowest priced. Allow 25% yearly depreciation, estimate the upkeep and operating expenses, and allow 6% interest on investment. Add the charges and arrive at the monthly operating cost of each.

Table showing the legal rate of interest and the maximum rate allowed in each of the several states and territories.

States and Territories	Legal Rate	Maximum Rate	States and Territories	Legal Rate	Maximum Rate
Ala.....	8%	8%	Neb.....	7%	10%
Alaska.....	8%	12%	Nev.....	7%	any
Ariz.....	6%	12%	N. H.....	6%	6%
Ark.....	6%	10%	N. J.....	6%	6%
Calif.....	12%	12%	N. Mex.....	6%	12%
Colo.....	8%	any	N. Y.....	6%	6%
Conn.....	6%	12%	N. C.....	6%	6%
Del.....	6%	6%	N. D.....	7%	12%
Dist. of Col.....	6%	6%	Ohio.....	6%	8%
Fla.....	8%	10%	Okla.....	6%	10%
Ga.....	7%	8%	Ore.....	6%	10%
Hawaii.....	8%	12%	Pa.....	6%	6%
Ida.....	7%	12%	P. Islands.....	6%	any
Ill.....	5%	7%	Porto Rico.....	6%	12%
Ind.....	6%	8%	R. I.....	6%	any
Ia.....	6%	8%	S. C.....	7%	8%
Kans.....	6%	10%	S. D.....	7%	12%
Ky.....	6%	6%	Tenn.....	6%	6%
La.....	5%	8%	Tex.....	6%	10%
Me.....	6%	any	Utah.....	8%	12%
Md.....	6%	6%	Vt.....	6%	6%
Mass.....	6%	any	Va.	6%	6%
Mich.....	5%	7%	Wash.....	6%	12%
Minn.....	6%	10%	W. Va.....	6%	6%
Miss.....	6%	8%	Wis.....	6%	10%
Mo.....	6%	8%	Wyo.....	8%	12%
Mont.....	8%	any			

The Legal rate* is used unless a higher rate has been agreed upon in writing.† By charging interest in advance the banks have often evaded the law concerning a maximum rate. 8% charged on \$1,000,000 and the interest reloaned in advance is equal to $8\frac{17}{25}\%$ on the principal.

*Legal rate is fixed by the statutes of different states.

†Some few states, however, forbid contracting for a higher rate than the rate set by law.

THE FIVE KINDS OF INTEREST COMPARED

Kind of Interest	No. of Days for Year	How Find Time	Method Used	When Used and by Whom
Simple	360	*Comp. Sub.	Bankers 60 day 6%	By the courts where other interest is not specified, and among business men.
Bankers	360	Count days	Bankers 60 day 6%	By banks in discounting notes and by some business men.
Accurate or Exact	365	Count days	Special	For interest on Treasury Notes and Bonds; balances between banks and by some business men.
Compound	360	Comp. Sub.	Bankers 60 day 6%	By written agreement only.
Annual	360	Comp. Sub.	Bankers 60 day 6%	By written agreement only.

For illustration and comparison, the figures for all the kinds of interest are shown for the following problem:

Interest on \$840 May 1, 1918, to October 4, 1921, at 6% = ?

Simple	Accurate or Exact
1921—10—4	\$840.00
1918— 5—1	.06
<u>3— 5—3 = 1233 days</u>	50.40 Int. 1 year
	<u>1252 No. of days</u>
5) \$840 = 6000 days	10080
168 = 1200 days	250200
4.20 = 30 days	10080
.42 = 3 days	<u>5040</u>
\$172.62	365) 63100.80 (\$172.88 = Accu- rate Int.

*Compound Subtraction.

Bankers

May 1, 1918, to May 1, 1921 = $365 \times 3 = 1095 + 1$ for leap year = 1096.

3 years = 1096

May, 1921.....	30	\$840	= 6000 days
June, 1921.....	30	168	= 1200 days
July, 1921.....	31	4	20 = 30 days
August, 1921.....	31	2	80 = 20 days
Sept., 1921.....	30		28 = 2 days
Oct., 1921.....	4		
Total.....	1252	175	28 Banker's Interest

Compound*

\$840	\$943.82
.06	56.63
50.40 1st yr.	1000.45
840.	20 0090 Int. 4 mo.
890.40	5 002 " 1 mo.
.06	50 " 3 days
53.4240 2d yr.	25 51 " 5 mo. 3 days
890.40	1000 45
943.82 3d yr.	1025 96 Amount
.06	840
\$56.6292	185 96 Comp. Int.

ANNUAL INTEREST*

\$840 at 6% for 1 year = \$50.40, if not paid, it draws interest the remaining time, or 2 years, 5 mo., 3 days. At the end of the second year, another \$50.40 is due and this draws 6% for the time then left of 1 year, 5 months, 3 days. At the end of the third year still another \$50.40 becomes due and draws interest for the little time then left or 5 months, 3 days; combining these periods of time—for the rate and base are the same—(6% and \$50.40), we have

2 yr. 5 mo. 3 days	a total of 6% on \$50.40 for 4 years, 3 mo., 9 days
1 yr. 5 mo. 3 days	or 1539 days. $(4 \times 360) + 150 + 9 = 1539$
5 mo. 3 days	
4 yr. 3 mo. 9 days	

*NOTE: See practice problems page 218.

<u>\$50.40</u>	= 6000 days
<u>12.60</u>	= 1500 days
.252	= 30 days
.050	= 6 days
.025	= 3 days
<u>12.927</u>	Int. on the Interest
<u>172.62</u>	Simple Interest (See 1st illustration) on \$840
\$185.55	Annual Interest or (Simple Interest-interest on interest unpaid.)

The following table is frequently used in finding the exact number of days between any two dates:

TABLE OF TIME DIFFERENCES

FROM ANY DAY OF	TO THE SAME DAY OF THE NEXT											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January.....	365	31	59	90	120	151	181	212	243	273	304	334
February.....	334	365	28	59	89	120	150	181	212	242	273	303
March.....	306	337	365	31	61	92	122	153	184	214	245	275
April.....	275	306	334	365	30	61	91	122	153	183	214	244
May.....	245	276	304	335	365	31	61	92	123	153	184	214
June.....	214	245	273	304	334	365	30	61	92	122	153	183
July.....	184	215	243	274	304	335	365	31	62	92	123	153
August.....	153	184	212	243	273	304	334	365	31	61	92	122
September.....	122	153	181	212	242	273	303	334	365	30	61	91
October.....	92	123	151	182	212	243	273	304	335	365	31	61
November.....	61	92	120	151	181	212	242	273	304	334	365	30
December.....	31	62	90	121	151	182	212	243	274	304	335	365

To find the exact number of days from any day of any month to the corresponding day of any later month, within a year, find the column of the last month directly opposite the line of the first month. From March 3 to June 3 is 92 days; from October 4 to December 4 is 61 days; from February 3 to November 16 is 273+13 or 286 days. In leap years, add one day if February 29 is between the two dates given.

(144)

INTEREST PROBLEMS CHANGED TO PERCENTAGE

If you will observe closely you will discover that interest when computed for a year is in all respects the same as percentage. In fact, interest is percentage with time element involved. When less or more than a year is given, a corresponding change is made in the rate or principal.

Illustration:

Interest on \$480 for 9 mo. at 6% = ?

9 mo. = $\frac{3}{4}$ of a year.

$480 \times (\frac{3}{4} \text{ of } .06)$ or $.04\frac{1}{2} = \$21.60$, Ans.

$(\frac{3}{4} \text{ of } 480)$ or $\$360 \times .06 = \$21.60$, Ans.

Or 6% of \$480.00 = \$28.80, or Interest for one year. $\frac{3}{4}$ (9 months) of \$28.80 = \$21.60, Ans.

Follow these changes mentally:

The Original Problem	The Principal Changed	The Rate Changed
1. 6% on \$ 960 for 4 mo. =	6% of \$ 320 =	2 % of \$ 960 =
2. 6% on 1200 for 3 mo. =	6% of 300 =	$1\frac{1}{2}$ % of 1200 =
3. 4% on 1500 for 6 mo. =	4% of 750 =	2 % of 1500 =
4. 3% on 900 for 10 mo. =	3% of 750 =	$2\frac{1}{2}$ % of 900 =
5. 9% on 1800 for 2 mo. =	9% of 300 =	$1\frac{1}{2}$ % of 1800 =
6. 6% on 2400 for 18 mo. =	6% of 3600 =	9 % of 2400 =
7. 4% on 280 for 15 mo. =	4% of 350 =	5 % of 280 =

(145)

Drill in changing Interest problems to Percentage problems.

1. \$1200 for 8 mo. at 6% = $*(1200 \times 4\%)$ or $(800 \times 6\%) =$
2. 960 for 3 mo. at 4% =
3. 480 for 1 mo. at 12% =
4. 560 for 3 mo. at 3% =
5. 840 for 9 mo. at 8% =
6. 720 for 10 mo. at 6% =
7. 420 for 4 mo. at 9% =
8. 500 for 180 da. at 6% =
9. 240 for 90 da. at 8% =
10. 960 for 40 da. at 9% =

*8 months = $\frac{2}{3}$ of a year; $\frac{2}{3}$ of 6% = 4%; Therefore $4\% \times 1200 =$ Answer.
Or $\frac{2}{3}$ of \$1200 = 800.00; Therefore $800.00 \times 6\% =$ Answer.

All interest table rates are worked out in this way:

SIMPLE INTEREST TABLE
(\$100.00 on a 360-Day Year Basis)

Time	2½%	3%	3½%	4%	4½%	5%	5½%	6%	6½%	7%
1 da.	.0069	.0083	.0097	.0111	.0125	.0139	.0153	.0167	.0181	.0194
2 da.	.0139	.0167	.0194	.0222	.0250	.0278	.0306	.0333	.0361	.0389
3 da.	.0208	.0250	.0292	.0333	.0375	.0417	.0458	.0500	.0542	.0583
4 da.	.0278	.0333	.0389	.0444	.0500	.0556	.0611	.0667	.0722	.0778
5 da.	.0347	.0417	.0486	.0556	.0625	.0694	.0764	.0833	.0903	.0972
6 da.	.0417	.0500	.0583	.0667	.0750	.0833	.0917	.1000	.1083	.1167
7 da.	.0486	.0583	.0681	.0778	.0875	.0972	.1069	.1167	.1264	.1361
8 da.	.0556	.0667	.0778	.0889	.1000	.1111	.1222	.1333	.1444	.1556
9 da.	.0625	.0750	.0875	.1000	.1125	.1250	.1375	.1500	.1625	.1750
10 da.	.0694	.0833	.0972	.1111	.1250	.1389	.1528	.1667	.1806	.1944
11 da.	.0764	.0917	.1069	.1222	.1375	.1528	.1681	.1833	.1986	.2139
12 da.	.0833	.1000	.1167	.1333	.1500	.1667	.1833	.2000	.2167	.2333
13 da.	.0903	.1083	.1264	.1444	.1625	.1806	.1986	.2167	.2347	.2528
14 da.	.0972	.1167	.1361	.1556	.1750	.1944	.2139	.2333	.2528	.2722
15 da.	.1042	.1250	.1458	.1667	.1875	.2083	.2292	.2500	.2708	.2917
16 da.	.1111	.1333	.1556	.1778	.2000	.2222	.2444	.2667	.2889	.3111
17 da.	.1181	.1417	.1653	.1889	.2125	.2361	.2597	.2833	.3069	.3306
18 da.	.1250	.1500	.1750	.2000	.2250	.2500	.2750	.3000	.3250	.3500
19 da.	.1319	.1583	.1847	.2111	.2375	.2639	.2903	.3167	.3431	.3694
20 da.	.1389	.1667	.1944	.2222	.2500	.2778	.3056	.3333	.3611	.3889
21 da.	.1458	.1750	.2042	.2333	.2625	.2917	.3208	.3500	.3792	.4083
22 da.	.1528	.1833	.2139	.2444	.2750	.3056	.3361	.3667	.3972	.4278
23 da.	.1597	.1917	.2236	.2556	.2875	.3194	.3514	.3833	.4153	.4472
24 da.	.1667	.2000	.2333	.2667	.3000	.3333	.3667	.4000	.4333	.4667
25 da.	.1736	.2083	.2431	.2778	.3125	.3472	.3819	.4167	.4514	.4861
26 da.	.1806	.2167	.2528	.2889	.3250	.3611	.3972	.4333	.4694	.5056
27 da.	.1875	.2250	.2625	.3000	.3375	.3750	.4125	.4500	.4875	.5250
28 da.	.1944	.2333	.2722	.3111	.3500	.3889	.4278	.4667	.5056	.5444
29 da.	.2014	.2417	.2819	.3222	.3625	.4028	.4431	.4833	.5236	.5639
1 mo.	.2083	.2500	.2917	.3333	.3750	.4167	.4583	.5000	.5417	.5833
2 mo.	.4167	.5000	.5833	.6667	.7500	.8333	.9167	1.0000	1.0833	1.1667
3 mo.	.6230	.7500	.8750	1.0000	1.1250	1.2500	1.3750	1.5000	1.6250	1.7500
4 mo.	.8333	1.0000	1.1667	1.3333	1.5000	1.6667	1.8333	2.0000	2.1667	2.3333
5 mo.	1.0417	1.2500	1.4583	1.6667	1.8750	2.0833	2.2917	2.5000	2.7083	2.9167
6 mo.	1.2500	1.5000	1.7500	2.0000	2.2500	2.5000	2.7500	3.0000	3.2500	3.5000

To use the interest table multiply the principal by the rate and point off two places.

Use a calculator if convenient to multiply. See appendix for a description of calculating machines.

1. Interest on \$725.40 from June 1 to November 3 at 7% = ?

11—3 Rate on \$100 for 5 mo. (see table) = \$2.9167

6—1 Rate on 100 for 2 days (see table) = .0389

5—2 Total rate 2.9556

$\$2.9556 \times \$725.40 = \$2143.99$

$2143.99 \div 100 = \$21.44$, Ans.

The Banks have interest tables for all principals, and get the desired result by adding partial interests rather than by adding partial rates and multiplying.

Illustration

Int. on \$700.	5 months	7%	by table one	=	\$20.42
Int. on 700.	2 days	7%	by table two	=	.27
Int. on 25.	5 months	7%	by table three	=	.73
Int. on 25.	2 days	7%	by table four	=	.01
Int. on .40	5 months	7%	by table five	=	.01
Int. on .40	2 days	7%	by table six	=	.00
Total					\$21.44

(146)

Using the interest table work these problems:

No.	Prin.		Time	Rate	No.	Prin.		Time	Rate
2	\$ 840	25	3 mo., 3 d.	6 %	9	\$ 725	75	43 d.	6 %
3	17	50	4 days	5 %	10	960	50	1 d.	6½ %
4	4500	00	3 mo., 9 d.	4½ %	11	425	24	27 d.	5½ %
5	6000	00	17 days	2½ %	12	80000	00	5 mo., 20 d.	5 %
6	375	12	7 days	7 %	13	7500	00	1 mo., 10 d.	4 %
7	333	33	4 months	3½ %	14	18000	00	18 d.	3 %
8	250	00	1 mo., 1 da.	6 %	15	8425	50	6 mo., 29 d.	7 %

THE 60-DAY, 6% INTEREST METHOD

Used very generally in business and should be taught in preference to other methods.

It is based on the discovery that the interest on \$1.00 at 6% for

6000 days is \$1.00 (Interest = Principal)

600 days is .10 (Point off 1 place)

60 days is .01 (Point off 2 places)

6 days is .001 (Point off 3 places)

All other periods of time are considered as multiples or factors of 6, 60, 600, and 6000 days, or as combinations of these.

THE METHOD ILLUSTRATED

Principal \$3600

Using Factors of 6000	Using Factors of 600
\$3600 = Interest 6000 days	\$360 = Interest 600 days
1800 = Interest 3000 days	180 = Interest 300 days
1200 = Interest 2000 days	120 = Interest 200 days
900 = Interest 1500 days	90 = Interest 150 days
720 = Interest 1200 days	72 = Interest 120 days
600 = Interest 1000 days	60 = Interest 100 days
Using Factors of 60	Using Factors of 6
\$36 = Interest 60 days	\$3.60 = Interest 6 days
18 = Interest 30 days	1.80 = Interest 3 days
12 = Interest 20 days	1.20 = Interest 2 days
9 = Interest 15 days	.60 = Interest 1 day
7.20 = Interest 12 days	
6 = Interest 10 days	

Using Multiples of 6

\$3.60 = Interest 6 days

7.20 = Interest 12 days

10.80 = Interest 18 days

14.50 = Interest 24 days

Multiples of 60 and 600 days may be used also as opportunity offers. In general, though, use 60 days as your basis; if the time is very long use 600 days or 6000; if very short use 6 days as the basis. If time is in years, months and days, as 3 years 1 month and 14 days, convert it all to days, using 1124 days ($1000+100+20+3+1=1124$) or ($600+300+150+60+12+2$).

\$3600	= 6000 days
600	= 1000 days
60	= 100 days ($\frac{1}{10}$ of 1000)
12	= 20 days ($\frac{1}{5}$ of 100)
1	80 = 3 days ($\frac{1}{2}$ of 6 days \$3.60)
	60 = 1 day ($\frac{1}{3}$ of above)
\$674	40 = Interest 6% for 1124 days

Interest on \$960.40 for 3 mo. 8 days.

\$9	60	40 = 2 mo. (60 days)
4	80	2 = 1 mo.
	96	= 6 days or $\frac{1}{10}$ of 60 days
	32	= 2 days or $\frac{1}{3}$ of 6 days
\$15	68	60 = Interest 6% for 98 days

FOR OTHER RATES THAN SIX

If 3% is wanted take a half of 6%.

If 2% is wanted take a third of 6%.

If $1\frac{1}{2}\%$ is wanted take a fourth of 6%.

If 1% is wanted take a sixth of 6%.

If 5% or 7% is wanted deduct or add a sixth of 6%.

If 4% or 8% is wanted deduct or add a third of 6%.

If $4\frac{1}{2}\%$ or $7\frac{1}{2}\%$ is wanted deduct or add a fourth of 6%.

If 9% is wanted add a half of 6%.

If 10% is wanted divide 6% by 6, and set result one place to the left.

TO ILLUSTRATE:

\$250.00 at 6%, 60 days = \$2.50. If 3% for 60 days is desired, take $\frac{1}{2}$, or \$1.25, and so on as stated above.

What combinations would you use in figuring 6% interest for

2 mo. 10 days	2 years	79 days
3 mo. 12 days	1 year 2 mo.	51 days
5 mo. 6 days	1 yr. 1 mo. 10 days	47 days
5 mo. 15 days	6 months	105 days
1 mo. 3 days	6 days	242 days
5 days	6 years	1 mo. 13 days
9 days	1 year	3 mo. 5 days
95 days	27 days	11 days
24 days	54 days	17 days
1 day	45 days	1 mo. 1 day

CANCELLATION METHOD

This method is used by some, but is not so rapid as the 60-day method. For example, take \$250.00 at 6% for 75 days. Multiply the principal by the rate and number of days. Divide by 360 or 365 if exact interest is required. To cancel we have:

$$\begin{array}{r} 62.50 \quad 5 \\ \$250.00 \times 75 \times .06\% = 3.13 \\ \hline 360 \\ .60 \\ 4 \end{array}$$

(147)

MENTAL DRILL IN INTEREST

1. Interest 6% on—

\$480 for 60 days, 30 days, 20 days, 15 days, 12 days, 10 days.

\$360 for 6 days, 3 days, 2 days, 1 day, 5 days, 4 days.

\$900 for 60 days, 120 days, 180 days, 240 days, 300 days, 360 days.

\$240 for 6000 days, 3000 days, 2000 days, 1200 days, 100 days.

\$720 for 1500 days, 4500 days, 7500 days, 750 days, 450 days.

\$1200 for 27 days, 33 days, 57 days, 63 days, 9 days.

\$1800 for 28 days, 32 days, 58 days, 62 days, 8 days.

(148)

ORAL

(Use 6%)

\$ 480.00 for 15 days	45 days	3 mo.	8 days	1 day
1240.00 for 9 days	30 days	4 mo.	75 days	2 days
500.00 for 10 days	5 mo.	420 days	3 days	1 mo.
24.00 for 4 days	7 mo.	90 days	81 days	33 days
4000.00 for 11 days	3 mo., 9 days	18 days	4 mo., 12 days	750 days

Just as 7 bushels at \$8.00 is the same as 8 bushels at \$7.00, so days and dollars may be interchanged in figuring interest and an advantage often gained thereby.

(149)

ORAL

Illustrating Interchange of Principal and Time

†\$6000 for 111 days	*\$4800 for 3 mo. 14 days
1500 for 219 days	300 for 9 mo. 26 days
750 for 38 days	100 for 5 mo. 5 days
200 for 472 days	9000 for 1 mo. 19 days
90 for 87 days	330 for 764 days

†\$111.00 for 6,000 days equals \$111.00.

*Reduce to days first.

(150)

WRITTEN

(Simple Interest at 6%)

1. \$450.80 for 2 mo. 17 days	11. \$805.00 for 4 days
2. 421.15 for 83 days	12. 1224.83 for 41 days
3. 2040.50 for 3 mo. 7 days	13. 462.72 for 3 mo. 23 days
4. 980.80 for 173 days	14. 1724.00 for 7 mo. 1 day
5. 750.00 for 1 mo. 29 days	15. 8400.00 for 2 yrs. 2 mo.
6. 856.40 for 636 days	16. 6000.00 for 1 yr. 1 mo. 1 day
7. 97.25 for 98 days	17. 4500.00 for 51 days
8. 124.50 for 513 days	18. 1212.00 for 2 mo. 1 day
9. 222.22 for 2 mo. 2 days	19. 13.24 for 1 mo. 1 day
10. 3.00 for 3 mo. 3 days	20. 22.25 for 7 days

(151)

WRITTEN

(Interest at Other Rates Than 6%)

Principal			Time	Rate	Principal			Time	Rate
1.	\$924	80	3 mo., 18 days	5%	\$750	50	33 days	3 %	
2.	37	45	1 mo., 20 days	7%	840	00	1 mo., 24 days	9 %	
3.	1260	18	48 days	5%	74	25	5 months	2 %	
4.	428	15	1 yr., 3 mo.	7%	150	00	3 mo., 1 day	1 %	
5.	560	42	15 days	4%	18	50	75 days	10 %	
6.	345	50	43 days	8%	440	40	90 days	12 %	
7.	312	48	57 days	4%	24	75	36 days	4½%	
8.	112	70	96 days	8%	316	50	12 days	7½%	
9.	91	21	4 days	3%	1540	00	27 days	11 %	
10.	97	50	1 day	9%	500	00	2 mo., 6 days	5 %	

(152)

WRITTEN

(Where Time is to be Found by Compound Subtraction)

Principal		From	To	Rate
1.	\$724 80	May 14	October 11	6 %
2.	1246 75	June 1, 1920	July 7, 1921	6 %
3.	474 20	March 16, 1921	September 24, 1921	7 %
4.	98 40	November 4	December 1	9 %
5.	705 50	January 28, 1920	January 26, 1921	4 %
6.	630 00	May 17	May 30	4½%
7.	1333 33	October 16, 1921	July 6, 1922	3 %
8.	457 25	February 28	March 13	5 %
9.	300 00	May 31	September 5	10 %
10.	1260 00	April 24, 1919	March 6, 1920	2 %

(153)

WRITTEN

(Simple, Banker's and Accurate Interest)

Principal		Rate	Time		Kind of Interest
			From	To	
1. \$ 740	80	6 %	May 3	Oct. 14	Banker's
2. 1560	00	4 %	Mar. 18, 1920	Jan. 11, 1921	Simple
3. 960	00	6 %	June 30	Sept. 2	Accurate
4. 5000	00	8 %	May 2, 1920	July 17, 1921	Accurate
5. 650	00	8 %	Oct. 14	Dec. 27	Banker's
6. 2400	00	3 %	Aug. 23, 1921	Dec. 26, 1922	Simple
7. 1750	00	4 %	July 29	Sept. 11	Accurate
8. 975	50	4 %	Jan. 3, 1920	Mar. 1, 1920	Banker's
9. 800	00	5 %	Dec. 18, 1921	Apr. 14, 1922	Accurate
10. 1450	25	4½ %	July 18	July 25	Banker's

(154)

WRITTEN

(Simple, Banker's and Accurate Interest Compared)

(Find the Difference Between)

Kind of Interest	On		From	To	Rate
1. Simple and Banker's	\$ 720	00	May 21	Nov. 30	6 %
2. Simple and Accurate	640	00	June 30	Dec. 31	7½ %
3. Banker's and Simple	1740	90	Nov. 20, 1921	Jan. 27, 1922	4½ %
4. Banker's and Accurate	550	50	Mar. 3	July 3	4 %
5. Accurate and Simple	45	90	July 1	July 31	6 %
6. Accurate and Banker's	180	00	Mar. 2	Mar. 29	3 %

(155)

MISCELLANEOUS INTEREST PROBLEMS

1. A note of \$840.20, dated May 16th, reading "80 days after date" and "Interest at 7%" was not paid until September 1. How much was paid?

2. If the above note had read "interest at 8% after maturity", how much would have been due?

3. Lawton owned Crontz \$240.50 for goods bought June 20, 1917. On September 16, 1920, the court gave Crontz a judgment against Lawton for the account with interest (simple) at 6% and court cost \$175. How much did Lawton pay?

4. A city issued 4 year bonds for \$500,000 to build a new high school. If the bonds sold at par and drew $4\frac{1}{2}\%$ interest, what did the high school cost the citizens?

5. Find the simple interest on \$24.50 from June 28, 1916, to December 31, 1922, at 5%.

6. Find the simple interest on \$1 from June 24 to November 9, at 8%.

7. Find the simple interest on \$527.96 at $3\frac{1}{2}\%$ from July 5 to November 26.

8. An old colored lady came into a bank on August 30, 1915, and asked for a bag of gold that she left there during the Civil War. After an almost fruitless search, the President remembered a little package that had been in the safe for years. It proved to be her money, \$80, left there April 20, 1863. Had it been drawing 5% simple interest what amount would the old lady have had? Refer to compound interest table and find what she would have drawn if interest had compounded.

9. John's father-in-law bought the young couple a good home for \$7500, John agreeing to pay interest at 7% on the investment. Taxes \$150 a year, insurance \$15 a year, and \$50 for repairs. What did John's house cost him per month?

10. Find the simple interest on \$3000 for 5 days at $4\frac{1}{2}\%$.

(156)

REVERSING THE OPERATIONS

You should have noticed also that every mathematical operation has its exact reverse. Just as Addition has its Subtraction, Multiplication its Division, Involution its Evolution, etc., so Interest has its reverse operations.

In Interest we have six elements to consider, Principal, Rate, Time, Interest, Amount and Proceeds. Having the Rate and Time given and any one other, we are able to find all the remaining terms, or having either the Rate or the Time given and any two of the others we are able to find the missing terms:

To illustrate your equations:

1. Principal $\times$ (rate for the time) = interest
2. Interest $\div$ (rate for the time) = principal.
3. Amount $\div$ (100 + rate for the time) = principal.
4. Proceeds $\div$ (100 - rate for the time) = principal.
5. Interest $\div$ (principal $\times$ rate for 1 day) = time (in days.)
6. Interest $\div$ (principal $\times$ 1% for given time) = rate used.

Substituting figures for the above you have:

1. \$1260 at 5% for 54 days = \$9.45.
2. \$9.45 $\div$.007 $\frac{1}{2}$ (interest on \$1 for 54 days at 5%) = \$1260.
3. \$1269.45 $\div$ \$1.007 $\frac{1}{2}$ (\$1 + .007 $\frac{1}{2}$) = \$1260.
4. \$1250.55 $\div$.992 $\frac{1}{2}$ (\$1 - .007 $\frac{1}{2}$) = \$1260.
5. \$9.45 $\div$.17 $\frac{1}{2}$ (interest on \$1260 for 1 day at 5%) = 54 days.
6. \$9.45 $\div$ \$1.89 (interest on \$1260 for 54 days 1%) = 5%.

Notice that where we wish to find the time, we use 1 day as the basis; to find the principal, \$1 as the basis. (See 2, 5, and 6 above.)

Copy and supply the terms indicated by the question mark.

Principal		Rate	Time	Interest		Amount		Proceeds	
1.	\$800	00	7 %	4 mo., 3 days	?	?	?	?	?
2.	?		6 %	3 mo., 3 days	\$ 32	55		?	?
3.	325	50	?	11 mo., 9 days	24	52		?	?
4.	4500	00	4 $\frac{1}{2}$ %	?	175	50		?	?
5.	1846	24	7 $\frac{3}{4}$ %	1 mo., 5 days	?			?	?
6.	?		5 %	33 days	3	08		?	?
7.	825	00	?	54 days	9	90		?	?
8.	960	00	5 %	?	18	80		?	?
9.	?		4 %	27 days	?		752	75	?
10.		..	8 %	2 mo., 6 days	?			?	?

(157)

GENERAL PROBLEMS

1. How long will it take \$1000.00 to produce \$15.00 at 6%?
2. A certain loan produced \$25.00 interest at 6% for 30 days. What was the amount of the loan?
3. A lends B \$2000.00 secured by a mortgage on his farm. The loan is to run for two years with interest at $6\frac{1}{2}\%$ to be paid at maturity. How much will A collect at the end of the loan?
4. At what rate must \$200.00 be loaned for three years and six months to produce \$56.00?
5. A certain principal produced \$612.00 in four months at 6%. What was the principal?
6. At what rate will \$500.00 yield \$25.00 in 8 months?
7. A speculator invested \$1000.00 in stock on January 1, 1922. On May 1, 1922, he sold the stock for \$1250.00. What rate of interest, per annum, did he receive on his investment?
8. To settle a debt of \$150.00 that had been due for 18 months, I wrote a check for \$161.25. What was the rate per cent. of interest?
9. A merchant gives his banker a 60 day note for such an amount that the proceeds less 6% interest will pay an invoice of \$500.00 less 2%. What is the amount of the note and what was saved by borrowing to pay the bill?
10. Assume that the above merchant could have waited 20 more days before paying the bill but at the same time could not have saved the 2% discount. What per cent. per annum did he earn by borrowing and paying the bill?

(158)

PROBLEMS IN ANNUAL INTEREST

See Illustration and Explanation, page 204

1. Find the annual interest on \$480 at 6% from August 18, 1920, to December 31, 1922.

2. Find the amount due at maturity on a four-year note of \$4000.00 dated May 20, 1916, interest to be paid annually at 4%, but remaining unpaid.

(159)

PROBLEMS IN COMPOUND INTEREST

See Illustration, page 204

1. Find the compound interest at 6% on a \$5200 note given January 11, 1910, and paid June 30, 1914. (Compound annually.)

2. Find the compound interest on \$670.50 at 6% from June 2, 1918, to July 12, 1921. (Compound annually.)

3. What is the interest at 4%, compounded semi-annually on \$500 deposits in a Savings Bank January 1, 1920, and withdrawn December 31, 1921?

4. Work problem three again compounding quarterly at 4%.

(160)

PROBLEMS COMPARING COMPOUND AND ANNUAL INTEREST

1. Find the difference between compound and annual interest on \$8400 from June 18, 1920, to July 31, 1923, at 6%. (Compound annually.)

2. Find the difference between compound and annual interest on \$6500 from January 24, 1915, to December 30, 1919, at 6%.

3. Find the difference between compound and annual interest on \$3000 at 6% from May 20, 1920, to May 20, 1922.

4. Find the simple banker's, accurate, compound, and annual interest on \$800 at 6% from June 20, 1916, to October 20, 1919.

The Use of the Compound Interest Table Illustrated :

Tables on following two pages.

Find the compound interest at 8% on \$864.50 for 4 years 4 months. By referring to the table, 8% column, fourth position (4 years) we find the rate \$1.360489. Multiplying by \$864.50 we have \$1176.14. Add four months interest (\$31.36) we have \$1207.50, the **amount**.

$$\$1207.50 - \$864.50 = \$343 \text{ Compound Interest.}$$

(161)

PROBLEMS IN COMPOUND INTEREST

(Use tables shown on pages 220 and 221.)

Principal			Rate	Time	Compounded
1.	\$ 725	80	6%	7 years	Annually
2.	460	20	8%	3 years, 6 mo.	Annually
3.	375	80	7%	12 years	Annually
4.	1500	00	4%	40 years	Annually
5.	2750	00	3%	5 years, 6 mo.	Semi-annually

6. What is the Compound Interest on \$10000.00 for 6 years with interest at 6% compounded annually?

7. What is the amount due at maturity on a one-year note for \$2000.00 if the interest is compounded quarterly at 4%?

8. Find the amount collected on a debt of \$200.00 due January 1, 1922 if interest is compounded on it semi-annually, at 5%, since January 1, 1917.

COMPOUND INTEREST TABLE

Showing what \$1.00 will earn at compound interest, at various rates per cent. in any number of years, from 1 to 50 inclusive.

Yrs.	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs.
1	1.0100 000	1.0150 300	1.0200 0000	1.0250 0000	1.0300 0000	1.0350 0000	1.0400 0000	1
2	1.0201 000	1.0302 250	1.0404 0000	1.0506 2500	1.0609 0000	1.0712 2500	1.0816 0000	2
3	1.0303 010	1.0456 784	1.0612 0800	1.0768 9062	1.0927 2700	1.1087 1787	1.1248 6400	3
4	1.0406 040	1.0613 636	1.0824 3216	1.1038 1289	1.1255 0881	1.1475 2300	1.1698 5856	4
5	1.0510 100	1.0772 840	1.1040 8080	1.1314 0821	1.1592 7407	1.1876 8631	1.2166 5290	5
6	1.0615 202	1.0934 433	1.1261 6242	1.1596 9342	1.1940 5230	1.2292 5533	1.2653 1902	6
7	1.0721 354	1.1098 450	1.1486 8567	1.1886 8575	1.2298 7387	1.2722 7926	1.3159 3178	7
8	1.0828 567	1.1264 926	1.1716 5928	1.2184 0290	1.2667 7008	1.3168 0604	1.3685 6905	8
9	1.0936 853	1.1433 900	1.1950 9257	1.2488 6297	1.3047 7318	1.3628 9735	1.4233 1181	9
10	1.1046 221	1.1605 408	1.2189 9442	1.2800 8454	1.3439 1638	1.4105 9876	1.4802 4428	10
11	1.1156 683	1.1779 489	1.2433 7431	1.3120 8666	1.3842 3387	1.4599 6972	1.5394 5406	11
12	1.1268 250	1.1956 182	1.2652 4179	1.3448 8882	1.4257 6089	1.5110 6866	1.6010 3222	12
13	1.1380 933	1.2135 524	1.2936 0663	1.3785 1104	1.4685 3371	1.5639 5606	1.6650 7351	13
14	1.1494 742	1.2317 557	1.3194 7876	1.4129 7382	1.5125 8972	1.6186 9452	1.7316 7645	14
15	1.1609 690	1.2502 321	1.3458 6334	1.4482 9817	1.5579 6742	1.6753 4883	1.8009 4351	15
16	1.1725 786	1.2689 855	1.3727 8570	1.4845 0562	1.6047 0644	1.7339 8601	1.8729 8125	16
17	1.1843 044	1.2880 203	1.4002 4142	1.5216 1126	1.6528 4763	1.7946 7555	1.9479 0050	17
18	1.1961 475	1.3073 406	1.4282 4625	1.5596 5872	1.7024 3306	1.8574 8920	2.0258 1652	18
19	1.2081 090	1.3269 507	1.4568 1117	1.5986 5019	1.7535 0605	1.9225 0132	2.1068 4918	19
20	1.2201 900	1.3468 550	1.4859 4740	1.6386 1644	1.8061 1123	1.9897 8886	2.1911 2314	20
21	1.2323 919	1.3670 578	1.5156 6634	1.6795 8185	1.8602 9457	2.0594 3147	2.2787 0807	21
22	1.2447 159	1.3875 637	1.5459 7967	1.7215 7140	1.9161 0341	2.1315 1158	2.3699 1879	22
23	1.2571 630	1.4083 772	1.5768 9926	1.7646 1068	1.9735 8651	2.2061 1448	2.4647 1555	23
24	1.2697 346	1.4295 028	1.6084 3725	1.8087 2595	2.0327 9411	2.2833 2849	2.5633 0417	24
25	1.2824 320	1.4509 454	1.6406 0599	1.8539 4410	2.0937 7793	2.3632 4498	2.6658 3633	25
26	1.2952 563	1.4727 095	1.6734 1811	1.9002 9270	2.1565 9127	2.4459 5856	2.7724 6979	26
27	1.3082 089	1.4948 002	1.7068 8648	1.9478 0002	2.2212 8901	2.5315 6711	2.8833 6858	27
28	1.3212 910	1.5172 222	1.7410 2421	1.9964 9502	2.2879 2768	2.6201 7196	2.9987 0332	28
29	1.3345 039	1.5399 805	1.7758 4469	2.0464 0739	2.3565 6551	2.7118 7798	3.1186 5145	29
30	1.3478 490	1.5630 802	1.8113 6158	2.0975 6758	2.4272 6247	2.8067 9370	3.2433 9751	30
31	1.3613 274	1.5865 264	1.8475 8882	2.1500 0677	2.5000 8035	2.9050 3148	3.3731 3341	31
32	1.3749 407	1.6103 243	1.8845 4059	2.2037 5694	2.5750 8276	3.0067 0759	3.5080 5875	32
33	1.3886 901	1.6344 792	1.9222 3140	2.2588 5086	2.6523 3524	3.1119 4235	3.6483 8110	33
34	1.4025 770	1.6589 964	1.9606 7603	2.3153 2213	2.7319 0530	3.2208 6033	3.7943 1634	34
35	1.4166 028	1.6838 813	1.9998 8955	2.3732 0519	2.8138 6245	3.3335 9045	3.9460 8899	35
36	1.4307 688	1.7091 395	2.0398 8734	2.4325 3532	2.8982 7833	3.4502 6611	4.1039 3255	36
37	1.4450 765	1.7347 766	2.0806 8509	2.4933 4870	2.9852 2668	3.5710 2543	4.2680 8986	37
38	1.4595 272	1.7607 983	2.1222 9879	2.5556 8242	3.0747 8348	3.6960 1132	4.4388 1345	38
39	1.4741 225	1.7872 103	2.1647 4477	2.6195 7448	3.1670 2698	3.8253 7171	4.6163 6599	39
40	1.4888 637	1.8140 184	2.2080 3966	2.6850 6384	3.2620 3779	3.9592 5972	4.8010 2063	40
41	1.5037 524	1.8412 287	2.2522 0046	2.7521 9043	3.3598 9893	4.0978 3381	4.9930 6145	41
42	1.5187 899	1.8688 471	2.2972 4447	2.8209 9520	3.4606 9589	4.2412 5799	5.1927 8391	42
43	1.5339 778	1.8968 798	2.3431 8036	2.8915 2008	3.5645 1677	4.3897 0202	5.4004 9527	43
44	1.5493 176	1.9253 330	2.3900 5314	2.9638 0808	3.6714 5227	4.5433 4160	5.6165 1508	44
45	1.5648 107	1.9542 130	2.4378 5421	3.0379 0328	3.7815 9584	4.7023 5855	5.8411 7568	45
46	1.5804 589	1.9835 262	2.4866 1129	3.1138 5086	3.8950 4372	4.8669 4110	6.0748 2271	46
47	1.5962 634	2.0132 791	2.5363 4351	3.1916 9713	4.0118 9503	5.0372 8404	6.3178 1562	47
48	1.6122 261	2.0434 783	2.5870 7039	3.2714 8956	4.1322 5188	5.2135 8898	6.5705 2824	48
49	1.6283 483	2.0741 305	2.6383 1170	3.3532 7680	4.2562 1944	5.3960 6459	6.8333 4937	49
50	1.6446 318	2.1052 424	2.6915 8803	3.4371 0872	4.3839 0602	5.5849 2686	7.1066 8335	50

COMPOUND INTEREST TABLE

Showing what \$1.00 will earn at compound interest, at various rates per cent. in any number of years, from 1 to 40 inclusive.

Yrs.	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs.
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 430	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 251	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5668 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 933	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5937 425	10
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 816	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 334	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7859 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9453 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 830	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 695	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 238	21.1137 768	32
33	4.2740 3018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1820 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434	38
39	5.5658 9908	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40

EXPLANATION

If the interest is to be compounded semi-annually, find, from the table, the amount of interest at one-half the rate for twice the number of years; if quarterly, at one-fourth the rate for four times the number of years. Thus, 7 years at 9% payable semi-annually is equivalent to 14 years at 4½% payable annually; and 4 years at 6% payable quarterly is equivalent to 16 years at 1½% payable annually.

PARTIAL PAYMENTS

Interest bearing papers are not always paid at maturity nor in one payment. By agreement, they may be paid in many small amounts at such times as the maker of the note may find it convenient, either before or after maturity. Each payment should be endorsed on the back of the note showing the date and amount as indicated in the following illustration:

Received on this note
June 25, 1920,
Ninety-six and 00/100 Dollars, \$96.00

When part payments are made on a note, it is very evident that at some future time a final settlement must be made, and on such principles as will bring no injustice to either party. There are two methods of adjustments. The United States Rule and the Merchant's Rule. Both should be understood; the second is more just and is more popular with business men.

The following problem is shown, solved by both methods, that you may carefully compare the operations:

A 90-day note of \$500, dated May 16, 1920, interest at 6% shows endorsements as follows: Aug. 14, \$125.00; Sept. 30, \$100.00; Nov. 24, \$50.00; Jan. 4, 1921, \$200. What would be paid as the balance on April 1, 1921?

**THE UNITED STATES RULE AND MERCHANT'S
RULE COMPARED****THE UNITED STATES RULE**

By compound subtraction find the time between the date of the note and the first payment, between the first and second payments, etc. Determine all the time periods before beginning the operation.

THE OPERATION: (U. S. RULE.)

1.	1920— 8—14	\$500	= Prin. drawing int. for 2 mo. 28 days
		7 33	= Int. to 1st payment.
	1920— 5—16	507 33	= Amt. due when 1st payment is made.
		125 00	= 1st payment (8—14).
2.	2—28 = \$125	382 33	= Prin. drawing int. for 1 mo. 16 days.
		2 93	= Int. due at time of 2d payment.
	1920— 9—30	385 26	= Amt. due at time of 2d payment.
	1920— 8—14	100 00	= 2d payment (9—30)
	1—16 = \$100	285 26	= Prin. drawing int. for 1 mo. 24 days.
		2 57	= Int. due at time of 3d payment.
3.	1920—11—24	287 83	= Amt. due at time of 3d payment.
	1920— 9—30	50 00	= 3d payment.
	1—24 = \$50	237 83	= Prin. drawing int. for 1 mo. 10 days.
		1 59	= Int. due at time of 4th payment.
4.	1921— 1— 4	239 42	= Amt. due at time of 4th payment.
	1920—11—24	200 00	= 4th payment.
	1—10 = \$200	39 42	= Prin. drawing int. for 2 mo. 27 days.
5.		57	= Int. due at final settlement.
	1921— 4—1	\$ 39 99	= Amt. due at final settlement.
	1921— 1—4		
	2—27 = Balance		

A chart that may help you to better understand the problem.

1.	2.	3.	4.	5.	
+Int. \$7.33	+\$2.93	+\$2.57	+\$1.59	+\$1.57	
\$500. Prin.	= \$382.33	= \$285.26	= \$237.83	= \$39.42	
for 2—28	1—16	1—24	1—10	2—27	
—Payment \$125	—\$100	—\$50	—\$200	—\$39.99 Bal. due	

NOTE: Where the payment does not equal the interest due at time of payment, do not add the interest or deduct the payment. Carry the payment forward to the next payment and combine the time period with the next.

THE MERCHANT'S RULE

By counting days find the time from date of note and each payment through to the date of final settlement—all to the same date.

1. The note (\$500) draws interest from May 16, 1920, to April 1, 1921 or 320 days.

2. The 1st payment (\$125) draws interest from August 14, 1920, to April 1, 1921, or 230 days.

3. The 2d payment (\$100) draws interest from September 30, 1920, to April 1, 1921, or 183 days.

4. The 3d payment (\$50) draws interest from November 24, 1920, to April 1, 1921, or 128 days.

5. The 4th payment (\$200) draws interest from January 4, 1921, to April 1, 1921, or 87 days.

A chart that may help you to better understand the problem. This adjustment takes the form of a book account.

May 16, 1920.	(1) \$500+Int. for 320 days=Amount owed 4/1/21=		\$526	67		
	Aug. 14, 1920.	(2) \$125+Int. for 230 days	=		129	79
		(3) \$100+Int. for 183 days	=		103	05
		(4) \$50+Int. for 128 days	=		51	07
		(5) \$200+Int. for 87 days=			202	90
	Sept. 30, 1920.					
	Mar. 24, '21					
	Jan. 4 '21	Bal. due 4/1/21.	=		(39	86)
				\$526	67	\$526 67

EXPLANATION

The maker of the note is charged with the face of the note plus the interest to maturity, or date of final settlement if not paid at maturity. He is then credited for each payment plus interest for such time as the maker of the note has held the payment.

PROBLEMS

Solve by both Rules unless otherwise directed.

\$2400⁰⁰ CINCINNATI, OHIO, Oct 14, 1918



Ninety days after date I promise to pay to
the order of J. P. Sharp
Twenty-four Hundred and ⁰⁰/₁₀₀ Dollars
at his office
Value received Int. 6%
No. 55 Due Jan 12, 1919 Richard K. Carter

(1) Find balance due on above note at maturity. See indorsements on back (next page).

\$4800⁰⁰ CINCINNATI, June 4, 1920



Six months after date we promise to pay to
the order of W. T. Briggs
Four Thousand Eight Hundred & ⁰⁰/₁₀₀ Dollars
at Fifth-Third Natl Bank
Value received with interest at the rate of 6 per cent per annum
No. 58 Due Dec 4, 1920 W. M. Berning
E. Ohlhauser

(2) Find balance due on No. 58 November 11 Payments on back.

\$40⁸⁰ CINCINNATI, June 16, 1920



On demand after date I promise to pay to
the order of L. H. Davis
Forty and ⁸⁰/₁₀₀ Dollars
at State National Bank
Value received with interest at the rate of 6 per cent per annum
No. 439 Due _____ W. K. Stewart

(3) Find balance due on above November 4.

Jan. 15, 1918 - \$850.00
 Dec. 30, 1918 - \$420.00
 Jan. 4, 1919 - \$1140.00

Dec. 4, 1920 - \$2000.00
 Mar. 18, 1921 - 20.00
 June 16, 1921 - 1800.00

July 5, 1920 - \$1540
 Aug. 12, 1920 8.12
 Sept. 30, 1920 2.25

Payments made on notes of previous page.

 \$650⁰⁰ CINCINNATI, OHIO. Mar. 10. 1918
August 1, 1918. after date I promise to pay to
 the order of H. M. Kern
Six Hundred Fifty & no $\frac{00}{100}$ Dollars
 at Union Savings Bank & Trust Co.
 For value received, without any relief whatever from Validation or Appraisalment Laws
 with 4% per cent. interest from date until paid and Attorney's Fees
 No. **459** Due Aug. 1, 1918 H. A. Kenkel

- (4) Find balance due at maturity on above note.

 \$900⁰⁰ CINCINNATI, Jan'y. 2. 1919
Oct. 30, 1919 after date I promise to pay to
 the order of V. M. Ruwert
Nine Hundred & no $\frac{00}{100}$ Dollars
 at City Hall Bank
 Value received with interest at 5%
 No. **78** Due Oct. 30, 1919 W. O. Crosswhite

- (5) What is the amount due on October 30?

QUESTIONS FOR DISCUSSION

1. What rate of interest does a note draw after date of maturity?
2. How long may a note run before it is outlawed, declared uncollectible? (See Statute of Limitations for your own state.)
3. Who is the maker of No. 459 above? No. 78?
4. Who is Payee No. 78?
5. Suppose there were no payments made on No. 78; how would you find the amount due October 30, 1919, or at maturity?

April 30, 1918 - \$1.50
June 3, 1918 - 250

May 8, 1919 - \$1.00
July 15, 1919 - 150
Aug. 12, 1919 - 200

FOR DISCUSSION

1. If you owed a note and made a part payment of \$25, would you consider it just for the holder of the note to keep your \$25 and use it until the time of the next payment, just because you paid a little less than the interest due?
2. Why does the government forbid him to add the interest and deduct the \$25 under those circumstances?
3. How could both difficulties be avoided?
4. Why should part payments be carefully receipted by endorsement on the back of the note?
5. How may an endorser escape responsibility of payment?
6. What is meant by protest?
7. Can a note past due be discounted at a bank?

CHAPTER XIV

BORROWING CAPITAL

Nearly all business concerns find it necessary at certain seasons or under certain conditions to borrow additional funds. They sometimes borrow indirectly of their customers by having them pay the present value (worth) of what they owe, or directly of other individuals, but more generally they depend upon the banks.

Selling goods on account is equivalent to lending money without interest, unless the selling price includes the interest charge, which it generally does. Selling on account is equivalent to lending without interest, or to lending and charging interest in advance (the interest to be received later, however.) Considered from this standpoint the man who sells on account runs the risk of losing both the principal and the interest. The tradesman who sells for cash keeps his capital for his own use. Less capital, therefore, is required to finance a Cash business than a Credit business.

The offering of conditional trade discounts, with a view to securing prompt payment on goods sold, may justly be considered as an indirect method of borrowing, an effort to get our capital back into the bank for use.

Illustration: If we sell goods for \$8000, terms 2/10 n/60, the \$8000 belongs to our customer for 60 days. When we offer him 2% discount for paying 50 days sooner, we are but borrowing the \$8000 of him for 50 days and permitting him to take the interest in advance. The one who bought the goods and is paying the account calls the interest Purchase Discount; the one who sold calls it Sales Discount.

The conditional discounts offered are generally a little higher than the current rate of interest charged by the banks. This is due to the element of risk-reduction. If Brown owes us \$4000 there is a chance that we lose it altogether, so the discount offered for prompt payment will be current interest plus what we consider it worth to have the risk removed. It is often to the business man's advantage, therefore, to borrow, if necessary, and take advantage of this excess of discount rate over interest rates.

NOTE: Conditional trade discounts are essentially an application of interest, but inasmuch as the rate is always stated without any reference to time except as a limit within which to pay, they are treated under the head of Applications of Percentage. See Chapter X, page 160.

PRESENT WORTH

Business men and accountants prefer to determine values under present date. Values given for future dates must be converted into Present values, the earning power of a dollar or the current rate of interest being used as the determining factor.

HOW THE PRINCIPLE APPLIES

It is evident to all, that any amount of cash in hand is better than someone's promise to pay the same amount one year from today. But how much better? This depends upon the current interest rate. If \$240 will earn 6% interest (\$14.40) in one year it would amount to $\$240 + \$14.40$ or \$254.40. Therefore \$254.40 due one year from today (a future value) is equivalent to \$240 cash in hand (a present value). The Present Worth of any amount due at some future date is such a sum which, if loaned under the current rate of interest, would amount to the value given for that future date. The proceeds of a note or acceptance discounted at a bank is the present worth of the amount due at maturity. The banker has converted a future value into a present value. The Present Worth of a debt past due, if properly secured, is the original debt plus interest at the legal rate (or other rate stated in writing), from maturity to the present time.

To illustrate: If Browning owes us \$360, 60 days past due, he owes $\$360 + 6\%$ interest (the legal rate generally) for 60 days, or \$363.60, i. e., \$363.60 is the Present Worth of \$360, 60 days past due.

The Present Worth of a debt not yet due is a sum just large enough to amount to the debt at its maturity.

To illustrate: If Thompson owes us \$727.20 due 60 days from today, he owes us $\$727.20 \div 1.01$ ($\$1 + \text{Int. on } \$1 \text{ for } 60 \text{ days}$) or \$720.

The business man who is ignorant of the principle of Present Worth, or fails to apply it in his computations is very sure to reach wrong conclusions.

THE PRINCIPLE APPLIED TO TRADE DISCOUNTS

To meet competition in the extension of credit, we are selling on account 60 days. Because of inadequate capital we are obliged to borrow extensively at the bank at 8%. What cash discount can we afford to allow our customers if bills are paid within ten days?

Explanation: $\$100 \div (\$1 - 8\% \text{ Int. for } 50 \text{ days}) .98\frac{8}{9} = \$101.12$.

Let \$101.12 represent both the selling price and the note. The discount at the bank must equal the cash discount allowed, therefore, the selling price and the note are equal.

\$101.12, the amount necessary to borrow at the bank to have \$100 left for use, interest being taken in advance. ($\$101.12 - \$100 = \$1.12$ interest).

$\$1.12 \div \$101.12 = 1\frac{1}{9}\%$, the rate of discount we could afford to offer rather than borrow. Our terms would then be, On a/c, $1\frac{1}{9}/10 \text{ n}/60$, unless we decide to offer a larger inducement because of the risk involved in carrying the account.

If the customer has no funds and cannot take advantage of the $1\frac{1}{9}\%$ discount in ten days, he is generally willing to accept a draft or sign a note for the amount he owes on account, and due when the account is due. This paper can then be used to advantage in borrowing at your bank. (See Bank Discount.)

(162)

PROBLEMS ILLUSTRATING PRESENT WORTH

1. If we sell goods for \$960.00 on 3 months' time, the customer owes \$960.00. How much can we afford to accept today in settlement of the account? As much money as will, if we put it to earning interest or income, produce \$960.00 at the end of 3 months.

Suppose we can use this money and make it earn 9%. \$1 would earn $2\frac{1}{4}\%$ in 3 months, $\$960.00 \div 1.02\frac{1}{4} = \$938.88$. To prove our operation, figure 9% interest on \$938.88 for 3 months and add (interest on \$938.88 at 9% for 3 months is \$21.12) $\$938.88 + \$21.12 = \$960$.

2. If partners O and J decide to admit Z as a third partner, and he invests cash \$4000, O. M. Gault's note \$2000, now three months past due, and I. T. Briggs \$3000 note without interest due in six months, what value should they allow him for the three items? Surely not \$9000. On an 8% interest basis \$2000 + 3 mo. interest (\$40) is \$2040, the present worth of Gault's overdue note. $\$3000 \div \$1.04$ (the amount of \$1 at 8% for 6 mo.) = \$2884.62 the present worth of Briggs' note. $\$4000 + \$2040 + \$2884.62 = \$8924.62$ the present value of Z's investment.

3. I can buy a piano for \$450 cash or \$475 on 6 months time. If I have money on deposit drawing 4%, which terms should I take? How much would I save?

4. Blue owes us \$1240 due in 87 days. What amount can we accept today in payment if money is worth 6% to us?

5. We hold a \$540 note due in 75 days. The note reads, Ninety days after date; is dated October 4 and bears 6% simple interest. What is its present worth if money is worth 8%?

6. A father told his 18 year old son that he could have \$50,000 the day he was 21 or the present worth of \$50,000 at once. The boy wishing to attend college took the second offer. What sum did he receive, allowing 7% simple interest?

7. What sum earning simple interest at $4\frac{1}{2}\%$ will amount to \$5000 in 1 year and 8 months? (Add interest for full time and proceed as before.)

NOTE: You will discover that the Present Worth corresponds with the Principal in figuring Interest and that present worth problems stated in interest terms would read:

Amount $\div$ (\$1.00 + Rate for time given) = Principal.

8. If money is being loaned at 6% and we have to borrow, what conditional discount could we offer for payments within ten days on goods sold on a/c 90 days?

9. By paying his bills promptly and receiving his discounts a business man saved \$1824.50. This is equivalent to 8% on what sum?

(163)

PRESENT WORTH ON A COMPOUND INTEREST BASIS

1. What is the present worth of \$800 due in four years, if money will earn 6% , compounded annually?

EXPLANATION

By referring to the Compound Interest table, the rate on \$1 for 4 years at 6% is found to be \$1.262477; dividing \$800 by \$1.262477 we have \$633.67. If \$633.67 be loaned at 6% compound interest for four years it would amount to \$800.

2. In problem 6 on page 232, what would the son have received had interest been allowed at 7% compounded annually?

3. What sum loaned at 5% interest at the time of a boy's birth will, if compounded annually and the interest reinvested, amount to the cost of a college education, \$5000, when he is 18 years old? (See table.)

4. A man deposited \$50 each January 1 and July 1 for 20 years in a bank which paid 4% compounded semi-annually. What did he draw out at the end of the 20 years? (Figure as a deposit of \$50 yearly for 40 years at 2% compounded annually, add the 2% column rates given in the table together (1 to 40 years) and multiply by \$50.)

BORROWING OF BANKS

Banks are organized for the purpose of dealing in cash, credit and securities; they receive cash on deposit for safe keeping, and make loans to those in need of funds, either collecting interest in advance or at maturity of the loan. Loans are made only on an adequate security basis to those who have a good credit rating; i. e., those who are known to be of good character, steady in their habits, sane in their methods of doing business, and not only potentially able, but willing to pay what they owe.

Occasionally a man of sterling character who owns property or conducts a prosperous business can secure a loan on his personal note made payable to the bank, but generally the bank requires additional security. This may take the form of endorsements, collateral or a mortgage. In the first instance business acquaintances sign the note with the maker, or endorse it, thereby making themselves responsible for payment. In the second instance the notes of others, stocks, bonds or warehouse receipts are attached to what is called a collateral note, or filed separately. In the third instance a mortgage is given on property, generally real estate. If the maker of the note fails to pay, the bank looks to the endorsers, sells the collateral, or proceeds to foreclose on the mortgage.

That a proper credit rating may be determined, certain information is usually required. See illustration on the following page. You will observe that this is a sworn statement, setting forth the exact condition of the Keystone Mfg. Co. at that date. If the report is favorable and the form of security offered is acceptable, the note or other written evidence of the loan is given to the bank, and the amount named therein placed at the borrower's disposal.

FINANCIAL STATEMENT To THE FIFTH-THIRD NATIONAL BANK

(Corporation)

CORPORATE NAME The Keystone Manufacturing CompanyBUSINESS Manufacturers of automobilesMAIN OFFICE Detroit, MichiganLOCATION OF PLANT Detroit, Mich.BRANCHES Cincinnati, OhioSt. Louis, Mo.

FOR THE PURPOSE OF PROCURING CREDIT FROM TIME TO TIME WITH YOU, FOR OUR NEGOTIABLE, PAPER OR OTHERWISE, WE FURNISH THE FOLLOWING AS A TRUE AND CORRECT STATEMENT OF OUR FINANCIAL CONDITION ON December 31 1919 AND HEREBY AGREE TO NOTIFY YOU IMMEDIATELY IN WRITING, OF ANY MATERIALLY UNFAVORABLE CHANGE IN OUR FINANCIAL CONDITION. IN THE ABSENCE OF SUCH NOTICE OR A NEW AND FULL WRITTEN STATEMENT, THIS IS TO BE CONSIDERED AS A CONTINUING STATEMENT, AND THAT OUR PECUNIARY RESPONSIBILITY HAS NOT FALLEN BELOW THE CONDITION HEREIN SET FORTH AND IT IS HEREBY EXPRESSLY AGREED THAT UPON APPLICATION FOR ADDITIONAL CREDIT OR RENEWAL, THIS STATEMENT AND INFORMATION SHALL HAVE THE SAME FORCE AND EFFECT AS IF DELIVERED AS AN ORIGINAL REPORT OF OUR FINANCIAL CONDITION AT THE TIME SUCH ADDITIONAL CREDIT OR RENEWAL IS REQUESTED.

ASSETS				LIABILITIES			
CASH ON HAND		105	00	BILLS PAYABLE (NOTES) FOR MERCHANDISE		5	000 00
CASH IN BANK		7	845 00	BILLS PAYABLE FOR BORROWED MONEY TO OWN BANKS			
BILLS RECEIVABLE OF CUSTOMERS, DUE WITHIN 90 DAYS		6	805 18	BILLS PAYABLE FOR BORROWED MONEY FOR PAPER SOLD			
BILLS RECEIVABLE OF CUSTOMERS, DUE BEYOND 90 DAYS		1	444 82	BILLS PAYABLE TO DIRECTORS OR STOCKHOLDERS			
ACCOUNTS RECEIVABLE, ALL GOOD, OWING FROM CUSTOMERS		23	000 00	OPEN ACCOUNTS NOT DUE		15	000 00
MERCHANDISE, FINISHED, AT ACTUAL PRESENT CASH VALUE		20	000 00	OPEN ACCOUNTS PAST DUE			
MERCHANDISE, UNFINISHED, AT ACTUAL PRESENT CASH VALUE		20	000 00	ACCOUNTS OWING DIRECTORS OR STOCKHOLDERS			
RAW MATERIAL		18	000 00	DEPOSITS AND OTHER TRUST FUNDS			
TRADE ACCEPTANCES OWNED		2	290 00	TRADE ACCEPTANCES PAYABLE		1	107 45
				Accrued Payroll		1	592 55
				Accrued Interest		8	000 00
TOTAL QUICK ASSETS		99	490 00	TOTAL CURRENT LIABILITIES		30	700 00
STOCKS, BONDS AND INVESTMENTS				MORTGAGES AND LIENS ON REAL ESTATE DUE			
MACHINERY AND FIXTURES		98	500 00	BONDS		116	000 00
REAL ESTATE		122	500 00	CHattel MORTGAGES			
ACCOUNTS AND BILLS RECEIVABLE, OWING FROM OFFICERS AND EMPLOYEES				DUE TO CONTROLLED OR ALLIED CONCERNS, For Merchandise			
BILLS RECEIVABLE, OWING FROM DIRECTORS OR STOCKHOLDERS				DUE TO CONTROLLED OR ALLIED CONCERNS, For Advances			
ACCOUNTS RECEIVABLE, OWING FROM DIRECTORS OR STOCKHOLDERS		1	000 00				
DUE FROM CONTROLLED OR ALLIED CONCERNS, FOR MERCHANDISE							
DUE FROM CONTROLLED OR ALLIED CONCERNS, FOR ADVANCES							
NOTES AND ACCOUNTS RECEIVABLE, 6 MONTHS OR MORE PAST DUE		1	235 65	CAPITAL STOCK OUTSTANDING PREFERRED			
Deferred Charges			764 35	CAPITAL STOCK OUTSTANDING COMMON		150	000 00
Goodwill		50	000 00	RESERVES { SURPLUS		12	790 00
				UNDIVIDED PROFITS		40	000 00
				DEPRECIATION		24	000 00
TOTAL		373	490 00	TOTAL		373	490 00

I, HEREBY SOLEMNLY DECLARE AND CERTIFY THAT THE ABOVE IS A TRUE STATEMENT OF OUR BUSINESS.

CORPORATE NAME Keystone Manufacturing Co.DATE SIGNED January 25 1919

BY

(Officer Title Must Be Given)

President

NOTE.—IF YOU HAVE ANY DIFFICULTY IN FILLING IN THE FOREGOING STATEMENTS AND QUESTIONARY, THE CREDIT DEPARTMENT OR ANY OTHER OFFICER OF THIS BANK IS PREPARED TO EXTEND YOU COURTEOUS AND INTELLIGENT SERVICE; IN FACT, WE SOLICIT YOUR CO-OPERATION AND ACQUAINTANCE WITH OUR CREDIT METHODS.

FORMS OF NEGOTIABLE PAPER OFFERED

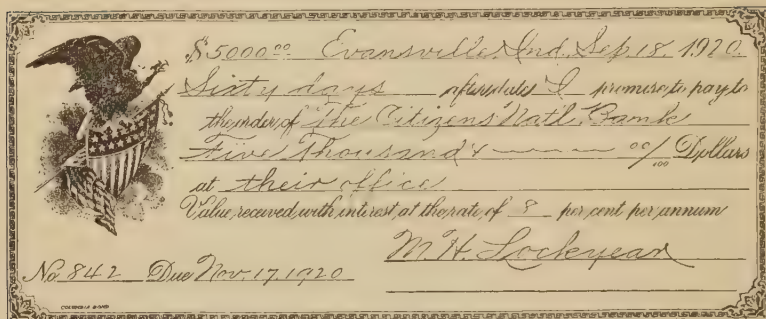


Illustration No. 11, Promissory Note.

Horace S. Mead

Wm. G. Craig

Illustration
 Personal Endorsements
 for Security

This note would be spoken of as Lock-year's note. He is the Maker. Craig and Mead are the Endorsers for security. The bank is the Payee. Some banks require interest to be paid in advance, others at maturity of loan.

The business man may offer for discount some note or acceptance not yet due, which he holds against his customers.

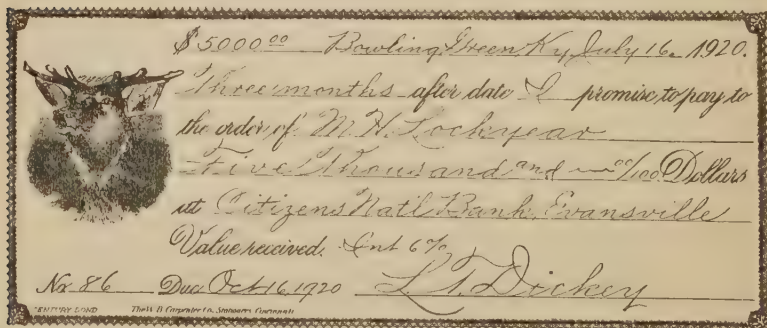


Illustration No. 12, Customer's Promissory Note.

Illustration 12 would be known as Dickey's note and would be endorsed for discount as follows:

*Discounted at the Citizens
National Bank, September 14, 1920,
M. H. Lockyear.*

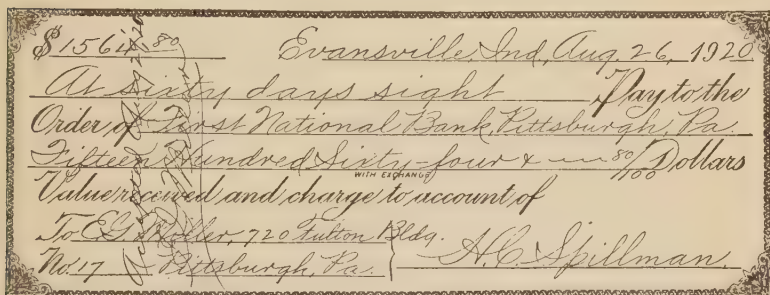


Illustration No. 13, Customer's Acceptance

This would be spoken of as Miller's acceptance; Miller is Drawee, First Nat'l Bank is Payee and Spillman is Drawer.

If the borrower holds no negotiable instruments like the two shown above, he may find it necessary to ask some of his customers to give him a note, or accept a draft drawn by him on them, for the amount of the account owed. To avoid the necessity of doing this, Trade Acceptances are coming into general use. See Illustration No. 14.

1. When does the above draft mature? (Count from date of acceptance.)

2. Why is it necessary to always give the address of the drawee?

A trade acceptance is a form of time draft drawn at the time of sale, and secured by the goods sold. This, rather than the book account, serves as evidence of the transaction, and at the same time affords an acceptable basis for borrowing at the bank. Trade Acceptances can be discounted at a Federal Reserve Bank if the local member refuses to discount them.

TRADE ACCEPTANCE			
No. <u>362</u>	Fort Wayne, Ind.,		April 15, 19__
To <u>John K. Renshaw, 1916 E. 77th St.,</u>	St. Louis, Mo.		
On <u>June 14</u>	Pay to the order of	<u>Ourselves</u>	
(DATE OF MATURITY)			
<u>Six Hundred Twenty and 20/100</u>		Dollars, (\$ <u>620.20</u>)	
The obligation of the acceptor hereof arises out of the purchase of goods from the drawer. The drawee may accept this bill payable at any bank, banker or trust company in the United States which he may designate.			
Accepted at <u>St. Louis</u>	on <u>April 16</u>	19__	
Payable at <u>Fourth Natl</u>	Bank	<u>Johnson, Carvel & Murphy</u>	
<u>John K Renshaw</u>	(SIGNATURE OF ACCEPTOR)	BY <u>C. M. Johnston</u>	
By			

Illustration No. 14, Trade Acceptance.

If collateral security is offered, the note given generally takes the following form:

\$ <u>5000.00</u>	BANGOR, MAINE, <u>Sept. 21,</u> 19__
<u>Six months</u>	AFTER DATE <u>I</u> PROMISE TO PAY TO
MERCHANTS NATIONAL BANK, OR ORDER	
<u>Five Thousand and no/100</u>	DOLLARS
FOR VALUE RECEIVED <u>I</u> HAVING DEPOSITED WITH THIS OBLIGATION	
AS COLLATERAL SECURITY	
<u>100 shares of Mission Wagon Mill Stock</u>	
<u>Certificate No. 4625 - Par value \$100.</u>	
WITH AUTHORITY TO SELL THE SAME WITHOUT NOTICE, EITHER AT PUBLIC OR PRIVATE SALE, OR OTHERWISE, AT THE OPTION OF THE HOLDER OR HOLDERS HEREOF ON THE NON-PERFORMANCE, OF THIS PROMISE, HE OR THEY GIVING <u>me</u> CREDIT FOR ANY BALANCE OF THE NET PROCEEDS OF SUCH SALE REMAINING AFTER PAYING ALL SUMS DUE FROM <u>me</u> TO THE SAID HOLDER OR HOLDERS, OR TO HIS OR THEIR ORDER PAYABLE AT THE MERCHANTS NATIONAL BANK, BANGOR, MAINE.	
<u>Erwin Laque</u>	
<u>154 3/4 49th St</u>	

Illustration No. 15, Collateral Note.

QUESTIONS TO BE ANSWERED BEFORE TAKING UP THE PROBLEMS IN BANK DISCOUNT

1. How does a bank make its profits?
2. To whom will a bank generally loan money?
3. What papers can be deposited at a bank?
4. What papers are left at the bank for collection?
5. What papers can be discounted?
6. What is bank discount?
7. What is a note? A sight draft? An acceptance?
8. What are the six steps in discounting a paper?
9. What is a trade acceptance?
10. What is meant by maturity?
11. What is meant by discount period?
12. Distinguish between interest and amount.
13. How do we distinguish a note from a draft?
14. When was the acceptance shown in illustration due?
15. Name the different kinds of security.
16. If you left Liberty Bonds or railroad stock at the bank as security on a note, who would receive the interest earned on the bonds and the dividends on the stock?
17. Why is the interest charged on a note sold to the bank called discount?
18. If you discounted J. W. Martin's note at the bank, and at maturity Mr. Martin could not pay it, what would the bank do?
19. What is meant by G. A. Deel's note? J. H. Minick's acceptance?
20. Under what conditions will the proceeds be larger than the face of the note?
21. Draw up in proper form a promissory note, using your name as maker and Myrtle Ebner as payee. Show two endorsements for security followed by endorsement for discount.
(Reference may be made to any text on Bookkeeping or Business Law.
"20th Century Bank Accounting" is especially recommended.)

PROCEDURE IN FIGURING BANK DISCOUNT

1. Have the paper approved by the bank.
2. Add interest (if interest bearing) for full time.
3. Find date of maturity.

NOTE: Count by months if paper reads months, by days if it reads days. Refer to illustrations of two notes on page 236, and also draft on page 237, and explain how the maturity dates are found. The latter you count from date of acceptance by Miller.

4. Count days from day discounted to maturity. (This gives the discount period.)

5. Compute Bank Discount (interest) on the amount (face + interest) for the time yet to run (discount period).

6. Subtract discount from the amount and the result is called Proceeds.

ILLUSTRATION

1. We will suppose that L. T. Dickey's note (see Illustration No. 12) was approved for discount at 8%, September 23, 1920.

(The numbers refer to the steps outlined above.)

- | | |
|---|---|
| <p>\$5000 = Face</p> <p>2. $\begin{array}{r} 75 = 3 \text{ mo. Int., } 6\% \\ \text{Amt.} = 50 \overline{) 75} = (60 \text{ d. Int., } 2 \text{ places}) \end{array}$</p> <p>$\begin{array}{r} 16 \overline{) 916} = 20 \text{ days, } 6\% \\ 2 \overline{) 537} = 3 \text{ days, } 6\% \end{array}$</p> <p>5. 3) $\begin{array}{r} 19 \overline{) 453} = \text{Int., } 6\% \\ 6 \overline{) 484} = 8\%, \frac{1}{2} \text{ greater than } 6\% \end{array}$</p> <p>$25.937 = 8\% \text{ Bank Discount.}$</p> <p>6. $\begin{array}{r} 5075.00 = \\ \\$5049.06 = \text{Proceeds.} \end{array}$</p> | <p>3. The due date is given on the note.</p> <p>4. Counting days from Sept. 23, the date of discount, to Oct. 16, the date of maturity, we have 23 days, the discount period.</p> |
|---|---|

(164)

PROBLEMS IN BANK DISCOUNT

1. We take our 60-day note, \$3000, to the bank, making it payable to them and leave Liberty Bonds, \$4000, as security. The interest rate is 8%. Find the proceeds.

2. We have Green's \$450 acceptance on hand, dated June 4, accepted June 5 and reading at thirty days sight. We discount this paper June 8 at 6%. Find the proceeds.

3. We hold Turbin's 90-day, 6% interest bearing note, \$2560, dated May 24. On June 17 we discount it at 6%. Find proceeds.

4. What would have been the proceeds had Turbin's note been discounted at 8%?

5. L. T. Dickey's note of \$5000 (see illustration, page 236) was discounted September 2 at 6%. Find the proceeds.

6. S. H. Kallenbach's 4-month note, dated June 4, face \$2540.80, interest at 4%, was discounted at 6% June 5. Find the proceeds.

7. E. G. Miller's acceptance of \$1564.80 (see Illustration No. 13) was discounted September 3 at 8%. Find proceeds.

8. C. E. Doner's \$820 note, 3-month, without interest, dated May 23, was discounted July 3 at $7\frac{1}{2}\%$. Find proceeds.

9. C. H. Krehbiel's acceptance for \$846.24, dated October 17, reading "Sixty days from date," and accepted October 18, was discounted by the holder Nov. 3 at 8%. Find proceeds.

10. A 90-day note for \$724.82, dated May 31, was discounted July 24 at 6%. Find proceeds.

(165)

DRAFTS DISCOUNTED

Find the proceeds of the following acceptances:

Face		Payable	Dated	Accepted	Discounted	At	
1. \$	450	50	30 days sight	Feb. 8/20	2/10/20	Mar. 3	6 %
2.	730	90	4 mo. after date	Sept. 9	Sept. 15	Sept. 16	8 %
3.	500	00	3 mo. after sight	July 3	July 5	Sept. 30	7½%
4.	48	24	90 days after date	Jan. 4	Jan. 4	Jan. 4	7 %
5.	4000	00	at 20 days' sight	June 11	June 14	June 20	6 %

CHAPTER XV

BOOKKEEPER'S PROBLEMS IN SETTLING ACCOUNTS

It is generally understood in business that any debt draws interest at the legal rate from date of maturity, and the courts will generally enforce such a claim. Few business men are so exacting as to require interest on items past due, unless it becomes necessary to bring suit in collecting. Occasionally, however, in making final settlements between firms, and often in college and C. P. A. examinations, you may be asked to average an account; to ascertain the exact amount due, including interest on each item since maturity, and allowing interest on all items not due. You may possibly be asked, also, to find the Equated or Average Date, the day when the balance may be paid without interest adjustment or loss to either party.

1. Finding the total amount owed where all items in the account are past due.

R. S. Olsen bought of us as follows (see Olsen's account): If nothing has been paid, what amount does he owe November 12? 6% interest on \$400 for 8 months 26 days (time from maturity February 16 to November 12)=\$17.73. 6% interest on \$250 for 4 months 8 days (time from maturity July 4 to November 12)=\$5.33. 6% interest on \$75 for 3 months 14 days=\$1.30. Total due=\$725+\$24.36 interest=\$749.36, Ans.

R. S. OLSEN

1921									
Jan.	17	a/c 30 d.	400						
June	4	a/c 30 d.	250						
June	28	a/c 30 d.	75						

2. Finding the amount owed where all items but one are past due and that item is subject to a trade discount.

Our account with J. Blair is as follows (see Blair's account):
He wishes to settle in full October 1. What is the account worth?

Int. 6%, Aug. 5 to Oct. 1 on \$2400 = \$22.40 + \$2400 = \$2422.40

Int. 6%, Sept. 29 to Oct. 1 on \$1800 = \$.60 + \$1800 = \$1800.60

\$960 less 2% discount allowed = \$960 - \$19.20 = 940.80

Value of account Oct. 1 = \$5163.80

JAMES BLAIR

1921										
July	6	a/c 30 d.	2400							
July	31	2/10 n/60	1800							
Sept.	26	2/10 n/60	960							

3. Finding the amount owed where some items are not yet due and others are past due.

What would the above account be worth October 16?

Int. 6%, Aug. 5 to Oct. 16 on \$2400 = \$28.40 + \$2400 = \$2428.40

Int. 6%, Sept. 29 to Oct. 16 on \$1800 = \$5.10 + \$1800 = 1805.10

Present worth of \$960 for 1 mo. 9 da. (time to maturity)

= \$960 ÷ 1.0065 (amount of \$1, 1 mo. 9 da. at 6%) = 953.80

Value of account Oct. 16 = \$5187.30

4. Finding the average date, when items are all on one side.

Butler Bros. have purchased of us as follows:

BUTLER BROS.

1921										
May	4	2/10, n/60	\$5000							
July	18	a/c	1600							
Oct.	4	a/c 30 d.	1200							

On Oct. 4 they gave us an 8% interest-bearing note for \$7800.00. What should be the date of the note?

Int. on \$5000 at 6%, July 3 to Nov. 3 (last due date) = \$100

Int. on \$1600 at 6%, July 18 to Nov. 3 (last due date) = 28

Total interest on items due previous to Nov. 3 \$128

$\$128 \div \$1.30$ (interest on \$7800 for 1 day, 6%) = 98, therefore, the note should be dated 98 days back from Nov. 3, the last due date shown in the account or Aug. 26, the average date.

5. Finding the average date when items appear on both sides of the account.

Our account with S. E. Leslie is as follows:

S. E. Leslie

Date	Explanation	L.F.	Date Due	Interest	Amount	Date	Explanation	L.F.	Date Due	Interest	Amount
1921						1921					
May 4	2/10 7/30		6-3	6 67	540 80	May 14	Cash		5-14	4 65	300
June 18	a/c 60 da		5-17		2400	14	Dis. Wabow		5-14	09	6 12
Aug 17	Bal of Int			6 76		June 25	Mdse. ret'd		6-25	69	80
						27	Cash		6-27	8 00	960
							Bal. of Int.				
						Aug 17	Brok. down				6 76
						17	Bal. of a/c				1587 92
				13 43	2940 80					13 43	2940 80

EXPLANATION

Any date may be selected as the focal date—the date to or from which we figure interest on each item—but the last maturity date shown is generally the best.

By figuring interest on each item of the account to August 17, we have a total debit interest of \$6.67 and a total credit interest of \$13.43, a credit balance of \$6.76. The balance of the account is a debit of \$1594.68. If the balance of interest and the

balance of the account are on the same side, it shows that the account was in balance at a date earlier than August 17; if the balances are on opposite sides, then the account will be in exact balance at a date later than August 17.

A. TO FIND THE DATE WHEN THE ACCOUNT BALANCED

Divide the balance of interest by the interest on the balance of the account for one day.

$\$6.76$ (balance of interest) $\div$ $\$.26578$ (interest on $\$1594.68$ for 1 day) = 26 days.

Counting forward 26 days from August 17, we have September 12, the average date, at which time the account could be paid without loss of interest to either party.

$\$1594.68 - \$6.76 = \$1587.92 =$ balance due August 17.

If the account is paid at a date later than the average date, find the balance owed by adding interest to the balance, $\$1594.68$ for the days intervening; to find the balance owed at a date earlier than Sept. 12, find the present worth of the balance, $\$1594.68$, for the number of days indicated.

TO ILLUSTRATE

B. What would be due on the above account November 18?

The time from September 16, the average date, to November 18 is 2 mo. 2 da. $\$1594.68 + \$16.48$ (interest for 2 mo. 2 da.) = $\$1611.16$.

C. What would be due on the same account July 12?

From July 12 to September 16 = 2 mo. 4 days

Interest on $\$1$ for 2 mo. 4 days, $6\% = .010\frac{2}{3}$

$\$1594.68 \div (1.00 + .010\frac{2}{3} \text{ or } 1.010\frac{2}{3}) = \$1577.85$ present worth value of $\$1594.68$ for 2 mo. 4 days.

(166)

PROBLEMS IN ADJUSTMENT OF ACCOUNTS

1. Frank Wright's account on our books is as follows: He sent us a New York draft on June 19 for \$1200. What is the balance remaining unpaid?

Frank Wright

1921										
May	20	2/10 2/30		600						
	20	Text		1420						
June	16	2/10 2/30		800						

NOTE: Balance remaining, after May 20th., items are paid, is to be divided by .98; every time he pays 98 cents he is to receive credit for \$1.

2. If \$250 is received on this account Oct. 30, what will balance the account Nov. 30? Dec. 30?

James T Lee

1921					1921					
Sept	4	a/c 30d		720	Sept	11	Rebate		1020	
Oct.	20	2/10 2/30		150		30	Cash		500	
	30	1/10 2/30		540						

NOTE: Lee gave no directions concerning the application of the \$250 payment, so we will apply it on the items in regular order. What would be due Nov. 30 if Lee had said, "This \$250 check is in payment of invoice of Oct. 20, balance to apply on invoice of Sept. 4?"

1. On what date should Wright pay the bill charged on May 20 to receive the 2% discount? (a) When is the amount due?

2. On what date is the item charged to Lee, on Oct. 20, due?

3.

Blue and Gray

1921									
Jan.	22	Mdse. @c						24675	
	30	Mdse. @ 60d						42580	
May	3	Mdse. @ $\frac{7}{10}$ 90						82560	
	3	Freight						720	

1. What is due on this account June 16?
2. Find the date when no interest need be added in making payment.
4. If \$1000 was sent to Blue and Gray on August 5,
 1. What would be the balance of interest?
 2. What would settle the account October 1?

5.

B. A. Thinker

1921									
July	18	Cash		56040	1921	June	20	Mdse. @ 30d	56040
Aug.	3	Cash		1000			28	Mdse. @c	1200
	15	Mdse		5425	July	31	Mdse. @ $\frac{7}{10}$ 30		53514

1. Find balance of account.
2. Find balance of interest on August 30.
3. Find balance of account due on focal date, Aug. 30.
4. Find the average date.
5. Suppose a check be given Thinker on August 8 in payment of the bill of July 31. Find the amount of the check.

6.

R. V. Busby

1921				1922			
<i>Apr.</i>	<i>6</i>	<i>Mdscr. 7/60</i>	<i>4500</i>	<i>June 15</i>	<i>Cash</i>	<i>1500</i>	
	<i>29</i>	<i>Mdscr. 7/60 7/60</i>	<i>1540</i>	<i>Aug 12</i>	<i>Cash</i>	<i>64080</i>	
<i>May</i>	<i>1</i>	<i>Fort. Chgd</i>	<i>7580</i>				

1. What additional amount would have settled this account Aug. 4?
2. What amount would have settled this account June 28?
3. When could the balance \$3975 have been paid without loss to either party?
4. What amount would have settled the account Oct. 3?

QUESTIONS IN REVIEW

1. What is meant by Terms of Credit?
2. What is meant by Focal Date?
3. What is the Equated Date?
4. Why is interest charged on items overdue?
5. How is the balance owed on an account determined when interest is to be charged and allowed?
6. What is meant by Balance of Interest?
7. How do we determine whether the Equated Date is previous to or later than the Focal Date taken for figuring the interest?
8. Will it make any difference what rate of interest is used?
9. What is meant by "2/10, n/60?"
10. If a part payment of \$600 is made before ten days expire, on an invoice bought on account 2/10, n/60, what credit is allowed?

PARTNERSHIP SETTLEMENT

1. If James and Henry invest \$5 and \$7 respectively in a supply of lemons, sugar and ice and sell \$18.60 worth of lemonade, what amount should each receive at the end of the day's work? Should each receive one-half or should James receive $\frac{5}{12}$ and Henry $\frac{7}{12}$ of the \$18.60?

Though it may often prove unfair, the law holds that unless there is an agreement to the contrary (this agreement should always be in writing) each partner should receive a half of the earnings, in which case James would receive $\$5 + \$3.30$ or $\$8.30$; and Henry would receive $\$7 + \$3.30$ or $\$10.30$. If the profits were divided in the same proportion as the capital invested then James would receive $\frac{5}{12}$ of the \$18.60 or \$7.75 and Henry would receive $\frac{7}{12}$ or \$10.85. To you, which settlement seems more just?

2. If Brown invested \$500 for 30 days and Green invested \$700 for 90 days and the profits from the business were \$435, what part should each partner receive? Would it be fair to give each partner half of the \$435? Why?

Would it be fair to give Brown $\frac{5}{12}$ and Green $\frac{7}{12}$ of the \$435? Why? The relation of their investments is disturbed by the fact that the time periods are different. We must now apportion the profits not on a dollar basis, nor on a day basis, but upon a dollar-day basis.

$$\$500 \text{ for } 30 \text{ days} = \$15000 \text{ for } 1 \text{ day}$$

$$700 \text{ for } 90 \text{ days} = 63000 \text{ for } 1 \text{ day}$$

Therefore Brown and Green should divide the \$435 profit in parts that are to each other as \$15,000 is to \$63,000. Brown would receive $\frac{\$15000}{78000}$ or $\frac{5}{26}$ and Green would receive $\frac{\$63000}{78000}$ or $\frac{21}{26}$ of the profits, or \$83.65 and \$351.35 respectively.

3. We will now suppose that Brown and Green are partners in a larger business, their net profit for the year \$5600, and their investment accounts as follows:

J. B. Brown, Capital.

Date	Explanation	L F	Time	Interest	Amount	Date	Explanation	L F	Time	Interest	Amount
1921 May 1	Withdrew		9mo.	60	1500	1921 Jan. 1	Invested	1 apr	300	5000	
						July 1	"	1/2 apr	60	2000	
							Total Cr. Int.			360	
							Deduct Dr. "			60	
							Bal. of Int.			300	

H. A. Green, Capital

Date	Explanation	L F	Time	Interest	Amount	Date	Explanation	L F	Time	Interest	Amount
1921 Oct. 16	Withdrew		75ds	50	1000	1921 Jan. 1	Invested	1 apr	420	7000	
						Oct. 1	"	90ds	50	2000	
							Total Cr. Int.			470	
							Deduct Dr. "			50	
							Bal. of Int.			420	

To count the days for each investment and withdrawal, multiply the days by the dollars and establish a dollar-day basis, for profit division here would be difficult, so we have used the interest method. We have found 6% interest on the investments and withdrawals of each partner from the date given to the end of the fiscal period, Dec. 31. Brown's balance of interest is \$300 and Green's is \$400. It is evident, therefore, that they should divide the earnings of the business in the same proportion; i. e., Brown should receive $\frac{300}{700}$'s of the \$5600, or \$2400, and Green should receive $\frac{400}{700}$'s of the \$5600, or \$3200.

1. Copy the above accounts on paper ruled by yourself, enter each partner's share.

A MORE DIFFICULT PROBLEM WORKED OUT

W. A. Shurtleff and W. H. Hubbard are partners. Their accounts show investments and withdrawals as follows, and a net profit for the year of \$5600.

W. A. Shurtleff, Capital

Date	Explanation	L. F.	Time	Interest	Amount	Date	Explanation	L. F.	Time	Interest	Amount
1921						1921					
Aug. 31	Withdrew		4 mo	30	1500	Jan. 1	Invested		1 yr	300	5000
Dec. 31	Bal. of Int.			328		July 7	"		5-24	58	2000
31	Net Cap.				8303.58		Total Contrib. \$5000				
							Deducted " 30				
							Balance 538				
						Dec. 31	Net Profit				2503.58
					9503.58						9503.58
						1922					
						Jan. 1	Net Capital				8003.58

W. H. Hubbard, Capital

Date	Explanation	L. F.	Time	Interest	Amount	Date	Explanation	L. F.	Time	Interest	Amount
1921						1921					
Nov. 1	Withdrew		2 mo	40	4000	Jan. 1	Invested		1 yr	420	7000
Dec. 31	Bal. of Int.			445.67		Oct. 14	"		2-17	2567	2000
31	Net Cap.				8096.42		Total Contrib. \$7000				
							Deducted " 420				
							Bal. of 40567				
						Dec. 31	Net Profit				3096.42
					44567 12096.42						44567 12096.42
						1922					
						Jan. 1	Net Cap.				8096.42

EXPLANATION

Having calculated the interest on each item from date given to December 31 (the end of fiscal period), we find that Shurtleff's balance of interest (Cr. Int. \$358—Dr. Int. \$30) is \$328, and Hubbard's balance of interest (\$445.67—\$40) is \$405.67. If we divide each balance of interest by \$.06 (the interest on \$1 for 1 year), we have \$5466.67 and \$6761.17 respectively, the average daily capital invested by each. We can now divide the \$5600 net profit on either an interest or capital basis, for they are in like relation to each other. The interest figures being smaller are preferred.

$\$328 + \$405.67 = \$733.67$ total balance of interest of which Shurtleff has $\frac{328}{733.67}$'s and Hubbard $\frac{405.67}{733.67}$'s, therefore,

* $\$5600 \div \$733.67$ and multiplied by $\$328$, or $\$2503.58 =$ Shurtleff's share. $\$5600 \div \$733.67$ and multiplied by $\$405.67$, or $\$3096.42 =$ Hubbard's share.

*Where there are two operations, one a division and the other a multiplication, perform the multiplication first and the division last. This will avoid needless fractions and very probable errors.

We have now divided the $\$5600$ between the partners in proportion to the capital invested, using the interest method, a method that is short and at the same time very fair to the partners.

An agreement is sometimes made between the partners that each shall maintain an equal part of the firm's capital, and that if any partner fails to do so, interest is to be charged to his account at a specified rate and the other partners credited accordingly. That no injustice be done any partner or a premium placed on withdrawing capital, this rate should approximate very closely the per cent. of net profit earned on the total capital invested, but should not be less than the legal rate.

EXPLANATION

In the Shurtleff and Hubbard problem previously shown, Shurtleff's interest balance was $\$328$ and Hubbard's $\$405.67$, a total of $\$733.67$, one-half of which would be $\$366.83$, the balance of interest each should have had to be equal partners. It is evident then that if 6% is what the business is earning, we can easily allow for the difference in capital by charging Shurtleff $\$38.83$, the amount of interest he falls short of the half, and crediting Hubbard for the same amount, or his excess over the half.

$$\$405.67 - 366.83 = \$38.84 \text{ (within 1 cent)}$$

$$366.83 - 328.00 = 38.83$$

If this adjustment is made, then the net profit may be divided equally.

(167)

PROBLEMS IN PARTNERSHIP SETTLEMENT

1. Labor, Think and Rest are partners with investments as follows: Jan. 1, Labor invested \$9600. Think \$7500 and Rest \$5000. On May 25 Think invested \$2000; on July 3 Rest invested \$3000; on Nov. 1 Labor withdrew \$2500. Their earnings for the year were \$4800 after all expenses and their salaries had been paid. Make proper division of profit by the interest method.

2. W. Harding, W. Wilson and H. Taft are partners in the American Adjustment Co. Harding invested, Jan. 1, \$15,000, and on August 3, \$12,000. On Nov. 5 he withdrew \$8000. Wilson invested, Jan. 1, \$25,000. On March 4 he withdrew \$20,000. Taft invested, Jan. 1, \$8400 and on July 12, \$16,200. Their net profit for the year is \$7500. Make adjustment by interest method.

3. Mary Just, Alice Thrift and Clara Skillful formed a partnership to conduct a millinery business. Mary Just invested \$1200 on Jan. 1 and withdrew \$150 July 3, also \$240 Aug. 7. Alice Thrift invested \$900 Jan. 1, and \$750 June 20. Clara Skillful invested \$1800 Oct. 14. Their net earnings for the year were \$3520. Make proper distribution of profit on an interest basis. (Use 10%.)

4. If in problem No. 3 they had agreed to allow each other 10% interest on all capital in excess of one-third of the total, and divide the \$3520 equally, how would you make the distribution?

NOTE: Add the three interest balances, divide the sum by 3, find the three differences between each partner's balance and the $\frac{1}{3}$ she should have had. Charge those having less than the required $\frac{1}{3}$ and credit those having more, then divide the \$3520 equally.

5. If Harding, Wilson and Taft (see problem No. 2) had agreed to allow interest on excess investments at the rate of 8%, and to divide the net earnings equally, what would each partner have in the business after the adjustment?

CHAPTER XVI

BUSINESS OWNERSHIP

Business involves the investment and use of property. The value of the property invested is known as the proprietorship. If ownership is vested in a single individual the business is spoken of as a sole-proprietorship; if two or more persons have merged their property-interests and ability for mutual profit, the business is known as a partnership; if the property invested ceases to belong to the individual and becomes the property of an artificial person created by the State, the business is referred to as a Corporation.

I. The Sole-Proprietorship—a business owned and controlled by one person—offers the owner these advantages: He is independent; his property interests are definite and determined; he is free from fear of disagreement, jealousy or unfaithfulness of others; he is free from hampering agreements; can set his own business standards; make his own decisions, and all the profit he makes is his own.

The disadvantages of the Sole-Proprietorship are these: The owner must depend on hired help or agents to assist him, and too often these persons are not vitally interested in building up the business; in case of illness the business is without direction; the capital, credit and personal force behind the business are so limited that progress against the competition of larger firms is quite impossible.

II. The Partnership offers the following advantages: An increase in capital and credit; added ability; and the safeguard arising out of consultation and discussion in matters of administration.

The disadvantages and limitations of a Partnership are these:

1. Its duration is uncertain. The death of a partner, expiration of the agreement, transfer of interest, or financial failure of any partner dissolves the partnership.

2. It is difficult to find more than a very few men who are both able and willing to take the risk of joining in a partnership agreement, and for this reason, the possible capital power of a partnership is limited.

3. In a partnership each partner is responsible for all the debts of the firm to the extent not only of what he has invested in the partnership, but to all his other property holdings also.

4. A man cannot as a rule borrow money on his interest in a partnership. He has nothing to prove a definite ownership to any particular property.

5. The partners have equal authority regardless of amount invested, the owner of the smallest interest often controls the business.

6. There is no higher authority, except the courts, to whom they can go for adjustment of differences.

7. Release from a partnership agreement is not easily secured.

III. The advantages of a Corporate Form of business ownership are these:

1. Practically a perpetual existence. Neither the death of a shareholder nor the transfer of stock affects the corporate existence.

2. Equal financial responsibility. No shareholder can lose more than the par value of the stock he agrees to take, if an original subscriber; or the amount paid for the stock, if purchased from a previous holder; with this exception that holders of stock in a national bank are responsible for double the par value of stock purchased.

3. The certificates of stock can generally be used as security for a loan, without loss of dividend.

4. Capital can be increased indefinitely. \$100,000,000 corporations are numerous. Comparatively few of the stocks listed on the New York Exchange are in amounts less than \$1,000,000. The number of stockholders may vary from the three or five required by different states for incorporation, to the more than 100,000 stockholders of the U. S. Steel Corporation.

5. A corporation, being under strict governmental control, affords a greater degree of safety to shareholders than partners in a partnership can have.

6. Corporate ownership makes it possible to issue bonds for certain legitimate purposes, and to list either stocks or bonds on the Exchange as investment opportunities.

7. It offers to all the opportunity of investment, with the minimum of individual risk and little or no responsibility of management.

Some of the disadvantages of holding stock in a Corporation:

1. The holders of over half of the stock can, by virtue of control, elect themselves to offices, fix their salaries and so manage affairs that dividends are low or impossible. This influences the small investor to sell at a loss, generally to the advantage of those in control who secretly make the purchase. This is called a Freeze-out or Squeeze-out.

2. The management of affairs is often in the hands of those who have little interest in the business beyond receiving a salary.

3. Through contracts made with companies organized on the outside by those in control of affairs, profits belonging to the parent concern with its many stockholders are often diverted to subsidiary firms owned by the few, who thereby increase their profits unfairly.

4. Heavy incorporation fees and special corporation taxes cannot be escaped.

HOW EACH IS ORGANIZED AND DISSOLVED

1. A Sole-Proprietorship business begins and ends at the pleasure of the individual. His interest in the business is evidenced by his capital account in the ledger, which is credited for investments and profits and debited for withdrawals and losses. The balance of the account shows the proprietor's Present Worth or Net Capital. If the Liabilities (what he owes) are greater than his Assets (what he owns) he is insolvent and his capital account will show a debit excess.

The first section of the following account shows a net profit, the second section shows a net loss, and the third shows that the whole investment, plus \$245.50 belonging to others, has been lost.

W. H. Hunter, Capital

1920	Nov. 12	Withdrew	C15	1500	1920	Jan 1	Invest.	J1	8400
	Dec 31	Net Cap.		1432520		June 5	"	C8	5000
						Dec 31	Net Profit for year	J14	242520
				1582520					1582520
1921	Dec 31	Net Loss for year	Jur	832520	1921	Jan 1	Net Cap		1432520
	31	Net Cap		6000					
				1432520					432520
1922	Dec 31	Net Loss for year	J65	624550	1922	Jan 1	Net Cap		6000
						Dec 31	Net Insol- vency		24550
				624550					624550
1923	Jan 1	Net Insol- vency		24550					

The value of all the proprietor's property and the amounts owed him, less what he owes others, should agree with the balance of this account when books are properly kept and closed. Compare the following statement with the first section of the account shown above.

Balance Sheet W. H. Hunter's, Dec. 31, 1920.

<u>Assets</u>			
Cash		42580	
Notes Rec. (See Schedule "A")		200	
Accts " " " " "B"		782450	
Mdse. Inv. " " "C"		591895	
Store Fix. " " "C"		56080	
Del'y Eqp't. " " "C"		820	
Total Assets			1575005
<u>Liabilities</u>			
Accts Payable (See Schedule "D")			142485
Present Worth Proprietorship			1432520
W. H. Hunter's Invest. \$13400			
" " " Deduct withdraw 1500			
" " " Net Invest. 11900			
Add Net Profit 242520			
" " Net Capital Dec. 31, 1920		1432520	
		1432520	432520

2. A Partnership is formed by the drawing of an agreement, called articles of Co-Partnership, giving definite information concerning the amount invested, distribution of earnings, salaries, withdrawals, duties, privileges, etc. A copy of this agreement is kept by each partner. The share owned by each partner is shown by the balance of his capital account on the firm's ledger. The sum of the partners' individual interests represents the total capital invested in the business. Each partner's capital account takes the same form as the sole-proprietor's capital account previously shown, and the liabilities, including the total proprietorship, must equal the total assets.

Financial Statement of Doctors Can and Will

<u>Assets</u>		<u>Liabilities</u>	
Cash	425 80	Accts. Payable	246 50
Accts Receivable	4560 24		
Notes Receivable	360	<u>Proprietorship</u>	
X-Ray Outfit	2420 40	W.E. Can, Invest \$4000	
Gas	560 80	" " " 1/2 Profit 100 59 1/2	
Office Eqpt. misc.	150 00	" " " Net Capital	500 59 3/4
Library	360		
Drugs	56 12	Orl. T. Hill, Invest \$4000	
Stationery	15	" " " 1/2 Profit 100 59 1/2	
		" " " Net Capital	500 59 3/4
	<u>10258 36</u>		<u>10258 36</u>

A partnership may be dissolved by law, court decision, or mutual agreement. For a full discussion of partnership see any good text on Business Law.

3. To form a Corporation, application is made to the State officials for blank Articles of Incorporation. These are filled out by those who are promoting the organization and returned with the required fee for approval. If approved, the papers are signed and returned for record in the county offices. When this has been done shares of stock are issued to those who have invested, officers are elected, and business begun.

The Stock Certificates, issued in from \$1 to \$100 amounts, are evidence of ownership, and can be offered as security or sold as the holder may wish. A Capital Stock or Stock Issued Account in the firm's ledger shows the par value of all stock sold.

The Capital Stock Account plus the Surplus Account represents the Proprietorship.

Capital Stock

					1918 May 15			100 000
--	--	--	--	--	----------------	--	--	---------

Surplus

1918 Dec. 31	For. Div.	5000	1918 Dec. 31		18000
			1919 Dec. 31		12500
			1920 Dec. 31		16400

In the illustration shown, the \$100,000 plus the \$41,900 (Balance of Surplus Account)=\$141,900, the firm's proprietorship. The Surplus Account represents undivided earnings.

Balance Sheet, Utility Artwork Co

<u>Assets</u>		<u>Liabilities</u>	
Cash	142580	Accts. Payable	412482
Accts Receivable	627125	Wages Owed	15040
Materials	1452480		
Machinery & Tools	800050	<u>Proprietorship</u>	
Plant	15000	Cap. Stock \$50000	
Finished Goods	15420	Div. Unpd. 5000.	
Office Furniture	1800	Surplus 329223	
Office Supplies	12510		5829223
	6256745		6256745

A corporation is dissolved by revocation of charter, court decision, adverse legislation, or mutual agreement of stockholders. Upon dissolution, shareholders receive their proportionate parts of the capital and surplus.

PROBLEMS

(168)

SOLE-PROPRIETORSHIP

1. On Jan. 1 G. E. Davis invested, cash, \$4500. On Dec. 31 his books show the following facts: Assets: Cash \$1725.80, Mdse. \$3840.20, Accts. Receivable \$1520.10, Office Equipment \$170.40, Delivery Equipment \$1500, Store Equipment \$400, Supplies \$96.50. Liabilities: Notes Payable \$2000, Accts. Payable \$1124.80.

- a. What is Davis' Present Worth at the end of the year?
- b. Has he gained or lost and how much? (Assets—Liabilities=Present Worth.)
- c. Complete the following capital account for G. E. Davis.

G. E. Davis

1921					1921				
Dec	31	Net Capital			Jan	1	Investment	4500	
					Dec	31	Net Profit		
					Jan	1	Net Capital		

d. If a \$1000 error was made in listing the mdse. (correct amount \$2840.20) what effect would it have on his worth at the end of the year?

e. Show his account as it would then appear.

2. Samuel Prout quits a position which paid him \$125 a month; takes \$1000 which has been earning him 6% interest and opens a store, the operating expense of which will be \$200 a month, the depreciation on property \$150 a year and direct losses \$250. What must his yearly profits on sales be before the investment works to his advantage?

(169)

PARTNERSHIP

1. On Jan. 1 Red, White and Blue each invested \$6000 cash in an auto repair business. At the end of the year their Balance Sheet shows the following facts:

Red, White and Blue. Balance Sheet, Dec. 31, 1921

<u>Assets</u>		<u>Liabilities</u>	
Cash	28625	Notes Payable	6000
Notes Rec.	15020	Accts Payable	398070
Accts Rec.	842450	Red's Present Worth	XXXXXX
Mdse. Inv.	1927250	White's " "	XXXXXX
Fur. & Fix.	78025	Blue's " "	XXXXXX
Store Egpt.	182575		
Del'y Egpt.	5000		
Real Estate	8000		
Office Supplies	8024		
Ins. Prepaid	100		
	XXXXXX		XXXXXX

a. What was the present worth of the firm Dec. 31?
(Assets—Liabilities always equals the Present Worth.)

b. What was the present worth of each partner?

c. If Red had invested \$4000, White \$5000 and Blue \$9000, what change would it show in the present worth of the firm? Of each partner?

d. What rate of income did each earn on the \$6000 invested?

2. Weaver, Baker and Miner sign articles of co-partnership. Weaver invests, cash, \$4000 and real estate, \$8000. Baker invests auto-trucks worth \$5600, and store fixtures worth \$3600. Miner invests mdse. worth \$10,000 and cash \$1550. At the end of one year, the assets and liabilities of the firm are as follows:

Assets: Cash \$275.80, Notes Rec. \$500, Accounts Rec. \$11,840.21, Real Estate \$8000, Delivery Equipment \$5200, Store Fixtures and Equipment \$3400, Mdse. \$15,400, Supplies on Hand \$153.50. (Liabilities next page.)

Liabilities: Notes Payable \$8500, Accounts Payable \$4500, Interest to date on note \$20.50.

- a. What was the original capital of the firm?
- b. What was the capital at the end of the year?
- c. Did they gain or lose and how much?
- d. If each received $\frac{1}{3}$ of the net profit what would his capital then be?

3. Forest, Lake and Hill have a present worth at the end of six months' business of \$75,250.20. Their net profits for the period were \$15,750.15. They invested $\frac{2}{5}$, $\frac{1}{3}$ and $\frac{4}{15}$ respectively. What amount does each now have in the business if profit is divided according to investment?

4. M. N. Marx began the year's business with assets amounting to \$27,450.60 and liabilities \$16,247.80. At the end of the year he owes \$18,287.40 and his net worth is \$8540.25.

- a. Did he gain or lose and how much?
- b. What per cent. of his capital?
- c. Show his capital account for the year.

5. Henry Graff made a net profit in 1917 of \$4842.20; in 1918, \$3725.80, in 1919, \$5250.40 and in 1920 a loss of \$740.50. His resources were then \$29,426.80 and his liabilities \$10,450.11.

- a. What was his original investment?
- b. What per cent. profit did he make the last year?
- c. If his resources, Jan. 1, 1917, were \$12,725.18, what did he owe?

(170)

CORPORATIONS

1. A, B, C, D and E incorporate for the purpose of manufacturing furniture. They agree to take 200, 190, 120, 110 and 80 shares, respectively, at \$100 each and pay cash. They make application for a charter, or letters patent, on blanks furnished by the State officials, asking the privilege of selling \$100,000 worth of stock and establishing a furniture business under the firm name of The A. B. Furniture Co. The request being granted, and their application approved, they pay the incorporation fee, have their charter recorded at the county courthouse, and proceed to do business.

No. <u>17</u>	SHARES \$100.00 EACH	Shares <u>10</u>
This Certifies That <u>James O. Whiteman</u> is the owner of <u>Ten</u> Shares of the Capital Stock of J. A. Whitney & Co. transferable only on the Books of the Corporation in person or by Attorney on surrender of this Certificate. In Witness Whereof, the duly authorized officers of this Corporation have hereunto subscribed their names and caused the Corporate Seal to be hereto affixed at _____ on this <u>third</u> day of <u>August</u> A. D. 19<u>17</u> <u>J. Martin</u> Secretary <u>J. A. Whitney</u> President		

Stock Certificate, Illustration No. 16.

As previously stated, ownership in a corporation is divided into shares, sometimes called stock. These shares or stock may be issued in units ranging from \$1.00 to \$100.00. Most shares, however, are issued in amounts of \$50.00 and \$100.00; probably the \$100.00 share is the most common. As a rule, the par value of the stock is shown on its face as indicated by the illustration here given. You will note that at the top of this illustration you have the words, "Shares \$100.00 each." This does not necessarily imply that the share is worth exactly \$100.00, although this is known as the par value. If the corporation is making large profits and paying large dividends, the share will likely sell for more than \$100.00. If the business is not profitable and small dividends or no dividends are paid, the shares may sell for considerably less than par. When stock sells below par it is said to sell at a discount; when above par it is said to sell at a premium.

Shares are of two kinds, namely: Common Stock and Preferred Stock. The Preferred Stock usually has printed on the face in bold type "Preferred Shares." The preferred shares are usually preferred as to dividend, that is a guarantee of a certain rate of income per year. The income from Common Stock depends on the profits of the corporation.

Study the above form carefully. It is a simple form, yet contains all the information usually found on highly engraved shares. The stub is not here shown, but should contain the same information as contained in the certificate.

If your father owns shares in a corporation, bring one of these shares to class for study and discussion.

QUESTIONS

- a. What is the capital stock of the firm?
 - b. What per cent. of the stock was issued? What per cent. not issued?
 - c. A and B, own what per cent. of the stock issued?
 - d. What is the capital, or present worth, of the firm as organized?
2. At the end of the year, the books of the A. B. Furniture Co. show:

A. B. Furniture Co. Balance Sheet, Dec 31, 1921.

<i>Assets</i>		<i>Liabilities</i>	
<i>Cash</i>	<i>25840.10</i>	<i>Accts Pay.</i>	<i>136450.20</i>
<i>Accts Rec</i>	<i>24500.40</i>	<i>Notes Pay.</i>	<i>50000</i>
<i>Materials</i>	<i>94270</i>	<i>Proprietorship</i>	
<i>Finished Goods</i>	<i>52960.10</i>	<i>Capital Stock \$100000.</i>	
<i>Work in the making</i>	<i>32425.80</i>	<i>Less unissued " 30000.</i>	
<i>Real Estate</i>	<i>15450</i>	<i>Stock issued</i>	<i>70000</i>
<i>Machinery</i>	<i>72800</i>	<i>Div. 10% on stock</i>	<i>xxxxxx</i>
<i>Tools</i>	<i>1850</i>	<i>Surplus</i>	<i>xyxxxx</i>
<i>Deferred Charges</i>	<i>2540.20</i>		
<i>Accrued Assets</i>	<i>47275</i>		

QUESTIONS

- a. What is the present worth of the firm?
 - b. What will be the amount of a 10% dividend on issued stock?
 - c. What was the firm's net profit for the year?
 - d. What amount of the profit will remain as a surplus after the dividends are paid?
 - e. The net profit was what per cent. of the investment?
 - f. Copy the above statement, supply the dividend, surplus and footings.
 - g. What amount of dividend did B receive?
3. The directors of the A. B. Furniture Co. decide to issue the remaining 300 shares of stock as preferred stock, to earn 8%. X buys 150 shares, Y 80 shares and Z 70 shares at \$102½ a share.

QUESTIONS

a. What will the 300 shares bring if brokerage $12\frac{1}{2}c$ a share is deducted by the bank for selling?

b. What dividend will the preferred stock require yearly?

c. If the net earnings for the second year are \$13,720.90 what will be left to surplus after paying the preferred dividend and a common stock dividend of 12%?

d. E and Y each hold 80 shares. Which received the larger dividend and how much?

e. Would you rather have 8% and be more free from risk and the cares of management or take the risk and worry and share what is left after the preferred dividends are paid?

4. X held his 150 shares 2 years, received two 8% dividends and sold the stock at \$120 a share, without brokerage charge.

QUESTIONS

a. What was his profit and income on the 2-year investment?

b. What per cent. on the investment did he make?

c. What rate of income per year did he make on the investment?

5. Financial Statement of the Wells Transfer Co., Dec. 31, 192...:

ASSETS—20 taxicabs, cost \$36,000, valued at $\frac{3}{4}$ cost; four trucks, cost \$1800 each, valued at \$1500 each; tires and miscellaneous equipment, \$1500; office furniture \$250; office supplies \$27.40; cash on hand \$84.25; bank account \$2542.84; rent prepaid \$300; insurance prepaid \$540.

LIABILITIES—Accounts payable \$1724.20; notes payable \$1800; salaries owed \$275.50; interest accrued on note \$92.50; judgment for damages, favor Mrs. Trainer, \$5000; capital and surplus \$29,352.29.

QUESTIONS

a. If the firm was authorized to sell \$40,000 worth of stock and only 240 shares were sold, what is the surplus?

b. If the surplus account at the beginning of the year was \$2750 and the dividend declared for the year was $8\frac{1}{2}\%$, what were the total net earnings for the year?

c. What per cent. was earned on capital invested?

d. What per cent. would have been earned had it not been for the law suit, the expense of which was \$275 and the resulting judgment against them \$5000?

6. The capital stock of a corporation is \$200,000; the surplus \$50,000; the number of shares, 2000, all of which were sold.

QUESTIONS

a. What is the par value of each share?

b. What is the book value of each share?

c. If the dividend declared was $7\frac{1}{2}\%$, what was the amount of the dividend?

d. If the stock was sold originally at \$90 per share, what is the rate of income on investment? (Income divided by investment-cost equals rate of income on investment.)

e. What would be the rate of income on investment if I bought 10 shares of the above stock at \$120, and the $7\frac{1}{2}\%$ dividend rate was maintained?

f. If the man who sold me this stock had received four yearly dividends of $7\frac{1}{2}\%$, what was his profit and income on the investment?

g. If he held the stock for $4\frac{1}{2}$ years, what was his rate of yearly income on investment?

h. If you had money in the bank drawing only 4% , what could you afford to bid on the above stock? (Divide the income on each share by the rate your money is now earning.)

i. How much could a bank afford to bid whose money can be loaned at 8% ?

7. A corporation capitalized at \$100,000 pays a dividend of \$8000 to the stockholders.

QUESTIONS

a. If the par value of each share is \$50 and all of the shares have been issued, how much does each share yield?

b. If Carter paid \$75 each for his 12 shares, what will his dividend be?

c. What rate of dividend will Carter receive?

d. What rate of income on his investment?

e. At what price would we have to buy shares of this stock to return 8% on the investment? (Divide the income per share by .08, the rate desired.)

f. If the corporation bought back 500 shares and was able to pay the same amount of dividend, what increase in dividend rate would result?

g. What increase in income would Carter receive?

8. The assets of the S. Y. Hyde Elevator Co., Dec. 31, 1921, were \$724,840.20; the liabilities, \$119,426.70. The authorized capital stock was \$600,000, of which only \$500,000 had been sold. The Surplus Account, Jan. 1, 1921, was \$54,800; the profits distributed during 1921 were \$82,500.

QUESTIONS

a. What was the net profit for 1921?

b. What was the balance of the Surplus Account Dec. 31, 1921?

c. What was the capital of the firm Jan. 1? Dec. 31?

d. What rate of dividend was paid for 1921?

e. How much did the book value of each share increase during the year?

f. Including book value increase, what profit did Hugh Black make on 15 shares?

g. What rate of income on investment did he receive if he paid \$90 a share?

h. If the remaining stock was distributed to the shareholders in proportion to their old holdings, without cost to them, how much would it decrease the rate of dividend the following year if the same amount were distributed?

i. If the same rate of dividend were maintained, what amount of earnings would have to be distributed?

j. If you have money earning 8%, what would you offer for 20 shares of stock in the above company?

(171)

STOCK AND BOND PROBLEMS

STOCKS

Ownership in a corporation is represented by Stock certificates. These are transferable on the company's books, and dividends are paid to owners as recorded.

Common Stock carries with it the right to vote, hold office and share in the earnings. Preferred Stock releases the holder from responsibility of management and gives him a stated yearly dividend. Both common and preferred stocks may be listed on the Exchange.

Unless the market is being manipulated, the market prices respond naturally to current interest rates and the earnings (both past and prospective) of the business represented. Large dividends, the accumulation of a surplus, and low interest rates tend to raise the price of stocks; low dividends, absence of a surplus, and a high rate of interest tend to lower the price of stocks.

Brokerage is charged by the one who buys or sells stock for others, usually at the rate of $\frac{1}{8}\%$ or $12\frac{1}{2}$ cents per share—8 shares for \$1.

1. What will 72 shares of United Fruit Co. Stock cost at \$104 $\frac{1}{2}$, if brokerage is $\frac{1}{8}\%$?

$$\$104\frac{1}{2} + \frac{1}{8} = \$104\frac{5}{8}$$

$$104\frac{5}{8} \times 72 = \$7533.00$$

or

$$\$104\frac{1}{2} \times 72 = \$7524$$

$$\text{Com. } (\frac{1}{8} \text{ of } 72) = \underline{\quad 9 \quad}$$

$$\underline{\$7533}$$

Find the cost or proceeds of the following purchases and sales:

No.	No. Shares	Kind of Stock	Par	Market	Brokerage	Find
2	96	Gen. Elec. Co.	\$100	123 $\frac{1}{2}$	$\frac{1}{8}\%$	Cost
3	40	Can. Pacific	100	113 $\frac{3}{8}$	$\frac{1}{8}\%$	Cost
4	12	Union Pacific	100	120 $\frac{1}{2}$	$\frac{1}{8}\%$	Cost
5	32	Union Pacific, pfd.	100	65	$\frac{1}{8}\%$	Cost
6	120	Peoples G & C.	100	53 $\frac{1}{2}$	$\frac{1}{4}\%$	Proceeds
7	42	Studebaker	100	78 $\frac{3}{4}$	15c each	Proceeds
8	75	Chase National	100	322	$\frac{1}{10}\%$	Cost
9	4	Elkhorn Coal	50	40	$\frac{1}{8}\%$	Proceeds
10	245	Continental Oil	100	112	$\frac{1}{8}\%$	Proceeds

11. Through my banker I sold 454 shares of Retail Candy Stock at $6\frac{1}{2}$ (par \$10) and bought 50 shares of United Retail Stores Stock at $56\frac{1}{4}$ (par \$50).

a. What was the balance due me, if brokerage was $\frac{1}{8}\%$ on the sale and $\frac{1}{10}\%$ on the purchase?

b. What increase in dividends will I receive if the Candy Stock paid 4% and the Stores Stock pays $7\frac{1}{2}\%$?

c. What increase in rate of income on investment? (The few dollars left over, less than cost of one share, are counted as earning 4%.)

(172)

1. Reproduce this chart, supplying the missing items.

No.	Shares	Kind of Stock	Par	Div. Rate	Amount of Income	Market	Broker- age	Rate on Invest.
1	70	Crucible Steel	\$100	5 $\frac{1}{2}\%$	?	94 $\frac{1}{4}$	..	?
2	124	Ill. Central	100	5 %	?	94	..	?
3	35	Utah Copper	10	4 $\frac{1}{2}\%$	?	4 $\frac{3}{8}$	$\frac{1}{8}\%$	?
4	92	Gen. Mo., Deb	100	7 %	?	73 $\frac{1}{2}$	$\frac{1}{8}\%$	?
5	110	Am. T. & T.	100	(?)	\$660.00	105 $\frac{1}{8}$	..	?
6	(?)	Am. Tob. Co.	100	7 $\frac{1}{2}\%$	112.50	122 $\frac{1}{2}$	$\frac{1}{8}\%$	?
7	82	Arkansas Gas	100	$\frac{1}{10}\%$	?	8 $\frac{1}{8}$	..	?
8	36	Pullman Cars	100	(?)	648.00	96 $\frac{1}{2}$	$\frac{1}{8}\%$	?

NOTE: Dividends are always figured on par value regardless of market fluctuations. A 6% dividend on 200 shares of stock means a \$1200 income. If the stock cost \$75 a share, the purchaser has 8% on his investment; $\$6$ (income per share) $\div$ $\$75$ (cost of share) = 8%; if he paid \$120 for it he has but 5% on his investment; $\$6 \div \$120 = 5\%$.

(173)

2. What can you afford to offer for the following stocks under the conditions given, provided the risks are equal?

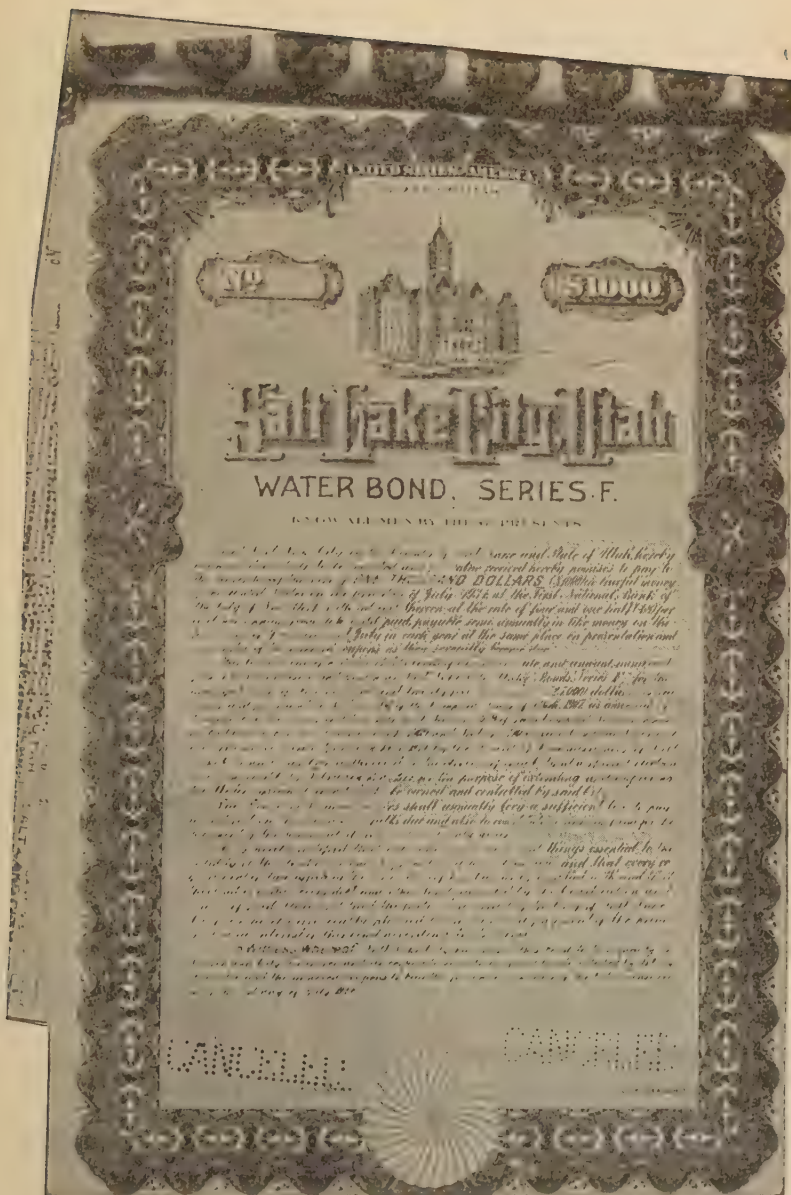
No.	Kind of Stock	Div. Rate	Div. Paid	Par	If Money is Worth
1	Sears Roebuck	4½%	yearly	\$100	6 %
2	B. & S. Mining Co.	2 %	half yearly	10	5 %
3	Westinghouse Mfg.	1½%	quarterly	50	4 %
4	Wells Fargo & Co.	6 %	yearly	100	7 %
5	New York Central	5 %	yearly	100	6½%

NOTE: The yearly income to be derived, divided by the rate money is worth, determines the maximum price for the purchaser and the minimum price for the one who sells. If we have money on deposit at 4½% and can buy stock paying 7% dividends we can afford to bid \$155½; \$7 (the income per share) ÷ .045 (the rate money is earning) gives \$155½.

3. A company is capitalized at \$750,000.00 and has profits set aside for the payment of dividends amounting to \$60,000.00. How much should P. E. Jones receive, who has 50 shares he purchased at \$110.00 each, par value \$100.00? What is his return on investment?

BONDS

Bonds are written agreements to pay on demand or at a certain date, a specified sum of money with interest at regular intervals and at a specified rate. They are issued by the U. S. Government, States, Counties, Townships, Cities, and business corporations. Government and municipal bonds are secured by the stability of the government and its power to levy and collect taxes. Business bonds are generally secured by mortgages on real estate. Bonds are issued as a means of borrowing capital in large sums for immediate purposes, distributing payment over the future years. Coupon bonds have interest certificates attached. These are clipped off and presented for payment by the holder at maturity-date marked thereon. The interest on registered bonds is paid by check issued to the person listed on the books as owner. Bonds are issued in denominations of \$100, \$500, \$1000 or more, and like stocks may be bought and sold.



Coupon Bond.

(174)

The following is a bill for Bonds bought of a Cincinnati Bank Dec. 1, 1920:

\$400 U. S. $4\frac{1}{4}\%$ 3d L. L. Bonds Due	
9/15, 1928, at \$88.12 =	\$352.48
Int. accrued 76 days.....	3.58
Commission.....	.80
	\$356.86

Bill for sales of above bonds June 24, 1921:

\$400 U. S. $4\frac{1}{4}\%$ 3d L. L. Bonds at	
\$91.30 =	\$365.20
Interest.....	4.68
	\$369.88
Less Commission.....	.80
Credit Account.....	\$369.08

QUESTIONS

If a purchaser received dividends, on above bonds amounting to \$12.48, what was his rate of income on investment?

NOTE: Income divided by cost of bond-investment gives the rate of income on investment, unless we take into account the market fluctuations and exact time stock is held. If $87\frac{1}{2}$ is paid for M. K. & T. $4\frac{1}{2}$'s, the rate on investment is $5\frac{1}{7}\%$. $\$4.50$ (the income per share) divided by $\$87.50$ (the cost per share) = $5\frac{1}{7}\%$.

If on May 18 we purchase 80 shares of Twin City Rapid Transit Stock at $75\frac{1}{2}$ (brokerage included), receive a dividend of 5% on July 6 and sell the stock at $79\frac{1}{4}$ on Sept. 6, what is the rate of income on investment?

$79\frac{1}{4} - 75\frac{1}{2} = \$3\frac{3}{4}$ profit on each share.

$80 \times \$3\frac{3}{4} = \$300$ profit made on rising market.

$80 \times \$5 = \$400$ dividend received.

\$700 (total income) for the 2 months, means \$4200 per year.

$\$4200 \div \$6040$ (cost) = $69\frac{1}{2}\%$ + interest on investment.

(175)

What is the rate of income on investment on the following bond investments:

No.	Kind of Bonds	No. Bought	Par	Cost	Div. Rec'd		Time Held	Selling Price
1	Liberty Bonds	1	\$100	87½	\$ 2	12	4 mo.	90
2	Liberty Bonds	4	5000	85½	..	..	10 da.	87½
3	Liberty Bonds	3	100	92	7	50	6 mo.	90
4	U. S. Rubber Co.	12	100	53½	8	50	3 mo.	58½
5	Am. Locomotive	8	1000	85¾	51	20	2 mo.	96½

INSOLVENCY AND BANKRUPTCY

When the assets exceed the liabilities, the business is said to be Solvent; if the assets exactly equal the liabilities, the business is Solvent without Capital; if the liabilities exceed the assets, the business is Insolvent.

Insolvent firms may still continue business and, under favorable credit and market conditions, earn enough to make up the deficiency and again be solvent. If there is no hope of financial recovery, the proprietor or any creditor is privileged to ask the Court for a discharge in bankruptcy. The business is then taken over by the court and placed in charge of a receiver or a trustee who is generally under bond. He may be ordered to conduct the business until it is re-established or to close it out and distribute the resources among the creditors, paying the firm's debts in the order prescribed by law, as follows:

1. All taxes due the United States, State, County or City.
2. The actual necessary expense of keeping the property in good condition from time petition is filed.
3. Expense of any legal proceeding in connection therewith.
4. The expenses of the Receiver or Trustee and his salary for services rendered.
5. Wages owed to those employed by the business during the previous few months, not exceeding \$300 for each person.

6. Judgments and interest thereon, not outlawed.
7. Notes and Accounts Receivable with interest, not outlawed.
8. Capital invested by the proprietors.

When final settlement has been made, the trustee's reports are reviewed, and, if approved, the bankrupt is discharged. He is then free to re-engage in business.

(176)

PROBLEMS IN INSOLVENCY AND BANKRUPTCY

1. Creditors request the Court to appoint a receiver for J. M. Rhodes, who they claim is insolvent and unable to pay what he owes. An auditor finds the condition of affairs as follows:

ASSETS			LIABILITIES		
Cash	\$ 127	80	Accts. Payable	\$5640	25
Accts. Rec.	1824	70	Notes Payable	6000	00
Notes Rec.	725	80	Salaries owed	200	00
Mdse.	3640	00	Taxes owed U. S.	50	20
Office Furn.	600	00			
Dely. Eqpt.	1200	00			
Deferred Items	35	40			

QUESTIONS

- a. Is Rhodes insolvent?
- b. If his capital account at last closing showed a present worth of \$3750.80, what amount has he lost since?
- c. Do you think he could pay his debts if allowed to continue?
- d. If he owed you \$800 would you rather take what he can pay now and close the account, or give him a chance to continue the business and, perhaps, in time pay you all?
- e. What amount should you receive on an immediate settlement, if the receiver first takes out \$500 for his services and there are no other preferred claims?
- f. What is the total loss to his creditors if the court declares him bankrupt?

2. The Stark Mfg. Co. asks the court for a discharge in bankruptcy. Their assets are \$84,292.50 and their liabilities \$125,640.35. They have been closed for three months due to overproduction. Their loss is due to a 40% fall in market value on goods manufactured when prices on labor and material were high. S. O. Blue, appointed receiver, charges \$2000 for his services, and sells the business upon the approval of the court to J. Brady for \$77,000 cash.

QUESTIONS:

a. What per cent. of its liabilities can the firm pay?

b. If their report last year showed Capital \$150,000 and Surplus \$11,240, what was the loss to the owners during the year?

c. What was the firm's total loss for the year?

d. What amount did the creditors lose?

e. Brady reorganized the business as the Brady Woodware Works, turned the property over to the new firm for \$80,000 cash, sold 1400 shares at \$90 each, keeping the other 100 shares for his services. The earnings the first year were \$17,250. What did he make out of the venture if after a 10% dividend was paid, he sold his interest at \$102 per share?

f. If Brady held the property only 4 months what rate of income did he receive on the investment?

3. Dorr Bros. failed in business with liabilities amounting to \$42,000. Their assets were disposed of to highest bidder for \$19,200. If the expense of settlement was \$1200 how much would you receive on a claim of \$240?

4. Carter Bros. were declared bankrupt, owing creditors A \$5600, B \$7500 and C \$6400 and D \$12,500. Their assets consisted of Cash \$300, Merchandise and Furniture inventoried at cost, \$8250, Accounts Receivable due \$3600, and Real Estate at cost \$4800. The Furniture and Merchandise was sold at a 20% discount, Accounts Receivable realized \$3300 and the Real Estate sold at cost \$4800. The expense of selling being \$800, what amount was received by each creditor?

***QUESTIONS FOR DISCUSSION**

1. Name the three forms of business ownership.
2. What are the advantages and disadvantages of owning your own business?
3. What are the advantages and disadvantages of forming a partnership?
4. Why have so many large corporations been organized during the last fifty years?
5. What advantage has a shareholder in a small corporation over a partner in a similar business?
6. What is meant by proprietorship?
7. How is the value of a proprietorship determined?
8. What evidence does a partner have that he has an interest in the business?
9. What does a stockholder in a corporation have as evidence of his investment?
10. What determines the market price of a share of stock?
11. What is meant by Common Stock? Watered Stock? Preferred Stock? Treasury Stock? Capital Stock?
12. Define Par Value, Dividend, Surplus, Charter, Franchise.
13. What is the usual procedure in organizing a corporation?
14. What facts should be set forth in Articles of Copartnership?
15. Distinguish between Capital and Capital Stock.
16. How do we determine the amount we can afford to bid on stocks and bonds offered for sale?
17. What may cause stock to sell below par?
18. Which is usually the safer investment, stocks or bonds? Why?
19. Explain why a government bond is usually safe. Are all government bonds safe?

*Opportunity should be given the class to look these up in the library. Any good text on corporation law will furnish the information desired.

CHAPTER XVII

DENOMINATE NUMBERS

1. To reduce a Simple Denominate Number to higher terms divide by the number of units required to make a unit of the next higher order.

ILLUSTRATION

$$1728 \text{ pts.} \div 2 = 864 \text{ qts.}$$

$$864 \text{ qts.} \div 8 = 108 \text{ pks.}$$

$$108 \text{ pks.} \div 4 = 27 \text{ bu.}$$

2. To reduce a Compound Denominate Number to higher terms divide the lowest of the series by the number of units required to make a unit of the next higher order, carry the result as a fraction, add the result to the number of next higher units given and repeat the operation.

ILLUSTRATION

7 gal. 3 qts. 1 pt. 3 gills = how many gallons?

$$3 \div 4 = \frac{3}{4} \text{ pt.} + 1 \text{ pt.} = 1\frac{3}{4} \text{ pt.}$$

$$1\frac{3}{4} \div 2 = \frac{1\frac{3}{4}}{2} = \frac{\frac{7}{4}}{2} = \frac{7}{8} \text{ qts.} + 3 \text{ qts.} = 3\frac{7}{8} \text{ qts.}$$

$$3\frac{7}{8} \div 4 = \frac{3\frac{7}{8}}{4} = \frac{\frac{31}{8}}{4} = \frac{31}{32} \text{ gal.} + 7 \text{ gal.} = 7\frac{31}{32} \text{ gal., Ans.}$$

3. To reduce a Simple Denominate Number to lower terms multiply by the number of next lower units contained in one unit. of the quantity given.

ILLUSTRATION

$$2\frac{1}{2} \text{ miles} \times 320 = 800 \text{ rods}$$

$$800 \text{ rods} \times 16\frac{1}{2} = 13200 \text{ feet}$$

$$13200 \text{ feet} \times 12 = 158400 \text{ inches.}$$

4. To reduce a Compound Denominate Number to lower terms multiply the highest of the series by the number of units of the next lower contained in it, add in the number of next lower units given, then proceed in like manner to the next of the series.

ILLUSTRATION

4 sq. yds. 7 sq. ft. 3 sq. in. = how many inches?

$$4 \times 9 = 36 \text{ sq. ft.} + 7 \text{ sq. ft.} = 43 \text{ sq. ft.}$$

$$43 \times 144 = 6192 \text{ sq. in.} + 3 \text{ sq. in.} = 6195 \text{ sq. in.}$$

5. To add Compound Denominate Number series arrange the series with all like measures in columns. Add the lowest term column, reduce result to units of the next higher order, carry the result and leave the remainder. Report the operation for each column.

ILLUSTRATION

Add 7 bu.	1 pk.	2 qts.	1 pt.	
3 bu.	2 pks.	1 qt.		
	3 pks.	7 qts.	$1\frac{1}{2}$ pts.	
5 bu.		3 qts.	1 pt.	
				$3\frac{1}{2}$ pts. = 1 qt. + $1\frac{1}{2}$ pts., remainder.
				13 qts. + 1 qt. = 14 qts. = 1 pk. + 6 qts., rem.
				6 pks. + 1 pk. = 7 pks. = 1 bu. + 3 pks., remainder.
				15 bu. + 1 bu. = 16 bu.
Result: 16 bu., 3 pks., 6 qts., $1\frac{1}{2}$ pts.				

6. To subtract Compound Denominate Numbers arrange the two series so that like quantities are in columns. Subtract the lower-term quantities first, borrowing a unit of the next higher order when necessary.

ILLUSTRATION: A man born March 3, 1783, died February 28, 1845. What was his age at death?

$$\begin{array}{r}
 1845-2-28 \\
 1783-3-3 \\
 \hline
 61-11-25
 \end{array}
 \begin{array}{l}
 (3 \text{ months cannot be taken from 2 months so} \\
 \text{a year is borrowed and added, making} \\
 14 \text{ months}-3 \text{ months}=11 \text{ months.})
 \end{array}$$

7. To multiply Compound Denominate Numbers multiply the lower-term units first, reduce to the next higher order, leaving the remainder, and repeat the operation for each quantity of the series.

ILLUSTRATION: A square field is 28 rods, 7 ft., 9 in. each way. Find the perimeter.

28 rods	7 ft.	9 in.
		4
36 in. = 3 ft. no inches.		
28 ft. + 3 ft. = 31 ft. $\div 16\frac{1}{2} = 1 \text{ rod } 14\frac{1}{2} \text{ ft.}$		
112 rods + 1 rod = 113 rods.		

Result: 113 rods, $14\frac{1}{2}$ ft.

8. To divide Compound Denominate Numbers divide the highest-term units first, reduce the remainder to the next lower term, add, and divide as in the first instance.

ILLUSTRATION

4)9 gals. 3 qts. 1 pt. 3 gills

2 and 1 gal. over, or 4 qts.

$(4+3 \text{ qts.}) \div 4 = 1 \text{ and } 3 \text{ qts. over, or } 6 \text{ pts.}$

$(6+1 \text{ pts.}) \div 4 = 1 \text{ and } 3 \text{ pts. over, or } 12 \text{ gills}$

$(12+3 \text{ gills}) \div 4 = 3\frac{3}{4}$

Result: 2 gals., 1 qt., 1 pt., $3\frac{3}{4}$ gills.

ORAL REVIEW QUESTIONS

Drill on these until the whole class is very sure of the answers. Change the order each time.

1. How many square feet in 2 square yards?
2. How many square feet in 2 yards square?
3. How many rods in a mile?
4. How many rods around a half section?
5. How many cu. inches in a gallon?
6. How many lbs. make a bushel of wheat? potatoes? beans?
7. How many feet in a meter?
8. How many ounces in a gram?
9. How many quarts in a liter?
10. What is the relation of the circumference of any circle to its diameter?
11. What is the relation of the area of any square to the circle inscribed therein?
12. How many lbs. (nearly) in a gallon of water?
13. How many lbs. (nearly) in a cu. ft. of water?
14. Give three rules for dividing the circumference of a circle, when the radius and diameter are given.
15. How many gallons (nearly) in a cu. ft.?
16. How is the area of a circle found?
17. How many feet in a fathom?
18. How many acres in a section?
19. How many sections in a township?
20. How many quarts in a bushel?
21. How many quarts in a gallon?
22. How many barley-corns in a foot?
23. Distinguish between ton and tun.
24. How many pounds in a long ton?
25. How many cwt. in a ton?
26. What is the par value of a Franc in U.S. money? Pound?
27. How many square rods in an acre?
28. How many cu. in. in a cu. ft?
29. How many days has each of the 12 months?
30. How many sheets of paper in a ream?

(177)

MISCELLANEOUS PROBLEMS IN MEASUREMENTS

DISTANCE PROBLEMS

1. What will it cost to build a three wire fence around a section of land if posts set 2 rods apart cost 45 cents each and a single wire costs $\frac{3}{4}$ of a cent a foot?

2. The distance around a square field is 146 rods. How many acres does it contain?

3. The circumference of a bicycle wheel is 7 ft. 8 in. If the rider's right foot makes 20 strokes a minute and the wheel makes $2\frac{1}{2}$ turns to each stroke of his foot, how long will it take him to travel a mile?

4. There are 276 rows of corn in a field. If the outside rows are four feet from the edge of the field, and the rows are $3\frac{3}{4}$ feet apart, how many rods wide is the field?

5. A building is 50 feet long. If 5 windows each 3' 6" wide, are to be set into the side wall, an equal distance apart with no window closer to the end of the building than 8 ft., how far apart should the windows be placed?

6. The perimeter of a field twice as long as it is wide, is $760\frac{1}{2}$ rods. Find its dimensions.

7. A boat making $9\frac{1}{2}$ miles an hour, is forced to stop, retrace its course $2\frac{1}{2}$ miles and then resume its journey. If $1\frac{1}{2}$ minutes loss of time is allowed for each turn and the distance between the ports was $15\frac{3}{4}$ miles, how long did it take for the trip?

8. A field 40 rods square is to be fenced off into stock yards each 4 rods square. What will it cost to build the outside and cross-section fences at \$5 a rod?

9. If 5 clothes line poles are set up in a straight line and spaced 7 yards apart, how many feet of clothes line will they afford?

10. How many layers of brick will it take to make a wall 51 feet high if each brick is $1\frac{7}{8}$ inches thick and $\frac{1}{4}$ inch is allowed for mortar?

(178)

SURFACE PROBLEMS

1. A field $40\frac{1}{4}$ rods by $78\frac{1}{2}$ rods is planted to corn. If a bushel plants $7\frac{1}{2}$ acres, what will the seed cost at \$2.50 a bushel?

2. If hills of corn are $3\frac{1}{2}$ feet apart each way, how many hills represent one acre?

3. What will it cost for blocks to pave a street 1520 feet long and $45\frac{3}{4}$ feet wide if each block covers a space $4\frac{1}{2}" \times 8"$ and they cost \$37.50 per M?

4. A rug 12 feet square is placed in a room $16\frac{1}{2}' \times 18'$. What will it cost to polish the remainder of the floor at 30c a square yard?

5. A boy plowed a strip 2 rods wide around a field 48 rods $\times$ 72 rods. What did he earn at \$4.50 an acre?

6. All of a twenty-two story building $204' \times 78'$ inside measure is occupied by the Marx Mfg. Co. How many acres of floor space are occupied?

7. The area of South Dakota is 77615 sq. miles. Sections 16 and 36 of each township are set aside for school purposes. What is the value of the South Dakota school-lands at an average price of \$35.00 an acre?

8. How many sq. yds. in the walls and ceiling of a room $16\frac{1}{2}$ feet by 14 feet, and $8\frac{1}{3}$ feet high? Allow for 1 door $4\frac{1}{2}' \times 6\frac{1}{2}'$ and 4 windows $3\frac{1}{2}' \times 6'$.

9. A tract of land in the form of a right-angle triangle is 310 rods long and 45 rods wide at the base. How many acres does it contain?

10. If this field is divided equi-distant from the ends, in a line parallel with the base, how many acres will each part represent? (Work by inspection of drawing.)

(179)

VOLUME PROBLEMS

1. How many cu. yds. of dirt must be removed for a basement $20\frac{1}{2}' \times 36\frac{2}{3}'$ and $7\frac{1}{2}'$ deep?

2. A can 8 inches square is 18 inches high. How many gallons will it contain? (Exact measure.)

3. A can 8 inches in diameter is 18 inches high. How many gallons will it contain? (Exact measure.)

4. A swimming pool 28 feet wide and 60 feet long is $4\frac{1}{2}$ feet deep at one end and $8\frac{1}{2}$ feet at the other. How many gallons (approximately) does it contain?

5. Approximate the number to tons of water in the above pool.

6. How many bushels (exact measure) of wheat will a box car $48' 6'' \times 7' 8''$ inside measurements hold, if filled to a depth of $4\frac{1}{2}$ feet?

7. A circular grain elevator is 20 feet in diameter (inside) and 42 feet high. Find value of contents when filled with wheat worth \$1.80 a bushel.

8. An oil tank is $38\frac{1}{2}$ feet in diameter and 20 feet deep. Find approximate value of contents at 22 cents a gallon.

9. A vat $2\frac{1}{2}' \times 3\frac{1}{2}' \times 1\frac{1}{2}'$ will hold how many quarts (dry measure)? Liquid measure?

10. A pump chain carrying 24 cups, each $8'' \times 4\frac{1}{2}'' \times 5''$ and making 54 revolutions an hour will lift how many bbls? ($31\frac{1}{2}$ gallons each) in 24 hours if cups carry only $\frac{7}{8}$ full?

(180)

WEIGHT PROBLEMS

1. Find the cost of a ton of mill feed made of equal parts (by weight) of oats and corn, if oats and corn weigh 32 and 56 lbs., and cost 75c and \$1.20 per bushel respectively.

2. If an empty 30 gallon bbl. weighs 26 lbs., what will be the freight at 12c a cwt., on 48 bbls., filled with molasses that weighs 12 lbs. per gallon?

3. A cu. in. of wrought iron weighs .281 lbs. What would be the weight in tons of 4000 straps of iron $2\frac{1}{4}'' \times \frac{1}{4}'' \times 20$ ft?

4. A cu. in. of platinum, weighs .819 lbs. and a cu. in. of aluminum weighs .096 lbs. How many cubic feet of aluminum is equal in weight to a cu. ft. of platinum?

5. A silver dollar weighs 412.5 grains troy. What would be the weight avoirdupois of 5000 silver dollars? Give answer in pounds.

6. The gross weight of a car load of coal was 152425 lbs. The car weighed 24000 lbs. Find the value of coal at \$4.20 long ton.

7. A shipment of sugar weighed 7126 lbs., gross. Find value at $8\frac{1}{3}c$ a lb., if tare was 198 lbs.

8. By using pasteboard cartons instead of wooden boxes a firm reduced the weight of their shipments $\frac{1}{5}$. If all shipments were made in units containing 1 dozen articles, and each unit weighed 76 lbs., when packed in wood, what was the saving per article to their customers if freight rates were \$1.10 a cwt?

9. Find the cost per ream of 720 reams of 32-lb. paper, at $16\frac{2}{3}c$ a lb., if freight on the shipment is 74 cents a cwt.

10. A beef sold at $7\frac{1}{2}$ cents a lb. live weight brought \$93. If $\frac{2}{5}$ of the weight is lost in killing and dressing, at what price per lb. must the meat be sold to bring \$120?

(181)

NUMBER PROBLEMS

1. What will 5 great-gross of pens sell for if sold 3 for 5c?

2. A carload of 3170 melons weighed 35426 lbs. If they sold at \$95 a C. and the freight charge was 80 cents a cwt., what were the net sales?

3. Bought 7250 cabbages at \$4.20 a dozen and sold them at \$42 a C. Find profit.

4. Lincoln in his famous Gettysburg address given in 1863 said, "Four score and seven years ago our fathers" To what year in history did Lincoln refer?

5. Farmer Gray bought 217 lbs. of lime at $37\frac{1}{2}c$ a hundred pounds and 1165 ft. of flooring at \$56.40 a thousand feet in exchange for which he gave 642 eggs at 47c a dozen. What was the balance owed?

6. If paper is bought at 35c a ream, how many sheets can be sold for 5c and a profit of $\frac{1}{5}$ on cost realized?

7. Bought oranges at \$2.75 a C. I wish to gain $\frac{1}{5}$ of cost. At what price will I sell per dozen? Each? How many will I sell for a dime? for a quarter? for a half dollar?

8. Bought candles at \$14.50 a M. Put them in boxes holding one dozen each, which cost \$6.25 a gross. At what price should I sell to gain $\frac{1}{4}$ on cost?

9. A pair of old boots each containing a half dozen sets (6=1 set) of old silver spoons were sold at auction for \$3.00. What was the price paid per spoon?

10. Twenty shoe buttons costing \$1.75 a M, are required for each shoe. What will the buttons cost for an order of 960 pairs of these shoes?

11. 50,000 sheets of paper were bought at a cost of 12 $\frac{1}{2}$ c per pound. If the weight was 30 $\frac{1}{2}$ lbs. to the ream what was the cost? (Count 500 sheets to ream.)

12. How many sheets of paper 8 $\frac{1}{2}$ " x 11" can be cut from 10 reams, the sheets being 22" x 34"?

13. What will it cost to rule 91,000 sheets of paper, at a ruling cost of 2 $\frac{1}{2}$ c per.pound, the weight being 29 $\frac{1}{2}$ lbs, to the ream.?

TABLES USED IN MEASURING

DISTANCE

(ONE DIMENSION)

Linear Measure		Surveyor's Linear Measure	
12 inches, in.	= 1 foot, ft.	7.92 inches	= 1 link
3 feet	= 1 yard, yd.	24 links	= 1 rod
5 $\frac{1}{2}$ yards	= 1 rod, rd.	4 rods	= 1 chain
16 $\frac{1}{2}$ feet	= 1 rod, rd.	66 feet	= 1 chain
320 rods	= 1 mile, mi.	80 chains	= 1 mile
(1 mile = 5280 ft.)			

MISCELLANEOUS: 3 barleycorns = 1 inch; 4 inches = 1 hand; 6 feet = 1 fathom; 9 inches = 1 span; 3 feet = 1 pace; 1.152 $\frac{2}{3}$ common miles = 1 geog. mi.; 3 geog. mi. = 1 league.

AREA

(TWO DIMENSIONS)

Square Measure

144 sq. in. = 1 sq. ft.

9 sq. ft. = 1 sq. yd.

 $30\frac{1}{4}$ sq. yds. = 1 sq. rod.

160 sq. rds. = 1 acre, A.

640 acres = 1 sq. mile or section.

36 sq. mi. = 1 township, twp.

Diagram of a Township and a Section

All sections in a township are numbered as follows:

Twp. 107 N. R. 60 W

, $\frac{1}{36}$ of a Twp. Section 8

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36



N. $\frac{1}{4}$ Section (320 A.)		
$\frac{1}{2}$ mi.	W. $\frac{1}{2}$ of S. E. $\frac{1}{4}$	N. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$
S. W. $\frac{1}{4}$ (160 A.)	S. E. $\frac{1}{4}$ (80 A.)	S. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$

----- 1 mi. -----

Sections 16 and 36 of each township were set aside in the western states for the support of schools and are called school sections.

Townships are numbered from a given point, and the parts of a section are known by fractional parts. To locate the S. W. $\frac{1}{4}$ of Section 8, Twp. 107N., Range 60W., one should go northward from the surveyor's starting point, 107 rows of townships or 6×107 miles, then westward 60 rows of townships or 360 miles. This would locate the township desired. Then by number, see chart above, Section 8 would be found, and the southwest fourth taken. Very few sections are exactly 640 acres, due to the convergence of meridian lines and inaccurate surveys. Deeds generally read "... acres more or less."

VOLUME, CAPACITY, AND WEIGHT

Cubic Measure

1728 cu. in. = 1 cu. ft.

27 cu. ft. = 1 cu. yd.

128 cu. ft. = 1 cord.

24 $\frac{3}{4}$ cu. ft. = 1 perch.

231 cu. in. = 1 gal.

2150.42 cu. in. = 1 stricken bu.

2747.71 cu. in. = 1 heaped bu.

A dry quart is 67.2 cu. in.

A liquid quart is 57.75 cu. in.

Dry Measure

2 pints, pt. = 1 quart, qt.

8 quarts = 1 peck, pk.

4 pecks = 1 bushel, bu.

In each State the weight of a bushel of grain is fixed by statute. The most general weights given are*:

Barley.....48 lbs.

Beans.....60 lbs.

Coal.....80 lbs.

Clover Seed.....60 lbs.

Corn, Shelled.....56 lbs.

Corn, in ear.....70 lbs.

Flax.....56 lbs.

Millet Seed.....50 lbs.

Oats.....32 lbs.

Onions.....57 lbs.

Potatoes.....60 lbs.

Rye.....56 lbs.

Timothy Seed.....45 lbs.

Wheat.....60 lbs.

*These weights may vary slightly in a few states.

Liquid Measure

4 gills, gi. = 1 pint, pt.

2 pints = 1 quart, qt.

4 quarts = 1 gallon, gal.

31 $\frac{1}{2}$ gal. = 1 barrel, bbl.

63 gal. = 1 hogshead, hhg.

NOTE: The size of bbls. and hhgs. often varies. A cu. ft. of distilled water weighs about 62 $\frac{1}{2}$ lbs. and contains about 7 $\frac{1}{2}$ gals. A gallon of water weighs about 8 $\frac{1}{3}$ lbs.

Weights of Metals

A cubic inch of

Platinum weighs 819

Gold weighs..... .696

Lead weighs..... .409

Silver weighs..... .378

Copper weighs..... .317

Brass weighs..... .307

Steel weighs..... .283

Iron weighs..... .281

Tin weighs 263

Aluminum weighs..... .096

Avoirdupois Weight

16 ounces, oz.	=1 pound, lb.
100 pounds	=1 hundredweight, cwt.
20 cwt.	=1 ton, T (2000 lbs.)
22.4 cwt.	=1 long ton (2240 lbs.)

Used in weighing all commodities, except fine metals, precious stones and medicines in powder form.

Troy Weight

(Used in measuring precious stones and fine metals.)

24 grains, gr.	=1 pennyweight, pwt.
20 pwt.	=1 ounce, oz.
12 ounces	=1 pound, lb.

A carat is about 3.2 grains troy. It also means a twenty-fourth part. An 18 carat ring is 18/24 or 18 parts pure gold.

Apothecaries Weight

(For measuring dry medicines)

20 grains	=1 scruple, sc.
3 scruples	=1 dram, dr.
8 drams	=1 ounce, oz.
12 ounces	=1 pound, lb.

Abbreviations used: R = recipe, or take; a or aa = equal quantities; ij = 2, iij = 3, jv = 4, etc., the final i in all Roman notations being changed to j.

Measures of Time

60 seconds	=1 minute, min.
60 minutes	=1 hour, hr.
24 hours	=1 day, da.
7 days	=1 week, wk.
365 days	=1 common year, yr.
366 days	=1 leap year.
100 years	=1 century, C.

For commercial purposes a year is generally considered as 12 months of 30 days each.

From a Helpful Old Rhyme

30 days hath September,
April, June and November,
All other months have 31,
Except February, which has 28,
Till leap year gives it 29.

MEASURES OF VALUE

Money is a medium of exchange and a measure of value. Gold, being the most constant in supply and demand and having more nearly all of the other essentials of a good money, has been made the standard practically the world over. (China, Mexico, Persia and some of the Central American States have a silver standard.)

The essentials of good money are:

1. Constancy in supply.
2. Durability.
3. Beauty.
4. Intrinsic value (commercial value).
5. Convenient denominations (decimal basis).
6. Convenient size.

Value is a relative term, the worth of one thing expressed in units of another; as 4 bu. of wheat are worth 2 days' work. By reducing all values to dollars, they become, in a sense, standardized, and exchange is greatly facilitated.

UNITED STATES MONEY

Sept. 1, 1920, The Federal Reserve Board estimated the general stock of money in the U. S. as follows:	
10 mills = 1 cent, ct.	In the Treasury.....\$ 485,884,277
10 cents = 1 dime	In the Fed. Res. Banks..... 2,031,514,938
10 dimes = 1 dollar, \$	In Circulation..... 5,479,681,605
10 dollars = 1 eagle	Total.....\$7,997,080,820

This represents about \$51 per capita in actual use. The public debt June 30, 1920, was \$24,062,500,285, or about \$200 per capita. The yearly interest per capita on the public debt was about \$9.00.

ENGLISH MONEY

4 farthings	= 1 penny, d.
12 pence	= 1 shilling, s.
20 shillings	= 1 pound, £

FRENCH MONEY

10 millimes	= 1 centime, ct.
10 centimes	= 1 decime, dc.
10 decimes	= 1 franc, fr.

VALUE OF FOREIGN COINS IN UNITED STATES MONEY

Country	Legal Standard	Unit Used	Value in U. S. Money
Argentine Republic.....	Gold	Peso	\$0.9648
Austria-Hungary.....	Gold	Krone	.2026
Belgium.....	Gold, silver	Franc	.1930
Bolivia.....	Gold	Boliviano	.3893
Brazil.....	Gold	Milreis	.5462
British Colonies in Australia and Africa.....	Gold	Pound sterling	4.8665
British Honduras.....	Gold	Dollar	1.0000
Canada.....	Gold	Dollar	1.0000
Chile.....	Gold	Peso	.3650
China.....	Silver	Tael and Dollar	(See note)
Costa Rica.....	Gold	Colon	.4653
Cuba.....	Gold	Peso	1.0000
Denmark.....	Gold	Krone	.2680
Ecuador.....	Gold	Sucre	.4867
Egypt.....	Gold	Lb. (100 piasters)	4.9431
Finland.....	Gold	Markka	.1930
France.....	Gold, silver	Franc	.1930
Germany.....	Gold	Mark	.2382
Great Britain.....	Gold	Pound sterling	4.8665
Greece.....	Gold, silver	Drachma	.1930
Guatemala.....	Silver	Peso	.6864
Haiti.....	Gold	Gourde	.2500
Honduras.....	Silver	Peso	.6864
India (British).....	Gold	Rupee	.3244
Indo-China.....	Silver	Piaster	.7413
Italy.....	Gold, silver	Lira	.1930
Japan.....	Gold	Yen	.4985
Liberia.....	Gold	Dollar	1.0000
Mexico.....	Gold	Peso	.4985
Netherlands.....	Gold	Builder (Florin)	.4020
Newfoundland.....	Gold	Dollar	1.0000
Nicaragua.....	Gold	Cordoba	1.0000
Norway.....	Gold	Krone	.2680
Panama.....	Gold	Balboa	1.0000
Paraguay.....	Gold	Peso (Argentine)	.9648
Persia.....	Gold	Achrefi	.0959
Persia.....	Silver	Kran	.1264
Peru.....	Gold	Libra	4.8665

VALUE OF FOREIGN COINS IN UNITED STATES MONEY—Continued

Country	Legal Standard	Unit Used	Value in U. S. Money
Philippines.....	Gold	Peso	.5000
Portugal.....	Gold	Escudo	1.0805
Roumania.....	Gold	Leu	.1930
Russia.....	Gold	Ruble	.5146
Salvador.....	Gold	Colon	.5000
Santo Domingo.....	Gold	Dollar	1.0000
Serbia.....	Gold	Dinar	.1930
Siam.....	Gold	Tical	.3709
Spain.....	Gold, silver	Peseta	.1930
Straits Settlements.....	Gold	Dollar	.5678
Sweden.....	Gold	Krona	.2680
Switzerland.....	Gold	Franc	.1930
Turkey.....	Gold	Piaster	.0440
Uruguay.....	Gold	Peso	1.0342
Venezuela.....	Gold	Bolivar	.1930

NOTE: There are 18 different values for the tael or dollar, ranging from \$0.7455 for the dollar and from \$1.0278 to \$1.1449 for the tael.

CIRCULAR MEASURE

60 seconds (") = 1 minute (') $\frac{1}{4}$ of a circle is a quadrant
 60 minutes = 1 degree (°) $\frac{1}{6}$ of a circle is a sextant
 360 degrees = 1 circle, cir. $\frac{1}{12}$ of a circle is a sign

A minute, $\frac{1}{60}$ of a degree, or $\frac{1}{21600}$ of the earth's circumference at the equator is called a nautical or geographic mile.

A degree or $\frac{1}{60}$ of the earth's circumference at the equator is equal to 60 geographic miles or $69\frac{1}{8}$ statute miles.

Counting

2 units = 1 pair, pr.
 6 units = 1 set, or half doz.
 12 units = 1 dozen
 12 dozen = 1 gross
 12 gross = 1 great gross
 20 units = 1 score

Paper

24 sheets = 1 quire, qr.
 20 quires = 1 ream, rm.
 2 reams = 1 bundle, bdl.
 5 bundles = 1 bale, bl.

Most paper mills, the western mills especially, are using 500 sheets to the ream instead of 480, as shown above. This places it on practically a decimal basis.

APPENDIX

METRIC MEASUREMENT TABLES

The Metric System is used in practically all of Europe and South America, and almost universally in science and mechanics. Its use in the United States is increasing rapidly.

It has many advantages over the old unrelated tables:

1. It is developed throughout on a decimal basis, making conversion to higher or lower terms very simple.
2. Each term used (there are three principal terms and seven prefixes as compared with our several dozen) indicates its value. See table below.
3. It is consistent and simple in its development.
4. It unifies all measures and makes for greater harmony and co-operation between the nations in matters of commerce and manufacture. During the World War the allied powers found it necessary to use the Metric System to a great extent.

METRIC TERMS EXPLAINED

Prefixes Used	Units Used	
milli. = one-thousandth.....	.001	
centi. = one-hundredth.....	.01	
deci. = one-tenth.....	.1	"meter" for distance
deka. = ten.....	10	"gram" for weights
hecto. = hundred.....	100	"liter" for capacity
Kilo. = thousand.....	1000	
myria. = ten thousand.....	10,000	

The exponent ² is sometimes used to indicate square measure, and the exponent ³ to indicate cubic measure;

$$46 \text{ sq. m.} = 46 \text{ m.}^2$$

and

$$842 \text{ cu. cm.} = 842 \text{ cm.}^3$$

METRIC MEASURES OF AREA

100 sq. millimeters	= 1 sq. centimeter (sq. cm.)	=	.0001 sq. meter
100 sq. centimeters	= 1 sq. decimeter (sq. dm.)	=	.01 sq. meter
100 sq. decimeters	= 1 sq. meter (sq. m.)	=	1 meter = 1 centare
100 sq. meters	= 1 sq. dekameter (sq. Dm.)	=	100 meters = 1 are
100 sq. dekameters	= 1 sq. hektometer (sq. Hm.)	=	10,000 meters = 1 hectare
100 sq. hektometers	= 1 sq. kilometer (sq. Km.)	=	1,000,000 meters

The "centare," "are," and "hectare" are used only in measuring land.

METRIC MEASURES OF VOLUME AND CAPACITY

10 milliliters (ml.)	= 1 centiliter (cl.)	=	.01 liter
10 centiliters	= 1 deciliter (dl.)	=	.1 liter
10 deciliters	= 1 liter (l.)	=	1. liter
10 liters	= 1 dekaliter (Dl.)	=	10. liters
10 dekaliters	= 1 hektoliter (Hl.)	=	100. liters
10 hektoliters	= 1 kiloliter (Kl.)	=	1,000 liters

METRIC MEASURES OF WEIGHTS

10 milligrams (mg.)	= 1 centigram (cg.)	=	.01 gram
10 centigrams	= 1 decigram (dg.)	=	.1 gram
10 decigrams	= 1 gram (g.)	=	1. gram
10 grams	= 1 dekagram (Dg.)	=	10 grams
10 dekagrams	= 1 hektogram (Hg.)	=	100 grams
10 hektograms	= 1 kilogram (Kg.)	=	1000 grams
10 kilograms	= 1 myriagram (Mg.)	=	10000 grams
10 myriagrams	= 1 quintal (Q.)	=	100,000 grams
10 quintals	= 1 tonneau (T.)	=	1,000,000 grams

Weights and Measures of the Metric system with their equivalent in the old.

	Meters	Inches	Feet	Yards	Miles
1 Millimeter	= .001	= 0.0393	=		=
1 Centimeter	= .01	= 0.3937	=		=
1 Decimeter	= .1	= 3.9371	=		=
1 Meter	= 1	= 39.3710	= 3.280	= 1.0936	=
1 Dekameter	= 10	=	= 32.809	= 10.936	=
1 Hektometer	= 100	=	= 328.090	= 109.363	=
1 Kilometer	= 1,000	=	= 3280.899	= 1,093.633	= 0.62138
1 Myriameter	= 10,000	=	=	= 10,936.331	= 6.21380

EQUIVALENT LENGTHS

Old weights and Measures with their equivalents in Metric system.

In.	Feet	Yards	Rods	Miles	Meters
1 =	.0833 =	.02778 =	.005 =	.0000158 =	.0254
12 =	1 =	.333 =	.0606 =	.0001894 =	.3048
36 =	3 =	1 =	.182 =	.000568 =	.9144
198 =	16½ =	5½ =	1 =	.003125 =	5.0291
63360 =	5280 =	1760 =	320 =	1 =	1609.315

EQUIVALENT AREA MEASURES

Square Measure

1 sq. inch =	6.452 sq. cm.	1 sq. cm. =	.155 sq. inches
1 sq. foot =	.0929 sq. m.	1 sq. dm. =	.1076 sq. feet
1 sq. yard =	.8361 sq. m.	1 sq. meter =	1.196 sq. yards
1 sq. rod =	25.293 sq. m.	1 are =	3.954 sq. rods
1 acre =	40.47 ares	1 hektare =	2.471 acres
1 sq. mile =	259 hectares	1 sq. Km. =	.3861 sq. miles

EQUIVALENT VOLUME MEASURES

1 cu. inch =	16.387 cu. cm.	1 cu. cm. =	.061 cu. inches
1 cu. foot =	28.317 cu. dm.	1 cu. dm. =	.0353 cu. feet
1 cu. yard =	.7645 cu. m.	1 cu. m. =	1.308 cu. yards
1 cord =	3.624 steres	1 stere =	.2759 cords

EQUIVALENT WEIGHTS

Avoirdupois

Drams	Oz.	Lbs.	Cwts.	Ton	Grams
1 =	.0625 =	.0039 =	.000035 =	.00000174 =	1.771846
16 =	1 =	.0625 =	.000558 =	.000028 =	28.34954
256 =	16 =	1 =	.00893 =	.000447 =	453.59
7168 =	448 =	28 =	.25 =	.0125 =	12,700
28672 =	1792 =	112 =	1 =	.05 =	50,802
573440 =	35840 =	2240 =	20 =	1 =	1,016,048

Troy

Grains	Pwts.	Oz.	Lb.	Grams
1 =	.0467 =	.00208 =	.0001736 =	.0648
24 = 1	= .05	= .004167 =		1.555
480 = 20	= 1	= .833	=	31.1035
5760 = 240	= 12	= 1	=	373.242

EQUIVALENT WEIGHTS

		Avoir. or				
	Gram	Troy	Avoir. Oz.	Avoir. Lbs.	Cwt.	
1 Milligram =	.001 =	.015 =	=	=	=	
1 Centigram =	.01 =	.154 =	=	=	=	
1 Decigram =	.1 =	1.543 =	=	=	=	
1 Gram =	1 =	15.432 =	.035 =	.0022 =	=	
1 Dekagram =	10 =	154.323 =	.35 =	.022 =	=	
1 Hectogram =	100 =	=	3.527 =	.2204 =	=	
1 Kilogram =	1,000 =	=	35.273 =	2.2046 =	.019	
1 Myriagram =	10,000 =	=	352.749 =	22.0462 =	.1968	

EQUIVALENT CAPACITY MEASURES

Pints	Gallons	Pecks	Bushels	Quarters	Cu. ft.	Liters
1 =	.125 =	.0625 =	.0152 =	.00195 =	.02 =	.5676
8 = 1	= .5	= .125 =	.0156 =	.1604 =	4.543	
16 = 2	= 1	= 25 =	.03125 =	.3208 =	9.082	
64 = 8	= 4	= 1	= .125 =	1.283 =	36.32816	

EQUIVALENT FLUID AND DRY MEASURES

	Liters	Pints	Quarts	Gallons	Bushels
1 Milliliter =	.001 =	=	=	=	=
1 Centiliter =	.01 =	=	=	=	=
1 Deciliter =	.1 =	.176 =	=	=	=
1 Liter =	1 =	1.761 =	.880 =	.22 =	.0275
1 Dekaliter =	10 =	17.610 =	8.80 =	2.2 =	.275
1 Hectoliter =	100 =	176.10 =	88 =	22 =	2.751
1 Kiloliter =	1,000 =	=	880 =	220 =	27.512
1 Myrialiter =	10,000 =	=	8800 =	2200 =	275.121

MACHINES OF AID TO THE BUSINESS MAN IN SIMPLIFYING ARITHMETICAL CALCULATIONS

The general use of machines for various purposes in the modern business office is a sufficient reason for the following illustrations and descriptions of a number of the leading machines now in general use.

The companies manufacturing the machines illustrated and described, were kind enough to co-operate in producing the following pages. If the teacher or pupils are interested in a further study of these machines, the home offices (if a branch is not maintained in your city) will gladly send you more information upon request.

The addresses of the companies follow:

Burroughs Adding Machine Company, Detroit, Michigan.

Felt & Tarrant Mfg. Co. (Comptometer), 1713 N. Paulina St.,
Chicago, Ill.

The Dalton Adding Machine Sales Company, Cincinnati, Ohio.

Elliott-Fisher Company, 342 Madison Ave., New York, N. Y.

The Tabulating Machine Co., 50 Broad St., New York, N. Y.

Lightning Coin Changer Co., 4401 Ravenswood Ave., Chicago, Ill.

The Marchant Calculating Machine Co., Oakland, California.

Monroe Calculating Machine Co., Woolworth Bldg., New York,
City.

Powers Accounting Machine Co., 150 Nassau Street, New
York City.

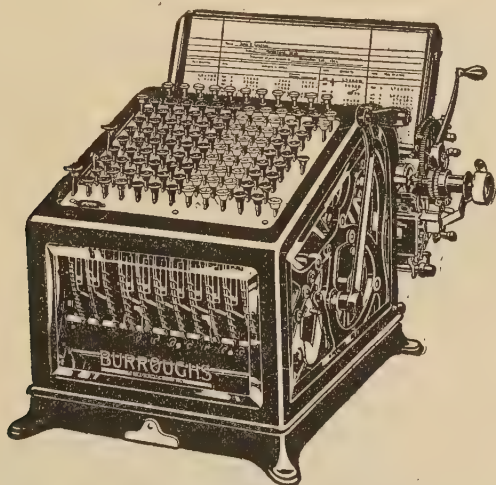
Remington Typewriter Co., Remington Bldg., 374 Broadway,
New York, N. Y.

Underwood Typewriter Co., Underwood Bldg., 30 Vesey St.,
New York, N. Y.

Wales Adding Machine Co., Wilkes Barre, Pennsylvania.

BURROUGHS MACHINES

Burroughs Machines are divided into three groups, namely: Adding, Bookkeeping and Calculating Machines.

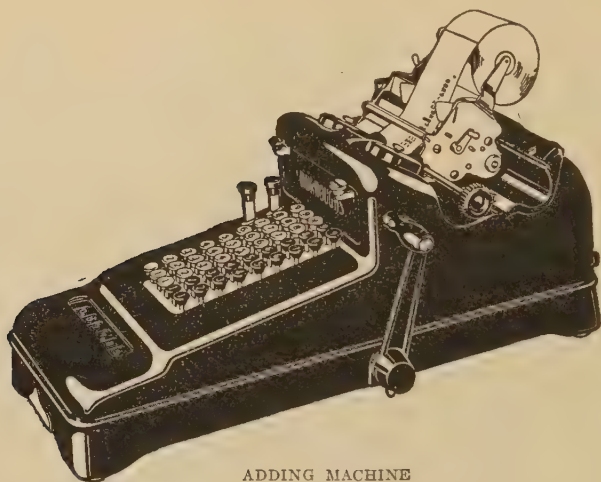


AUTOMATIC BOOKKEEPING MACHINES

Burroughs Automatic Bookkeeping Machines will post customers' ledgers, accounts payable, general ledger accounts and stock ledgers, whether card or loose leaf. These machines print dates and folio numbers automatically and proof of posting that is largely automatic. The Burroughs automatically adds debits, subtracts credits, computes balances, tabulates, indicates closed accounts, and plainly marks with red symbols all subtracted items, new balances and credit items.

ADDING MACHINES

Burroughs Adding Machines are used for any kind of figure work, such as preparing weekly and monthly statements, distributing sales by clerks and departments, compiling tax reports, figuring discounts and deductions, taking inventory and reconciling bank balances, besides handling many other figure jobs found both in the banking and commercial field. Users employ it to verify all their figure work. It saves time, prevents mistakes and pleases customers. (Illustrated at top following page.)

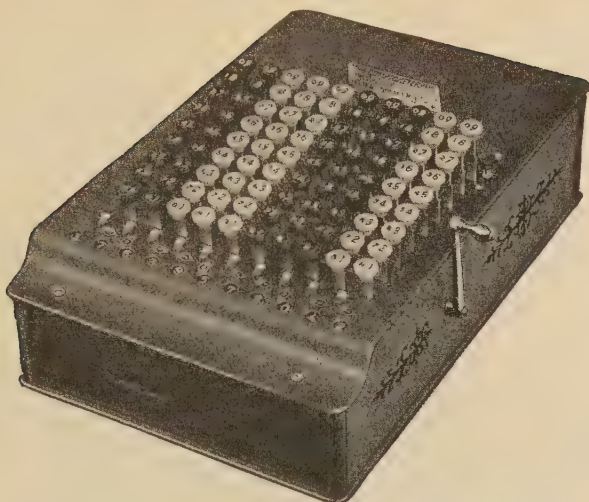


ADDING MACHINE



CALCULATING MACHINE

The Burroughs Calculator performs addition, subtraction, multiplication and division with wonderful ease, speed and accuracy. It is adapted to any kind of business. It is used to figure interest, single and chain discounts and selling costs, analyze accounts, extend and prove invoices and handle all miscellaneous figuring where totals are not required. It affords a faster and more accurate way of handling figure work than the old pen-and-brain method.

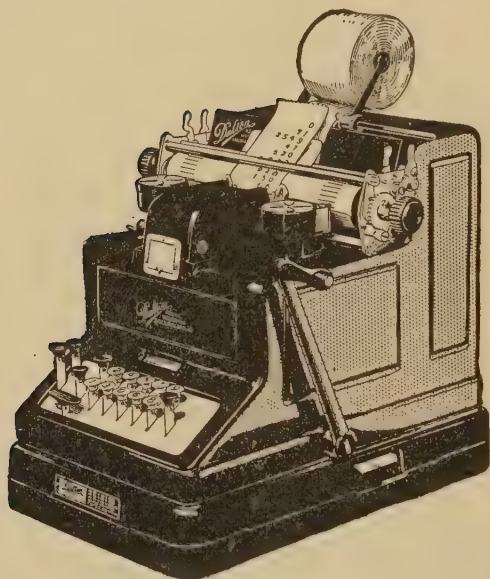


COMPTOMETER

While it is true that any arithmetical machine capable of performing addition can also be made to subtract, multiply, and divide, all such machines are not equally practical on each of these four basic operations. The flexibility needed for rapid all around work is found in machines of the key-driven type, of which the Comptometer is the outstanding example. Operation of this machine and others of the same type is exceedingly simple, since nothing more than depression of the keys is required to perform any operation—the cancelling lever having no other function than that of clearing the register and setting it back to zero.

It may be safely assumed that any machine of recognized merit is mechanically accurate. To guard against the danger of error from a partial or incomplete key-stroke the Comptometer is provided with the controlled-key, a system of mechanism which effectually blocks the possibility of error from this cause. In the event of a partial key-stroke, the keyboard automatically locks up in all columns save the one in which the fault occurs. The operator must then complete the unfinished stroke and touch the release key to unlock the keyboard before proceeding. Another Comptometer safeguard is the Clear Signal which notifies the operator that the register stands clear and ready for a new operation.

The wide field of usefulness of a machine of this type, capable of high speed on both adding and calculating, is indicated by the fact that the major portion of time spent on the figure work of accounting is devoted to adding and multiplying. The published result of an investigation to determine the relative proportion of time so spent, shows it to be approximately, for Addition 53%, Multiplication 41%, Division 4%, Subtraction 2%; or 94% for Addition and Multiplication.



DALTON ADDING MACHINE

The Dalton is more than an adding machine—it is a calculating machine that records every operation. It adds, subtracts, multiplies and divides.

The Dalton multiplies as easily as it adds; it figures pay rolls, interest and chain discounts, checks invoices, takes off trial balances, makes out statements, does all your figure work.

The great simplicity of the Dalton keyboard appeals to both operator and executive. Here is the whole world of figures in only 10 keys. No training required—no confusion in column selection—each figure is placed in its proper column by the machine automatically.

The construction of the Dalton keyboard has a further purpose than that of making it so simple that anyone can operate it immediately. It provides 100% efficiency in operation because this keyboard is scientifically arranged for Touch Method operation—a keyboard easily covered by the fingers of one hand and operated without looking at the keys.

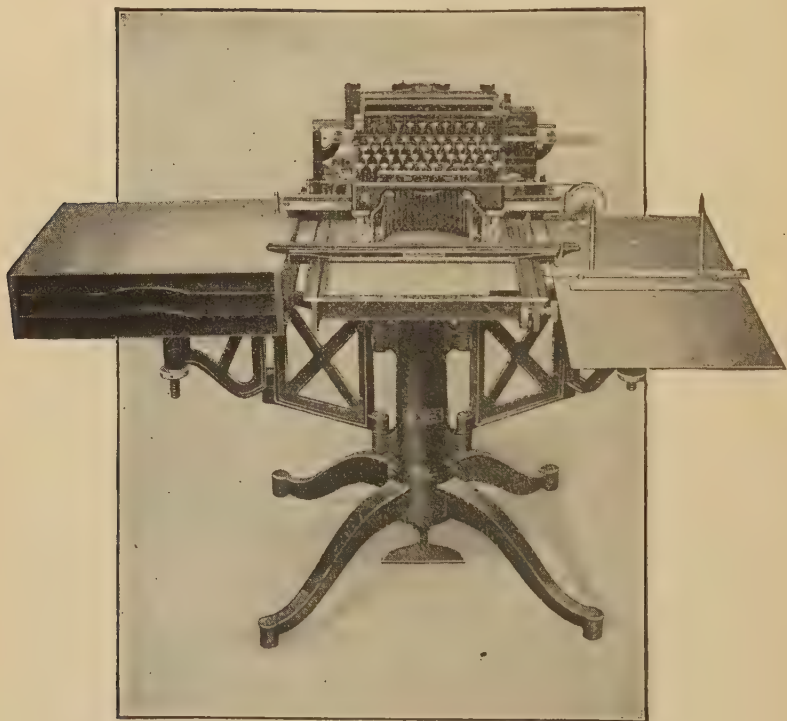


ELLIOTT-FISHER MACHINE NO. 10

Elliott-Fisher Machines, by employing the flat surface writing principle, make possible many new and valuable applications of the accounting and writing machine.

For instance, the Book Recording type, featured above, is particularly well suited for making entries in bound volumes, for the recording of minutes, government or civil records, etc.

Elliott-Fisher Writing Machines are made in various other sizes and capacities; from the simple machine for single copy work to the highly developed type used in conjunction with the most complex compound order and billing systems.



ELLIOTT-FISHER MACHINE NO. 14

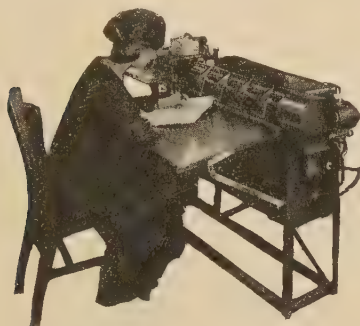
A further development of the Elliott-Fisher Machine—with its flat writing surface—is illustrated above.

The addition of registers, which total vertical columns and also add or subtract across the line, registering line balances, permits its adaptation to all classes of accounting work.

The flat writing surface is equipped with carbon rolls holding forms of different sizes and shapes in perfect register.

In all classes of work—on accounts receivable, accounts payable, sales distribution, stores records, expense distribution, cost records, statements, etc.—the machine will be found an invaluable help to economical office operation.

HOLLERITH ELECTRIC TABULATING MACHINES



TABULATING MACHINE

The Electric Tabulating Machine Method is the process of making analyses accurately, quickly and economically, through the medium of perforated cards and automatic machines.

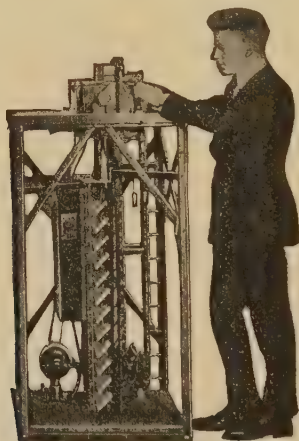


KEY PUNCH

Analyses of all kinds are greatly facilitated with the aid of Tabulating Machines, and many statistics impossible of compilation by manual methods, on account of the length of time required and the consequent expense, are, through the use of these machines, easily and rapidly assembled. This method is employed by many progressive concerns in making their Cost, Labor, Sales and Voucher Distribution analyses, as well as many others peculiar to each business.

The original data is transferred by means of the Key Punch to especially designed cards, which can be arranged to contain practically any data regarding the business.

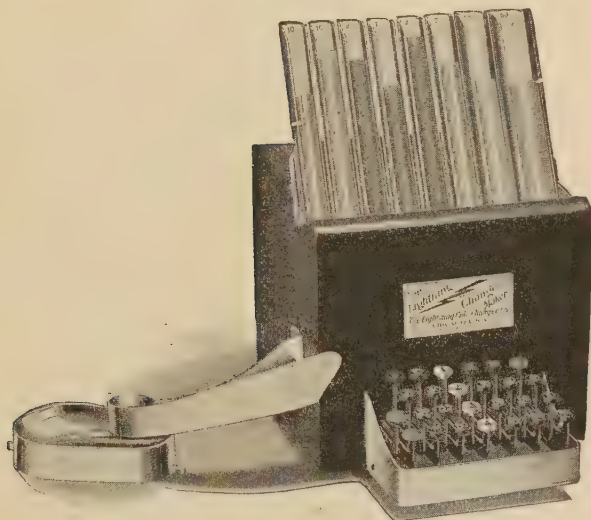
The Sorter automatically groups the cards into predetermined classifications. This wonderful mechanism, with its almost human capacity for selection and distribution, sorts the cards at the rate of 250 per minute.



HOLLERITH ELECTRIC SORTER

The recorded and selected data is then ready for the Tabulator, which is electrically controlled and automatically operated. This machine simultaneously adds from one to five columns of figures, at the rate of from 150 to 750 amounts per minute, according to the size of the Tabulator.

The many uses to which these machines can be put in the compilation of business statistics are almost without limit. The greatest satisfaction, however, comes from securing the desired information in an incredibly short time. In every case where a quick analysis is required, this system of obtaining results has been found so superior to previous manual methods that no comparison can be made.



LIGHTNING COIN CHANGER

The Lightning Changer, illustrated above, is used in every business where a big volume of change is handled. The purpose of this machine is to deliver or pay change accurately, rapidly, and conveniently from both the standpoint of the cashier as well as the customer.

It is used for theatres, banks, restaurants, cafeterias, markets, department stores, large corporations that make up pay rolls, public service corporations, etc. The operation of the Lightning is exceedingly simple and will not require any special instructions. You press the key to correspond with the amount you want to pay back. Instantly the coin is deposited in a convenient cup where the customer can pick it up with one grasp—even with the gloved hand.

Because it is accurate it eliminates mistakes and misunderstandings. It speeds up the handling of small change and eliminates delays.

The machine can be equipped with special spouts or attachments for making up pay rolls, or an attachment can be put on that will make the machine very practical for use in a department store in connection with air line and tube carrier systems.



The New Keyboard Marchant is a calculating machine on which you can add. It multiplies, divides, subtracts, adds, extracts square root and prorates. All of these operations are performed very rapidly, and a visible check is provided in three dials which show all factors of every operation.

Multiplication and division are very simple on the New Keyboard Marchant Calculator for they follow the same order of procedure on the machine as on paper. Addition is as simple as on an adding machine. It is performed with a forward turn of the operating handle, and a backward turn of the operating handle subtracts the number set on the keyboard.

The design of the New Marchant insures minimum mechanical trouble. In all of its details it is so simply constructed that adjustment can be made, parts replaced and new sections added right in the office in a few minutes' time.

Marchant Keyboard Calculators are made in three capacities—9 x 9 x 18, 8 x 8 x 16, and 6 x 6 x 12. The frames are the same size for all machines. The sectional construction feature makes it very simple to increase the capacity of either of the smaller machines, which is an important consideration for small but growing concerns.



THE MONROE CALCULATING MACHINE

Makes all figure-work as easy as turning the crank.

The Monroe, because of its simplicity of operation, clarifies the fundamental principles of arithmetic, and gives the student a practical grasp of the subject, affording a visual and muscular conception of what is done, thus stimulating the student's interest in figures.

Six outstanding features of the Monroe are:

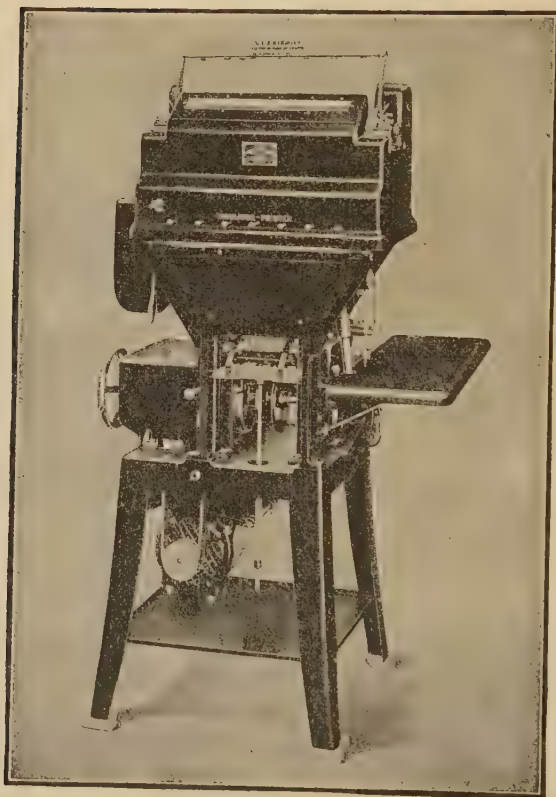
1. Accuracy—Giving a positive check and proof of each operation.
2. Simplicity—The two-way mechanism of the Monroe makes its operation simple and easy to learn. A forward turn of the crank to add or multiply; a reverse turn to subtract or divide. The problem is handled on the Monroe as it would be with pencil and paper—there are no arbitrary rules to learn in the operation of this machine.
3. Speed—Speed and directness are paramount in the operation of the Monroe, because of the two-way operating mechanism.
4. Directness—On the Monroe, "short-cut methods" are used to great advantage, all operations being direct.
5. Flexibility—The Monroe covers the exceedingly broad field of figure-work thoroughly, from simple addition, subtraction, multiplication and division, to and including Engineering Formula—extracting square root and cube root.
6. Durability—The high-grade materials and workmanship entering into the construction of the Monroe insure the machine standing up under the most rigid tests of service.

The Monroe Calculating Machine is supplied in three sizes: a 20-place machine (accommodating 20 digits in the result dial), 16-place, and 12-place.

General Offices of The Monroe Calculating Machine Company are located in the Woolworth Building, New York, N. Y.

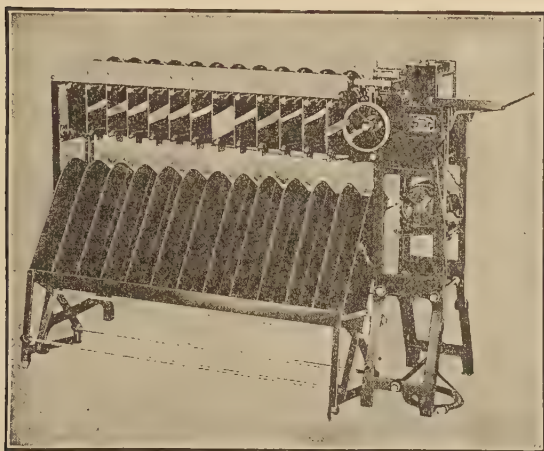
Factory at Orange, N. J.

POWERS ACCOUNTING MACHINES



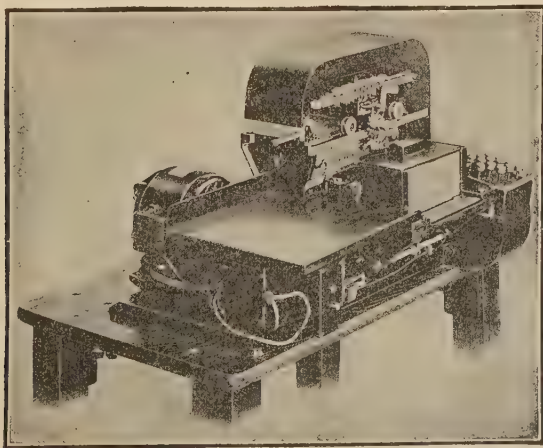
PRINTING TABULATOR

Powers Accounting Machines are tabulating machines using a thin flexible card $7\frac{3}{8}'' \times 3\frac{1}{4}''$ as a basis for their operation. This card can be designed for any class of accounting or statistical work. A battery of machines consist of a Punching Machine, through which medium the cards are perforated, a Sorting Machine, which automatically sorts the cards into desired facts, and the Printing-Tabulator, which mechanically selects and automatically adds and prints these facts on ruled sheets or roll paper. The Sorting and Tabulating operations are entirely controlled by the perforations in the cards.



AUTOMATIC SORTER

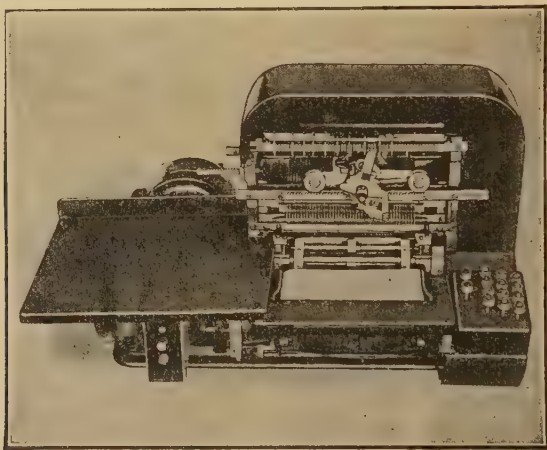
The Automatic Plain Sorter mechanically sorts the cards into desired groups of facts at a rate of 18,000 per hour, and its design affords the greatest ease of operation, and maximum efficiency.



AUTOMATIC KEY PUNCH

The Powers Printing Tabulator automatically prints final reports from the punched cards, and is equipped to add or designate in 7 units of 9 digits each, either printing each card with the totals or prints totals only with group designation, and makes carbon copies if desired.

Powers Machines are used for cost accounting, stock accounting, sales analysis, appropriation and expense distribution, finished payroll, labor and material distribution, and railroad freight accounting.



AUTOMATIC VISIBLE KEY PUNCH

The Automatic Key Punch is used to perforate cards, when the information to be punched in the card is secured from sources other than directly from the tabulating card itself. Blank cards are placed in the magazine in front of the machine. The operator depresses the keys on the keyboard, corresponding to the information desired. When the entire set-up is made, a touch of the release button automatically feeds a card from the magazine into the die punches of the machine, which perforates the card in its entirety; the card, when perforated, is automatically ejected into the receiving box. Repeated facts can be automatically "gang-punched" in any position on the card, in the same operation. Corrections can be made before a card is perforated. Machine is equipped with a carriage escapement, similar to that of a typewriter, with an indicator showing where the set-up is being made, and is also equipped with column skips adjustable to the design of the card used.

The Automatic Visible Key Punch (illustrated above) is used in the punching of visible or "dual" cards, when the perforations are desired in the same card on which the original information is written.



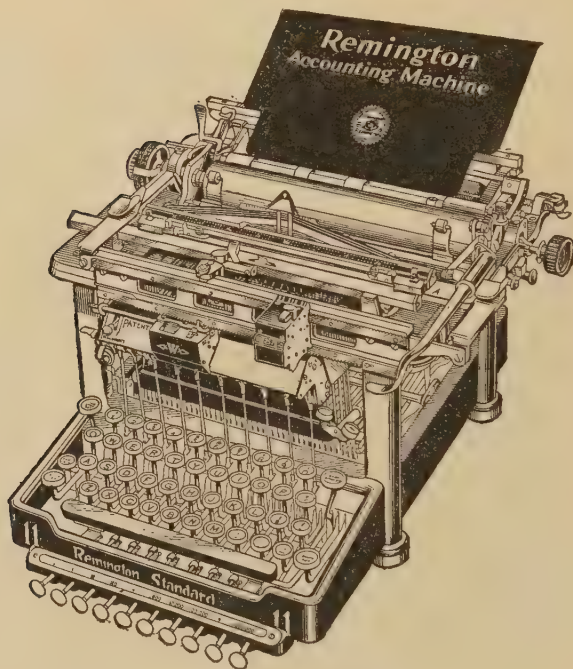
REMINGTON ACCOUNTING MACHINE

The Remington Accounting machine represents the union of the writing machine and the adding machine. Its primary use is for all purposes where typewritten descriptions and added columns appear on the same page. The combined machine is also available for writing alone or adding alone, as occasion requires.

The Adding and Subtracting Mechanism consists of two parts, the Actuating Mechanism and the Totalizer or Recording Mechanism. As many Totalizers may be used on the same machine as there are separate columns of figures to be written and added on the same sheet.

TABULATING REMINGTON

The Model No. 11, or Tabulating Remington, is the standard Remington model for all tabular and statistical forms. It is also equally efficient for all the ordinary forms of typewriting. The range of this machine is, in fact, so universal that it is known as the "complete typewriter." The outstanding feature of this model is the built-in, key-set decimal tabulator. This feature makes the change from one tabular form to another practically instantaneous, and thus makes the same machine available for the widest range of tabular and statistical tasks.

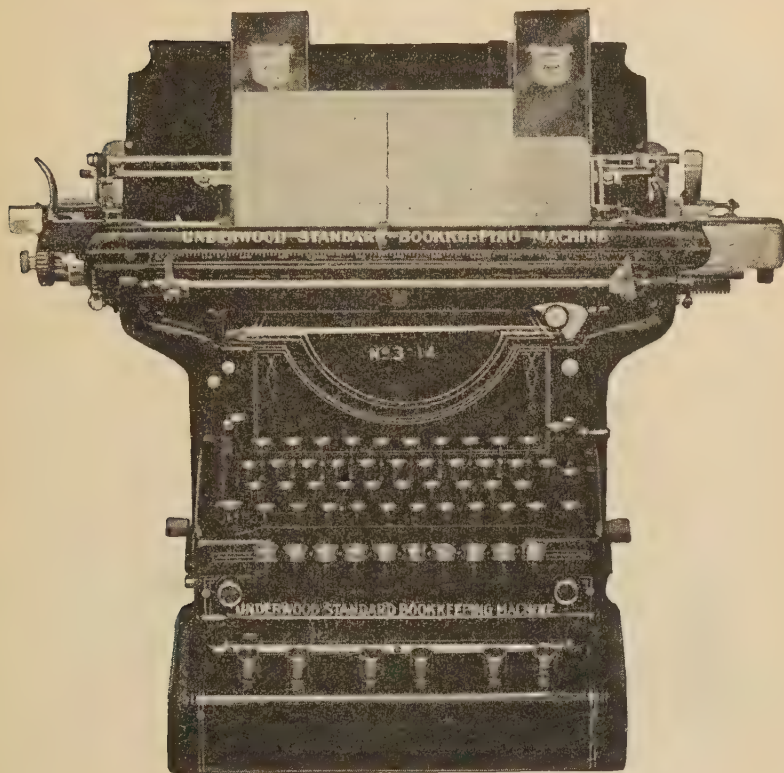


ACCOUNTING MACHINE

There are two variants of this machine; namely, the vertical accounting machine, which adds and subtracts vertical columns only, and the cross accounting machine, which includes, in addition to the mechanism on the regular vertical accounting machines, a cross actuator and totalizer, in which the different operations of the vertical totalizers are recorded.

The Remington Cross Accounting machine will operate in any combination desired to the user. Where some vertical columns are added and others subtracted, the cross totalizer shows the final net total. The machine will add in a vertical, while it subtracts in the cross totalizer, or vice versa. Individual columns may be added vertically and not "cross footed," or cross footed and not added vertically, according to the requirements of the work.

The Remington Accounting machine is in wide and growing use for billing, ledger posting, statement writing and the widest possible range of accounting tasks. This machine, by combining in one the two operations of writing and adding, eliminates absolutely the time and cost of the separate adding operations, and thus performs a most exceptional time saving service.



UNDERWOOD BOOKKEEPING MACHINE

This custodian of office records does your work your way. It simply applies to bookkeeping the scheme of typing records and mechanically calculating accounts. This it does through a minimum number of processes. It fits itself to any business or system.

Model No. 3 has three adding and subtracting registers and is equipped for Ledger Posting, double computed method. One register accumulates the total of the postings and at the end of the "run" this total should agree with the predetermined totals of the Sales Book, Cash Book or Journal, thereby proving that the correct amounts have been properly posted to the various accounts affected. The other two registers compute the new balances on the individual accounts and as each computation is arrived at independently of the other, and both registers at completion must show equal totals, the possibility of arriving at a wrong balance, through the mistake of picking up the previous balance incorrectly, is almost entirely eliminated.

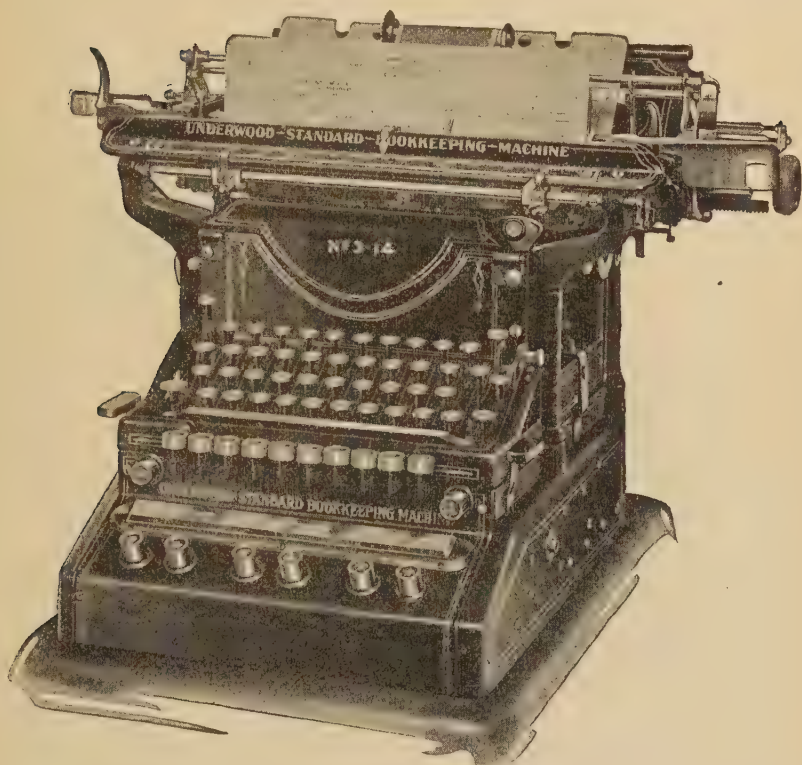


UNDERWOOD BILLING MACHINE

The model illustrated is equipped for condensed billing, the writing of the invoice and sales sheet (as many copies as are required) at one writing and condensing the entries on the sales sheet so that the space taken up on the invoice in the printing of headings may not be lost. One of the registers adds the amounts, subtracts the discounts or any deductions, and arrives at the total of each invoice. The other register accumulates the total of sales.

This model can also be used for unit billing. Any number of copies can be made at one writing. Billing machines are also made to handle Sterling Money, Fractions, Tons and Hundredweight, etc.

On account of the flexibility of the Underwood Billing and Bookkeeping Machines, billing and ledger posting can be handled on the same machine without loss of efficiency during either operation.



UNDERWOOD BOOKKEEPING MACHINE

In years that have passed, bookkeeping methods were never certain. In an office equipped with the Underwood Bookkeeping Machine this uncertainty disappears. The machine carries continuous balances from day to day and there is no time during the month that full information regarding the business cannot be obtained. Each account is balanced mechanically as the merchandise or debits, cash, credits and other entries are posted.

This model has three adding and subtracting registers and is equipped to assemble several forms so that the entries may be made on all at one operation. The forms may vary in length and width and the place where the entry is to appear may be at a different point on each, such as the top of the sheet on one and the bottom of the sheet on another.

For combined ledger and statement posting this machine permits rapid insertion and assembling of forms.

Underwood Bookkeeping Machines are equipped to furnish a Proof Sheet. The operation of the Proof Sheet is entirely automatic, eliminating the extra labor of aligning the ledger with the Proof Sheet.



WALES ADDING MACHINE

Daily uses—Daily proof of posting; Customers ledger; Daily ledger balance; Daily cash balance; Daily recapitulation of sales, cash and charge; Daily recapitulation of sales by kinds of goods; Daily summaries of cash receipts and disbursements; Checking invoices and freight bills; Making deposit slips; Proving daily inventory records; Adding labor and material tickets; Figuring the cost of labor and material to each job and overhead expenses; Figuring cost and selling prices; daily profit.

Weekly uses—Preparing pay rolls; Preparing weekly summaries of labor distribution; Preparing weekly summaries of material distribution; Preparing weekly summaries of sales by sales and departments, territories and commodities, etc.; Handling petty cash expenditures; Summaries of cost and selling prices, expenses and profit for the week.

Monthly uses—Footings ledger accounts; Making or footing customers statements; Taking off trial balance; Reconciling bank balance; Monthly summaries of cash receipts and disbursements; Monthly summaries of

merchandise sales and profits; Compiling labor costs; Compiling material supply costs; Preparing balance sheets.

Yearly uses—Calculating inventory extension; Footing inventory sheets; Compiling profit and loss statement; Preparing balance sheets; Comparative statements by progress made by departments, territories, commodities, etc.

What Wales adding and listing machines do—Every Wales adding and listing machine has two very definite and distinct functions, both performed simultaneously by the same mechanism. It is these two functions that form the foundation upon which the Wales work, however complex, is built.

1. It automatically and without possibility of error computes amounts that have been placed on the keyboard.

2. It neatly and legibly prints each item computed and is ready, at any time, by the mere pulling of the handle, to produce a printed result of the machine's infallibly correct automatic calculation.

In order to secure the greatest possible service from the use of a Wales it is necessary to know how best to adapt the machine to the different kinds of figure work entering into different lines of business. This company will furnish you a book upon request, giving valuable information for the use of the Adding and Listing Machine.

INDEX

	PAGE		PAGE
Abbreviations.....	36	Calculating Machines.....	296
Acceptance.....	237	Cancellation Method in Interest ..	211
Accounting Machines.....	296	Capacity.....	287
Accounts.....	58, 242	Capital.....	69
Account Sales.....	168, 170	Capital Stock.....	259
Accurate Interest.....	203	Cash Account.....	42, 64, 65
Adding Machines.....	298, 299, 300, 316	Cash Book.....	64, 65
Addition.....	14	Casting Out Nines.....	34
Check Figure Proof.....	34	Chain Discounts.....	49
Checking by Nines.....	34	Change Sheet.....	56
Civil Service.....	34	Check Figure Proof.....	34, 35
Group Method.....	19	Circular Graphs.....	175, 176, 185
Horizontal.....	24	Circular Measure.....	291
of Decimals.....	123	Circumference.....	291
of Fractions.....	83, 84	Coal Weight.....	287
Proof of.....	34	Coin Sheet.....	56
Two column.....	18	Coins of U. S.....	289
Adjusting Accounts.....	246	Collateral.....	234
Aliquot Parts.....	47, 48, 103, 145	Collateral Note.....	238
Analysis.....	101, 146, 153	Commercial Draft.....	237
Annexing Ciphers.....	126	Commission.....	159, 168
Annual Interest.....	203, 204, 218	Common Denominator.....	84
Application of Percentage.....	142	Common Fractions.....	77
Approximating Results.....	79, 99, 105	Common Stock.....	263, 268
Area.....	286	Comparison of Numbers.....	11, 98
Arithmetic of Bookkeeping.....	58	Comparison of Weights.....	294, 295
Articles of Copartnership.....	258	Complex Fraction.....	78
Assets.....	59, 69	Compound Interest.....	233
Averages, Modes and Medians....	41	Tables.....	220, 221
Averaging Accounts.....	242	Compound Subtraction.....	203
Avoirdupois Weight.....	288	Consignee.....	169
Axioms for Fractions.....	80	Consignment.....	168
Balance of Account.....	58, 242, 245	Corporations.....	255, 256
Balance Sheet.....	59, 69, 74	Cost by the 100 and 1,000.....	32, 33
Bank.....	234, 238, 240	Coupon Bond.....	270
Bank Discounts.....	240	Credit.....	58
Bankers' Interest.....	203, 204	Credit Rating.....	234
Bankruptcy.....	273	Customer's Acceptance.....	237
Base.....	142	Date of Maturity (notes) 224, 225, 227	
Billing.....	162	Debit.....	58
Board Foot.....	50	Decimals.....	121
Bonds.....	270, 271	Axioms.....	126
Bookkeeping.....	58	How to Use.....	122
Borrowing of Banks.....	234	Mixed.....	125
Borrowing Capital.....	229	Decimal Fractions.....	78, 121
Brokerage.....	268	Denominator.....	77, 121
Business Abbreviations.....	36	Denominate Numbers.....	277
Business Ownership.....	254	Addition.....	278
Business Period.....	59	Compound.....	278, 279

PAGE	PAGE
Division.....	279
Fractional.....	278
Multiplication.....	279
Quiz for Review.....	280
Reduction Ascending.....	277
Reduction Descending.....	278
Subtraction.....	279
Tables of.....	285
Development of Numbers.....	9
Discount.....	160, 236, 237, 240
Dividends.....	263, 264, 268
Divisibility, Tests of.....	85
Division.....	32, 33
Of Decimals.....	124
Of Denominate Numbers.....	279
Of Fractions.....	94, 95
Short Methods.....	33
Draft.....	237, 238
Drawee.....	237
Drawer.....	237
Dry Measure.....	287
English Money.....	289
Equated Date.....	241
Equation.....	100, 101, 102
Equation of Accounts.....	242
Exact Interest.....	203
Facility Drills.....	13
Factor.....	85, 209
Financial Statement.....	235
Fiscal Period.....	59
Focal Date.....	244
Foreign Coins, Value of.....	290
Fractional Equivalents.....	129
Fractions.....	77, 121
Addition.....	84, 89
Common Denominator.....	84
Division.....	94, 95, 96
General Observations.....	77, 78
Improper.....	78, 86
Lowest Terms of.....	79
Mixed.....	78, 87
Multiplication.....	91, 92, 93
Principles Applied.....	98
Proper.....	78
Reduction of.....	79
Short Method.....	86
Subtraction.....	88, 90
Fractional Relations.....	96, 97
Franc.....	289
French Money.....	289
Fundamental Operations.....	13
Graphs.....	183 to 198
Greatest Common Divisor.....	85
Gross Cost.....	168
Gross Sales.....	168
Grouping Figures.....	14, 19, 20
Horizontal Addition.....	24
How to Read Numbers.....	38
Improper Fractions.....	78, 86, 87
Indorsement.....	222, 226, 228, 236, 237
Insolvency and Bankruptcy.....	273
Insolvent.....	273
Interest.....	199
Accurate.....	203
Annual.....	203, 204, 218
Bankers.....	203, 204
Cancellation Method.....	211
Compound.....	203, 204, 219
Kinds of.....	203
Rates.....	202
Reversing the Operations.....	216
Short Methods.....	209
Simple.....	203, 207
60 Day Method.....	209
Tables.....	205, 207, 220, 221
Inventory.....	59, 71
Investment.....	60, 260, 272
Invoices.....	162
Journal.....	62
Land Measure.....	286
Least Common Denominator.....	84
Least Common Multiple.....	85
Ledger.....	59, 66, 67
Legal Rate of Interest.....	202
Liabilities.....	59, 69
Linear Measure.....	285
Liquid Measure.....	287
Loans.....	236
Lumber Measurement.....	50
Maker of Note.....	236
Making Change.....	39
Marking Goods.....	180
Means.....	101
Measures.....	285
Medians.....	41
Merchandise Inventory.....	59, 69, 71
Merchants' rule.....	224
Metric Measure.....	292
Mixed Decimals.....	123
Mixed Numbers.....	78, 87
Modes.....	41
Monetary Units.....	290
Money.....	199, 289
Multiplication.....	26, 31, 35, 43
Of Decimals.....	124
Of Denominate Numbers.....	279
Of Fractions.....	91
Proofs of.....	35

	PAGE		PAGE
Negotiable Paper.....	236, 238	Short Methods.....	43 to 52
Collateral Note.....	238	Signs.....	36
Promissory Note.....	225, 227, 236	Simple Interest.....	203, 207
Time Draft.....	237	Sole Proprietorship.....	254
Net Gain.....	69, 70	Solvent.....	273
Net Proceeds.....	168, 170	Statements.....	59, 69, 70
Nines, Casting Out.....	34	Stock Certificate.....	259, 263
Numerator.....	77	Stockholder.....	259
Origin of Figures.....	9	Stocks and Bonds.....	159, 268, 270
Paper.....	165, 285	Subtraction.....	17, 23
Partial Payments.....	222	By Addition.....	23
By Merchants' Rule.....	224	Of Dates.....	203
By U. S. Rule.....	222	Of Decimals.....	123
Indorsement.....	226, 228	Of Denominate Numbers.....	279
Partners.....	254, 261	Of Fractions.....	88
Partnership.....	254, 255	Supply and Demand.....	199
Partnership Contract.....	258	Surplus.....	259
Partnership Settlement.....	249, 253	Tables	
Par Value.....	263	Aliquot Parts.....	103, 145
Pay Rolls.....	53, 54	Apothecaries' Weight.....	288
Payee.....	227, 236	Avoirdupois Weight.....	288
Percentage.....	141	Circular Measure.....	291
Percentage and Its Applications		Compound Interest.....	220, 221
Compared.....	158	Cubic Measure.....	287
Preferred Stock.....	263, 268	Dry Measure.....	287
Premium.....	263	English Money.....	289
Present Worth.....	59, 69, 74, 230	Fractional Equivalent.....	129
Prime Cost.....	168	French Money.....	289
Principal.....	202, 206, 208	Legal Rates.....	202
Problems of a Merchant.....	171	Linear Measure.....	285
Proceeds of a Note.....	240	Metric.....	292
Of a Sale.....	170	Monetary Standards.....	290
Profit and Loss.....	58, 70, 158	Of Time Differences.....	205
Profit and Loss Statement.....	59, 73, 157	Review.....	280
Profit on Sales.....	173 to 179	Simple Interest.....	207
Promissory Notes.....	225, 227, 236	Square Measure.....	286
Proper Fractions.....	78	Time.....	288
Proportion.....	101	Troy Weight.....	288
Proprietorship.....	59	Weights of Grain, etc.....	287
Purchase Book.....	62	Term of Discount.....	160
Rate of Interest.....	200, 202, 216	Time Card.....	53
Reading Numbers.....	38	Time Clock.....	52
Receipted Bill.....	162	Time Discount.....	160
Receiver.....	273	Time Drafts.....	237, 238
Reduction of Fractions.....	79	Township.....	286
Relation of Numbers.....	98	Trade Acceptance.....	238
Resources.....	59, 69	Trade Discount.....	158, 160, 161, 231
Reversing the Operations.....	216	Trial Balance.....	59, 68, 74
Review Questions,		Troy Weight.....	288
76, 127, 182, 239, 248, 276		Turnover.....	171, 172
Sales Book.....	63	Two Figure Combinations.....	14
Scale.....	184	United States Rule.....	222
Section of Land.....	286	Volume.....	287
Shipment.....	168	Worth.....	69

Cost \$10 - gain 25% Selling Price?

$$\begin{array}{r} 10.00 \text{ Base} \\ + .25 \text{ Rate} \\ \hline 10.25 \text{ Percentage} \end{array} \quad B \times R = \text{Percentage}$$

Gain on article 2.50; cost \$10. Rate of gain?

$$10 \overline{) 2.50} \quad P \div B = R$$

Gain 2.50 Rate 25%; cost?

$$.25 \overline{) 2.50} \quad P \div R = B$$

Sold for 12.50, gaining 25%; Cost?

$$\text{Amt} \div (1 + R) = \text{Base}$$

$$\begin{array}{r} 100 - \\ .25 \\ \hline 1.25 \overline{) 12.50} \\ \hline 10 \end{array}$$

Sold 7.50; Lost 25%; cost?

$$\text{Diff} \div (1 - R) = B$$

$$\begin{array}{r} 100 - \\ .25 \\ \hline .75 \overline{) 7.50} \\ \hline 10 \end{array}$$

Cost \$10, Sold at gain of 25%; Selling Price?

$$\begin{array}{r} 100 - \\ .25 \\ \hline 1.25 \end{array} \quad \begin{array}{r} 1.25 \\ \hline 12.50 \end{array} \quad B \times (1 + R) = A$$

Sold article at loss of 25%; cost \$10; Selling Price?

$$\begin{array}{r} 100 - \\ .25 \\ \hline .75 \end{array} \quad \begin{array}{r} .75 \\ \hline 7.50 \end{array} \quad B \times (1 - R) = \text{Diff}$$



T5-CWS-776